Consensus of Multiagent Systems and Synchronization of Complex Networks: A Unified Viewpoint

Zhongkui Li, Zhisheng Duan, Guanrong Chen, *Fellow, IEEE*, and Lin Huang

***Abstract*—This paper addresses the consensus problem of multiagent systems with a time-invariant communication topology consisting of general linear node dynamics. A distributed observer-type consensus protocol based on relative output measurements is proposed. A new framework is introduced to address in a unified way the consensus of multiagent systems and the synchronization of complex networks. Under this framework, the consensus of multiagent systems with a communication topology having a spanning tree can be cast into the stability of a set of matrices of the same low dimension. The notion of consensus region is then introduced and analyzed. It is shown that there exists an observer-type protocol solving the consensus problem and meanwhile yielding an unbounded consensus region if and only if each agent is both stabilizable and detectable. A multistep consensus protocol design procedure is further presented. The consensus with respect to a time-varying state and the robustness of the consensus protocol to external disturbances are finally discussed. The effectiveness of the theoretical results is demonstrated through numerical simulations, with an application to low-Earth-orbit satellite formation flying.**

***Index Terms*—Consensus, graph theory, linear matrix inequality (LMI), multiagent system, synchronization, H_∞ control.**

I. Introduction

IN RECENT years, the coordination problem of multiagent systems has received compelling attention from various scientific communities due to its broad applications in such areas as satellite formation flying [4], cooperative unmanned air vehicles[2], scheduling of automated highway systems[3], and air traffic control[38]. One critical issue arising from multiagent systems is to develop distributed control policies based on local information that enables all agents to reach an agreement on certain quantities of interest, which is known as the consensus problem.

Consensus problem has a long-standing tradition in computer science. In the context of multiagent systems, recent years have witnessed dramatic advances of various distributed strategies that achieve agreements. Reference[43] proposed a simple model for phase transition of a group of self-driven particles and numerically depicted the complexity of the model. Reference[15] provided a theoretical explanation for the behavior observed in [43] by using graph theory. In [23], a general framework of the consensus problem for networks of dynamic agents with fixed or switching topologies and communication time delays was established. The conditions derived in[23] were further relaxed in[29]. Reference[1] proposed a passivity-based design framework to deal with the group coordination problem, where both fixed and time-varying communication structures were considered. References[12] and [13] considered tracking control for multiagent consensus with an active leader and designed a local controller together with a neighbor-based state-estimation rule. Predictive mechanisms were introduced in [44] to achieve ultrafast consensus. Reference[19] investigated the consensus problem for directed networks of agents with external disturbances and model uncertainties on fixed and switching topologies. Reference[7] characterized a distributed algorithm that asymptotically achieved consensus, and provided two discontinuous distributed algorithms that achieve max and min consensus, respectively, in finite time. For a relatively complete coverage of the literature on consensus, readers are referred to the recent surveys [24], [30]. One well-known problem in most existing works is that the agent dynamics are often restricted to be single or double integrators or structural high-order linear systems. Another common problem is that most proposed distributed consensus protocols are based on the relative states between neighboring agents, which, in many cases, is not available.

Another topic that is closely related to the consensus of multiagent systems is the synchronization of coupled nonlinear oscillators. In the pioneering work [26], the synchronization phenomenon of two master–slave chaotic systems was observed and applied to secure communications. References [22] and [27] addressed the synchronization stability of a network of oscillators by using the master stability function method. Recently, the synchronization of complex dynamical networks, such as small-world and scale-free networks, has been widely studied (see[5], [9],[17],[21], [28], [40], [42] and the references therein). Due to nonlinear node dynamics, usually, only sufficient conditions can be given for verifying the synchronization.

This paper is concerned with the consensus problem of multiagent systems under a fixed (time-invariant) communication topology, where each agent has general linear dynamics, which may also be considered as the linearized model of a nonlinear

Manuscript received October 30, 2008; revised March 05, 2009. First published June 02, 2009; current version published January 27, 2010. This work was supported in part by the National Natural Science Foundation of China under Grant 60974078,Grant 10832006 and Grant 60674093 and in part by the Key Projects of Educational Ministry under Grant 107110. This paper was recommended by Associate Editor M. Di Marco.

Z. Li, Z Duan, and L. Huang are with the State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing 100871, China (e-mail: zhongkli@pku.edu.cn; duanzs@pku.edu.cn).

G. Chen is with the State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing 100871, China, and also with the Centre for Chaos and Complex Networks and the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong.

Digital Object Identifier 10.1109/TCSI.2009.2023937

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network. A distributed observer-type consensus protocol based on relative output measurements is proposed, which can be regarded as an extension of the traditional observer-based controller for a single system. A new unified framework is introduced, which converts the consensus problem of multiagent systems with a communication topology having a spanning tree into the stability of a set of matrices with the same low dimension (as a single agent). The proposed framework is, in essence, consistent with the master stability function method used in the synchronization of complex networks and yet presents a unified viewpoint to both the consensus of multiagent systems and the synchronization of complex networks. The notion of consensus region is then introduced and analyzed with the help of the stability of matrix pencils. It is shown that there exists an observer-type protocol solving the consensus problem and meanwhile yielding an unbounded consensus region if and only if each agent is stabilizable and detectable. A constructive three-step consensus protocol design procedure is further proposed. The first two steps deal with only the agent dynamics and feedback gain matrices of the consensus protocol, while in the third step, the effect imposed by the communication topology of the agents on the consensus stability is managed by adjusting the coupling strength. One favorable feature of this approach is that the protocol designed to achieve consensus for one given communication topology can be directly used to solve the consensus problem for any other topology containing a spanning tree, where the only task is to select a suitable coupling strength.

Roughly speaking, there are two fundamental questions about the consensus problem of multiagent systems: "how to reach consensus" and "consensus on what." They are both addressed in this paper. The beginning part of this paper is mainly devoted to the first question. To that end, the second question, the consensus with respect to a time-varying state, is brought forward and discussed under the proposed framework in the last part of this paper. The robustness of consensus protocols to external disturbances will also be studied, where the communication topology is restricted to be undirected. As mentioned earlier, this paper intends to relate the consensus of multiagent systems and the synchronization of complex networks. Pertinent works along this line include [39] and [37], where synchronization problems of identical discrete- and continuous-time linear systems are studied. Differing from the protocols proposed in this paper, the coupling law used in [39] is essentially relative state feedback, and the dynamic coupling law in[37] requires the absolute output measurement of each agent, which is impractical in many cases. A typical instance is deep-space formation flying, where the measurement of the absolute position of the spacecraft is accurate only on the order of kilometers, which is thus useless for control purposes [35]. Other kinds of dynamic protocols have been proposed in different scenarios, e.g., in[10], [36]. To briefly highlight the main differences, this paper generalizes the existing results on consensus of multiagent systems in at least two aspects. First, the agent dynamics are extended to be in a general linear form, without limiting to integrators. The agent dynamics are excluded from having poles in the open right-half plane in order to reach a nontrivial consensus value. Second, observer-type consensus protocols based on relative output measurements between neighboring agents are proposed, which is more general than most existing models such as the protocol based on relative states and the static protocol used in [39]. For completeness, relative-state consensus protocols are also considered here as a special case of the present unified framework.

The rest of this paper is organized as follows. Section II introduces some basic notation and reviews some useful results of algebraic graph theory.Section III presents a new framework to tackle the consensus problem of multiagent systems and investigates possible applications of consensus algorithms to satellite formation flying in the low Earth orbit. The consensus problem with respect to a reference state is considered in Section IV. The robustness of the consensus protocol to external disturbances is formulated and analyzed in Section V.Section VI concludes this paper.

A. Notation and Preliminaries

Let $\mathbf{R}^{n\times n}$ and $\mathbf{C}^{n\times n}$ be the sets of $n\times n$ real and complex matrices, respectively. The superscript T means transpose for real matrices, and $*$ means conjugate transpose for complex matrices. I_N represents the identity matrix of dimension N, and I denotes the identity matrix of an appropriate dimension. Let $\mathbf{1}\in\mathbf{R}^p$ denote the vector with all entries equal to one. Matrices, if not explicitly stated, are assumed to have compatible dimensions. For $\zeta\in\mathbf{C}$, denote by $\mathrm{Re}(\zeta)$ its real part and by $\mathrm{Im}(\zeta)$ its imaginary part.$\mathrm{diag}(A_1,\ldots,A_n)$ represents a block-diagonal matrix with matrices A_i, $i=1,\ldots,n$, on its diagonal. The matrix inequality $A>B$ means that A and B are square Hermitian matrices and that $A-B$ is positive definite. For $A\in\mathbf{C}^{n\times n}$, $\bar{\sigma}(A)$ denotes its maximal singular value. A matrix $H\in\mathbf{C}^{n\times n}$ is Hurwitz (or stable) if all of its eigenvalues have strictly negative real parts. A matrix G is irreducible if there does not exist a permutation matrix P such that PGP^{T} is block triangular. The Kronecker product of matrices $A\in\mathbf{R}^{m\times n}$ and $B\in\mathbf{R}^{p\times q}$ is defined as

$$A\otimes B=\begin{bmatrix} a_{11}B & \cdots & a_{1n}B\\ \vdots & \ddots & \vdots\\ a_{m1}B & \cdots & a_{mn}B\end{bmatrix}$$

which satisfies the following properties:

$$\begin{aligned}(A\otimes B)(C\otimes D)&=(AC)\otimes(BD)\\ (A\otimes B)^{\mathrm{T}}&=A^{\mathrm{T}}\otimes B^{\mathrm{T}}\\ A\otimes B+A\otimes C&=A\otimes(B+C).\end{aligned}$$

A directed graph $\mathcal{G}$ consists of a node set $\mathcal{V}$ and an edge set $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$, in which an edge is represented by a pair of distinct nodes of $\mathcal{G}$:$(i,j)\in\mathcal{E}$, where i is the parent node, j is the child node, and j is neighboring to i. A graph with the property that $(i,j)\in\mathcal{E}$ implies that $(j,i)\in\mathcal{E}$ is said to be undirected. A path on $\mathcal{G}$ from nodes i_1 to i_l is a sequence of ordered edges in the form of (i_k,i_{k+1}),$k=1,\ldots,l-1$. A directed graph is said to be strongly connected if, for any pair of distinct nodes, there exists a path between them. A directed graph has or contains a directed spanning tree if there exists a node called root such that there exists a directed path from this node to every other node.

Suppose that there are m nodes in a graph. The adjacency matrix $A=(a_{ij})\in\mathbf{R}^{m\times m}$ is defined by $a_{ii}=0$, $a_{ij}=1$ if $(j,i)\in\mathcal{E}$ and 0 otherwise. Thus, $i,j,\ldots,$ represent both

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nodes and indices, which should not cause confusion from the context. The Laplacian matrix $\mathcal{L} \in \mathbf{R}^{m\times m}$ is defined as $\mathcal{L}_{ii} = \sum_{j\neq i} a_{ij}, \mathcal{L}_{ij} = -a_{ij}$ for $i \neq j$. Clearly, matrix $\mathcal{L}$ is symmetric if the graph is undirected.

Lemma 1: All the eigenvalues of $\mathcal{L}$ have nonnegative real parts. Zero is an eigenvalue of $\mathcal{L}$, with $\mathbf{1}$ as the corresponding right eigenvector [34].

Lemma 2: Zero is a simple eigenvalue of $\mathcal{L}$ if and only if graph $\mathcal{G}$ has a directed spanning tree [18], [29].

Lemma 3: If graph $\mathcal{G}$ contains a directed spanning tree, then, with proper permutation,$\mathcal{L}$ can be reduced to the Frobenius normal form [41]

$$\mathcal{L} = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1k} \\ 0 & L_{22} & \cdots & L_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{kk} \end{bmatrix}$$

where L_{ii}, $i = 1, \ldots, k-1$, are irreducible, each L_{ii} has at least one row with positive row sum, and L_{kk} is irreducible or is a zero matrix of dimension one.

II. Unified Approach to the Consensus Problem

Consider a group of N identical agents with general linear dynamics, which may be regarded as the linearized model of some nonlinear systems. The dynamics of the ith agent are described by

$$\dot{x}_i = Ax_i + Bu_i; \qquad y_i = Cx_i \tag{1}$$

where $x_i = [x_{i,1}, \ldots, x_{i,n}]^{\mathrm{T}} \in \mathbf{R}^n$ is the state,$u_i \in \mathbf{R}^p$ is the control input, and $y_i \in \mathbf{R}^q$ is the measured output. It is assumed that (A, B, C) is stabilizable and detectable.

The communication topology among agents is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of nodes (i.e., agents) and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges. An edge (i, j) in graph $\mathcal{G}$ means that agent j can obtain information from agent i but not conversely.

The relative measurements of other agents with respect to agent i are synthesized into a single signal in the following way:

$$\zeta_i = c\sum_{j=1}^{N} a_{ij}(y_i - y_j) \tag{2}$$

where $c > 0$ denotes the coupling strength,$a_{ii} = 0$, and$a_{ij} = 1$ if agent i can obtain information from agent j but 0 otherwise. An observer-type consensus protocol is proposed as

$$\begin{aligned} \dot{v}_i &= (A+BK)v_i + F\left(c\sum_{j=1}^{N} a_{ij}C(v_i - v_j) - \zeta_i\right) \\ u_i &= Kv_i \end{aligned} \tag{3}$$

where $v_i \in \mathbf{R}^{\mathbf{n}}$ is the protocol state, $i = 1, \ldots, N$, and $F \in \mathbf{R}^{\mathbf{q}\times\mathbf{n}}$ and $K \in \mathbf{R}^{\mathbf{p}\times\mathbf{n}}$ are the feedback gain matrices to be determined. The term $\sum_{j=1}^{N} a_{ij}C(v_i - v_j)$ in (3) denotes the information exchanges between the protocol of agent i and those of its neighbors. Note that protocol (3) is distributed, since it is based only on the relative information of neighboring agents.

With (3), system (1) can be written as

$$\dot{\xi}_i = \mathcal{A}\xi_i + c\sum_{j=1}^{N} \mathcal{L}_{ij}\mathcal{H}\xi_j, \qquad i = 1, \ldots, N \tag{4}$$

where

$$\xi_i = \begin{bmatrix} x_i \\ v_i \end{bmatrix}; \quad \mathcal{A} = \begin{bmatrix} A & BK \\ 0 & A+BK \end{bmatrix}; \quad \mathcal{H} = \begin{bmatrix} 0 & 0 \\ -FC & FC \end{bmatrix}$$

$\mathcal{L} \in \mathbf{R}^{N\times N}$ is the Laplacian matrix of $\mathcal{G}$, $\dot{\xi}_i = \mathcal{A}\xi_i$ is the augmented node dynamics, and $\mathcal{H}$ denotes the inner linking matrix. System (4) may be viewed as the linearized model of a complex nonlinear network.

One says that (3) solves the consensus problem if the states of system (4) satisfy

$$\xi_i(t) \to \xi_j(t) \qquad \forall\, i, j = 1, 2, \ldots, N, \quad \text{as} \quad t \to \infty. \tag{5}$$

Let $r^{\mathrm{T}} = [r_1, \ldots, r_N] \in \mathbf{R}^{1\times N}$ be the left eigenvector of $\mathcal{L}$ associated with zero eigenvalue, satisfying $r^{\mathrm{T}}\mathbf{1} = 1$. Introduce the following variable:

$$\delta(t) = \xi(t) - \left((\mathbf{1}r^{\mathrm{T}}) \otimes I_{2n}\right)\xi(t) \tag{6}$$

where $\xi = [\xi_1^{\mathrm{T}}, \ldots, \xi_N^{\mathrm{T}}]^{\mathrm{T}}$, and $\delta \in \mathbf{R}^{2Nn\times 2Nn}$ satisfies $(r^{\mathrm{T}} \otimes I_{2n})\delta = 0$. Similar to[23], δ is referred to as the disagreement vector. It can be verified that δ evolves according to the following so-called disagreement dynamics:

$$\dot{\delta} = (I_N \otimes \mathcal{A} + c\mathcal{L} \otimes \mathcal{H})\delta. \tag{7}$$

The following presents a necessary and sufficient condition for the consensus problem under dynamic protocol (3).

Theorem 1: For a directed network of agents with communication topology $\mathcal{G}$ that has a directed spanning tree, protocol (3) solves the consensus problem if and only if all matrices $A+BK$, $A+c\lambda_i FC$,$i = 2, \ldots, N$, are Hurwitz, where $\lambda_i, i = 2, \ldots, N$, are the nonzero eigenvalues of the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$.

Proof: First, it is to show that the consensus problem of network (4) is equivalent to the asymptotical stability problem of the disagreement dynamics (7). Rewrite (6) as

$$\delta = (\widehat{M} \otimes I_{2n})\xi \tag{8}$$

where

$$\widehat{M} = I_N - \mathbf{1}r^{\mathrm{T}} = \begin{bmatrix} 1-r_1 & -r_2 & \cdots & -r_N \\ -r_1 & 1-r_2 & \cdots & -r_N \\ \vdots & \vdots & \ddots & \vdots \\ -r_1 & -r_2 & \cdots & 1-r_N \end{bmatrix}.$$

By the definition of r, it is easy to see that zero is a simple eigenvalue of $\widehat{M}$, with $\mathbf{1}$ as the corresponding right eigenvector, and one is another eigenvalue with multiplicity $N-1$. Then, it follows from (8) that $\delta = 0$ if and only if $\xi_1 = \cdots = \xi_N$. That is, the consensus problem is solved if and only if $\delta(t) \to 0$, as $t \to \infty$.

Next, the stability of system (7) is discussed, which will solve the consensus problem indirectly. Let $Y \in$

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$\mathbf{R}^{N\times(N-1)}$,$W \in \mathbf{R}^{(N-1)\times N}$, $T \in \mathbf{R}^{N\times N}$, and upper triangular $\Delta \in \mathbf{R}^{(N-1)\times(N-1)}$ be such that

$$T = [\mathbf{1} \quad Y] \quad T^{-1} = \begin{bmatrix} r^{\mathrm{T}} \\ W \end{bmatrix} \quad T^{-1}\mathcal{L}T = J = \begin{bmatrix} 0 & 0 \\ 0 & \Delta \end{bmatrix} \tag{9}$$

where the diagonal entries of Δ are the nonzero eigenvalues of $\mathcal{L}$. Introduce the state transformation $\varepsilon = (T^{-1} \otimes I_{2n})\delta$, with $\varepsilon = [\varepsilon_1^{\mathrm{T}}, \ldots, \varepsilon_N^{\mathrm{T}}]^{\mathrm{T}}$. Then, (7) can be represented in terms of ε as follows:

$$\dot{\varepsilon} = (I_N \otimes \mathcal{A} + cJ \otimes \mathcal{H})\varepsilon. \tag{10}$$

On the other hand, it can be seen from (6) that

$$\varepsilon_1 = (r^{\mathrm{T}} \otimes I_{2n})\delta \equiv 0. \tag{11}$$

Note that the elements of the state matrix of (10) are either block diagonal or block upper triangular. Hence,ε_i, $i = 2, \ldots, N$, converge asymptotically to zero if and only if the $N-1$ subsystems along the diagonal, i.e.,

$$\dot{\varepsilon}_i = (\mathcal{A} + c\lambda_i\mathcal{H})\varepsilon_i, \qquad i = 2, \ldots, N \tag{12}$$

are asymptotically stable. It is easy to check that matrices $\mathcal{A} + c\lambda_i\mathcal{H}$ are similar to

$$\begin{bmatrix} A + c\lambda_i FC & 0 \\ -c\lambda_i FC & A + BK \end{bmatrix}, \qquad i = 2, \ldots, N.$$

Therefore, the stability of matrices $A + BK$, $A + c\lambda_i FC$,$i = 2, \ldots, N$, is equivalent to the case in which the state δ of (7) converges asymptotically to zero, i.e., the consensus problem is solved. ■

Remark 1: The importance of this theorem lies in the fact that it converts the consensus problem of a large-scale multiagent network under the observer-type protocol (3) into the stability of a set of matrices with the same dimension as a single agent, thereby significantly reducing the computational complexity. The observer-type consensus protocol (3) can be seen as an extension of the traditional observer-based controller for a single system to the one for multiagent systems. The separation principle of traditional observer-based controllers still holds in the multiagent setting. Communication topology $\mathcal{G}$ is directed and only assumed to have a directed spanning tree. Such an assumption is quite general and weak, as it is intuitively clear that consensus is impossible to reach if $\mathcal{G}$ has disconnected components. The effects of the communication topology on the consensus problem are characterized by the eigenvalues of the corresponding Laplacian matrix $\mathcal{L}$, which may be complex, rendering the matrices complex-valued in Theorem 1.

Remark 2: Theorem 1 generalizes the existing results on the consensus problem in at least two aspects. First, the agent dynamics are extended to be general linear but not limited to single-integrator, double-integrator, or structural high-order linear systems, as usually assumed in most existing papers [23], [29], [31]. Here, once again, the linear dynamics can be considered as the linearized dynamics of some originally nonlinear systems. Second, an observer-type consensus protocol is proposed, which is based on relative output measurements between neighboring agents, in contrast to [39] where a full-state coupling law is used and to [37] where the dynamic protocol requires the absolute output of each agent to be available. Compared to that of existing consensus protocols, a unique feature of the consensus protocol (3) is that a positive scalar called the coupling strength is introduced, similar to the complex network models studied in [8], [27], and[40]. With this parameter, the notion of consensus region can be brought forward, as detailed in the following section. It is also worth noticing that the method leading to Theorem 1 is, in essence, consistent with the master stability function method proposed in [22] and[27]. Therefore, one can deal with the consensus problem of multiagent systems and the synchronization problem of complex dynamic networks in a unified way.

Theorem 2: Consider the multiagent network (4) whose communication topology $\mathcal{G}$ has a directed spanning tree. If protocol (3) satisfies Theorem 1, then

$$x_i(t) \to \varpi(t) \triangleq (r^{\mathrm{T}} \otimes e^{At}) \begin{bmatrix} x_1(0) \\ \vdots \\ x_N(0) \end{bmatrix}$$
$$v_i(t) \to 0, \qquad i = 1, 2, \ldots, N, \quad \text{as} \quad t \to \infty \tag{13}$$

where $r \in \mathbf{R}^N$ is such that $r^{\mathrm{T}}\mathcal{L} = 0$ and $r^{\mathrm{T}}\mathbf{1} = 1$.

Proof: From (4), the network dynamics can be rewritten in compact form as

$$\dot{\xi} = (I_N \otimes \mathcal{A} + c\mathcal{L} \otimes \mathcal{H})\xi \tag{14}$$

where $\xi = [\xi_1^{\mathrm{T}}, \ldots, \xi_N^{\mathrm{T}}]^{\mathrm{T}}$. The solution of (14) can be obtained as

$$\begin{aligned} \xi(t) &= \mathrm{e}^{(I_N \otimes \mathcal{A} + c\mathcal{L} \otimes \mathcal{H})t}\xi(0) \\ &= (T \otimes I_{2n})\mathrm{e}^{(I_N \otimes \mathcal{A} + cJ \otimes \mathcal{H})t}(T^{-1} \otimes I_{2n})\xi(0) \\ &= (T \otimes I_{2n}) \begin{bmatrix} \mathrm{e}^{\mathcal{A}t} & 0 \\ 0 & \mathrm{e}^{(I_{N-1} \otimes \mathcal{A} + c\Delta \otimes \mathcal{H})t} \end{bmatrix} (T^{-1} \otimes I_{2n})\xi(0) \end{aligned} \tag{15}$$

where matrices T, J, and Δ are defined in (9). By Theorem 1,$I_{N-1} \otimes \mathcal{A} + c\Delta \otimes \mathcal{H}$ is Hurwitz. Thus

$$\mathrm{e}^{(I_N \otimes \mathcal{A} + c\mathcal{L} \otimes \mathcal{H})t} \to (\mathbf{1} \otimes I_{2n})\mathrm{e}^{\mathcal{A}t}(r^{\mathrm{T}} \otimes I_{2n}) = (\mathbf{1}r^{\mathrm{T}}) \otimes \mathrm{e}^{\mathcal{A}t}, \quad \text{as } t \to \infty.$$

It then follows from (15) that

$$\xi(t) \to (\mathbf{1}r^{\mathrm{T}}) \otimes \mathrm{e}^{\mathcal{A}t}\xi(0), \quad \text{as } t \to \infty$$

implying that

$$\xi_i(t) \to (r^{\mathrm{T}} \otimes e^{\mathcal{A}t}) \begin{bmatrix} \xi_1(0) \\ \vdots \\ \xi_N(0) \end{bmatrix}, \quad \text{as } t \to \infty \tag{16}$$

for $i = 1, \ldots, N$. Because$A + BK$ is Hurwitz,(16) directly leads to the assertion. ■

Remark 3: Some algebraic characteristics of the agent dynamics implied by Theorems 1 and 2 are now briefly discussed. First, the agent dynamics (1) are excluded from having poles in the open right-half plane; otherwise, the consensus value reached by the agents will tend to infinity exponentially. Therefore, it is assumed hereinafter that matrix A has no eigenvalues with positive real parts. Furthermore, if system (1) is asymptotically stable, i.e., if A is Hurwitz, then it follows from (13)

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that the consensus value reached by the agents is zero. Thus, the matrix A in (1) having eigenvalues along the imaginary axis is critical for the agents to reach consensus at a nonzero value under protocol (3). Typical examples of this case include the single and double integrators considered in the existing literature [23],[29], [31], [32]. It should be pointed out that similar results have been obtained for some special cases of this theorem, e.g., in[32] where the agent dynamics are assumed to be double integrators and in [39] where the consensus protocol is static.

A. Consensus Region Analysis

Given a protocol in the form of (3), the consensus problem can be cast into analyzing the following system:

$$\dot{\varsigma} = (\mathcal{A} + \sigma\mathcal{H})\varsigma = \begin{bmatrix} A & BK \\ -\sigma FC & A + BK + \sigma FC \end{bmatrix}\varsigma \qquad (17)$$

where $\varsigma \in \mathbf{R}^{2n}$ and $\sigma \in \mathbf{C}$.

The stability of systems(17) depends on parameter σ. The region $\mathcal{S}$ of complex parameter σ, such that (17) is asymptotically stable, is called the consensus region of network (4) in this paper. It follows from Theorem 1 that consensus is reached if and only if

$$c(\alpha_k + i\beta_k) \in \mathcal{S}, \quad k = 2, 3, \ldots, N$$

where $i = \sqrt{-1}$, $\alpha_k = \mathrm{Re}(\lambda_k)$, and $\beta_k = \mathrm{Im}(\lambda_k)$. For an undirected communication graph, its consensus region $\mathcal{S}$ is an interval or a union of several intervals on the real axis. However, for a directed graph, where the eigenvalues of $\mathcal{L}$ are generally complex numbers, its consensus region $\mathcal{S}$ is a region or a union of several regions on the complex plane.

It is worth noting that consensus region $\mathcal{S}$ can be seen as the stability region of the matrix pencil $\mathcal{A} + \sigma\mathcal{H}$ with respect to complex parameter σ. Thus, tools from the stability of matrix pencils will be utilized to analyze the consensus region problem. Before moving on, the following lemma is needed.

Lemma 4: Given a complex-coefficient polynomial [25]

$$p(s) = s^2 + (a + ib)s + c + id \qquad (18)$$

where $a, b, c, d \in \mathbf{R}, p(s)$ is stable if and only if $a > 0$ and $abd + a^2c - d^2 > 0$.

In the aforementioned lemma, only second-order polynomials are considered. Similar results for high-order complex-coefficient polynomials can also be given (see [25]). However, in the latter case, the analysis will be more complicated.

The following example has a bounded consensus region.

Example 1: The agent dynamics and consensus protocol are given by (1) and (3), respectively, with

$$A = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \quad 0]$$

$$F = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad K = [-1 \quad 2] \quad \sigma = x + iy.$$

Obviously,$A + BK$ is Hurwitz. The characteristic polynomial of $A + \sigma FC$ is

$$\det(sI - (A + \sigma FC)) = s^2 + (1 - x - iy)s + 3(x + iy).$$

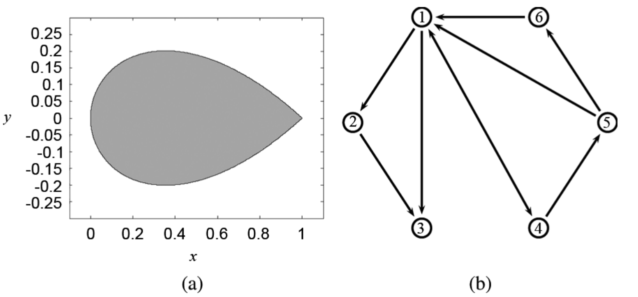

Fig. 1. (a) Bounded consensus region. (b) Communication topology.

By Lemma 4,$A + (x + iy)FC$ is stable if and only if $1 - x > 0$ and $-3y^2(1 - x) + 3x(1 - x)^2 - 9y^2 > 0$. The consensus region $\mathcal{S}$ in this case is shown in Fig. 1(a), which is bounded. Assume that the communication graph is given by Fig. 1(b), so the corresponding Laplacian matrix is

$$\mathcal{L} = \begin{bmatrix} 3 & 0 & 0 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

with nonzero eigenvalues $1, 1.3376 \pm i0.5623, 2, 3.3247$. It can be verified that consensus is achieved if and only if $0 < c \leq 0.3008$. Figs. 2 and 3 show the states of network (4) with different coupling strengths c's for this example.

Remark 4: The consensus region issue discussed in this section is similar to the synchronization region issue studied in[8], [17], [20], and [27]. The consensus region serves in a certain sense as a measure for the robustness of protocol (3) to parametric uncertainty. An example can be easily constructed such that the bound of the consensus region on the real axis for a certain protocol is rather narrow, e.g., [1, 1.005], and all the eigenvalues of $\mathcal{L}$ are one. Assume that the coupling strength c in (2) is subjected to multiplicative uncertainties, e.g.,(2) is changed to $\zeta_i = c(1 + \epsilon)\sum_{j=1}^{N} a_{ij}(y_i - y_j)$, where ϵ denotes the uncertainty. Clearly, if $|\epsilon| > 0.005$, then this protocol fails to solve the consensus problem. Therefore, given a consensus protocol, the consensus region should be large enough for the protocol to maintain a desirable robustness margin. As a matter of fact, the consensus region analysis given in this section provides a crucial basis for the consensus protocol design, which is the topic of the next section.

The following example has a disconnected consensus region.

1) Example 2: The agent dynamics and consensus protocol are given by(1) and (3), respectively, with

$$A = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad K = [-1 \quad 2] \quad \sigma = x + iy.$$

Obviously, $A + BK$ is Hurwitz. The characteristic polynomial of $A + \sigma FC$ is

$$\det(sI - (A + \sigma FC)) = s^2 + s - 2 + (x + 1)(x + 2) - y^2 + i(2y + 3)y.$$

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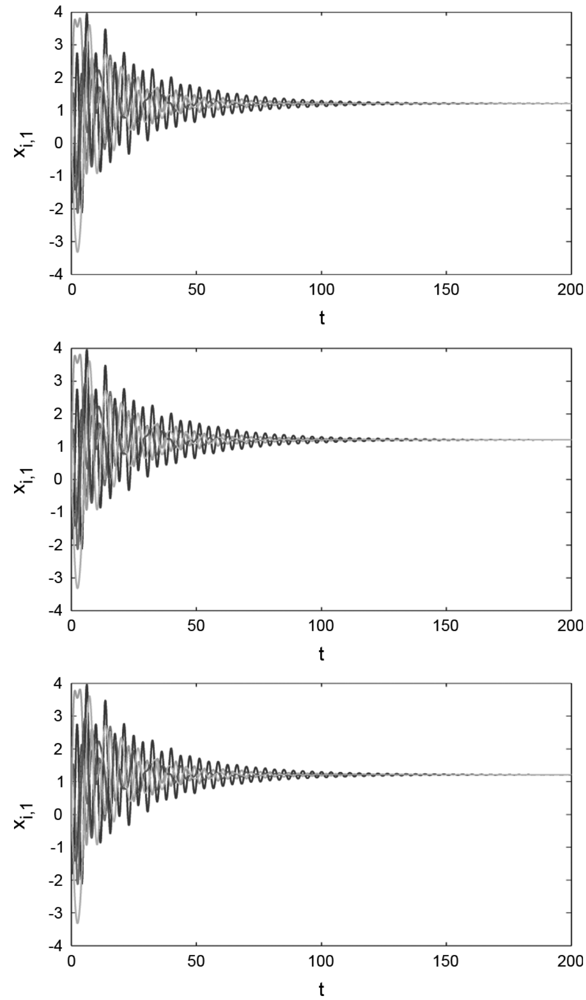

Fig. 2. States of six-agent network (4) when $c = 0.28$.

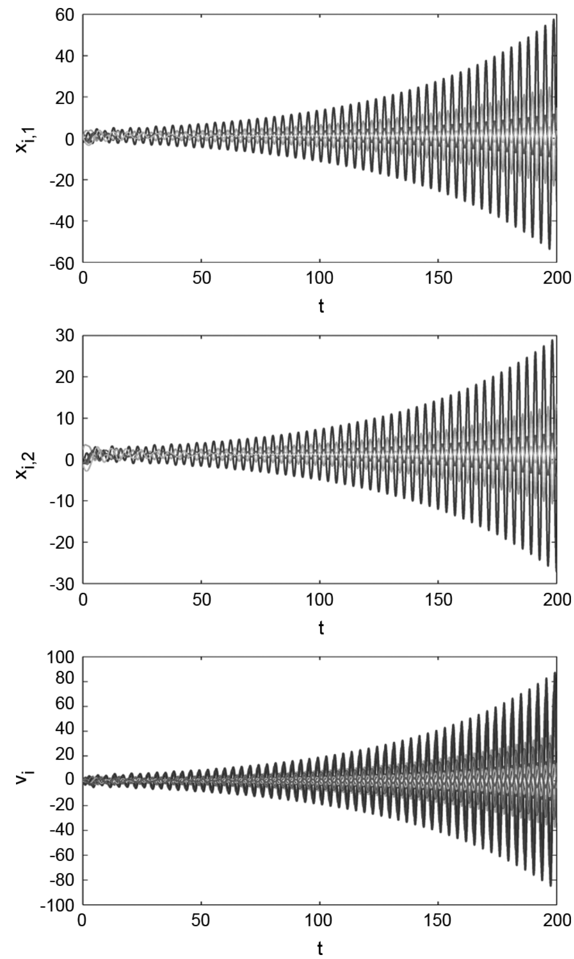

Fig. 3. States of six-agent network (4) when $c = 0.31$.

By Lemma 4, $A+(x+iy)BF$ is stable if and only if $x(x+3)-y^2-(2x+3)^2y^2>0$. The consensus region $\mathcal{S}$ in this case is shown in Fig. 4, which is composed of two disjoint subregions, both with unbounded real part and bounded imaginary part. If the communication graph is still given by Fig. 1(b), then consensus is achieved if and only if $0 < c \leq 0.3956$. If coupling strength c can be negative, then consensus will be achieved also when $c < -3$.

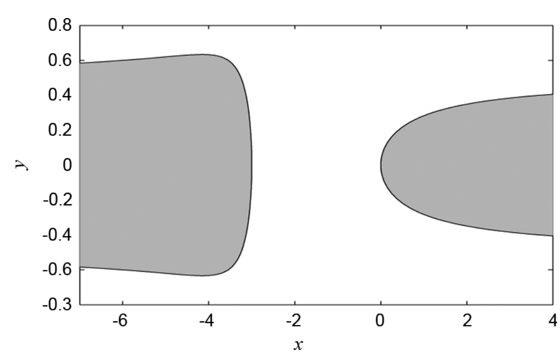

Fig. 4. Disconnected consensus region.

B. Consensus Protocol Design

In many cases, the protocols have to be designed so as to solve the consensus problem for various given communication topologies.

From the previous section, it can be seen that the cases with bounded consensus regions are more complicated than the cases with unbounded consensus regions. Hence, it is convenient to design a protocol such that the consensus region is unbounded.

Proposition 1: Given the agent dynamics (1), there exists a matrix F such that $A+(x+iy)FC$ is Hurwitz for all $x \in [1,\infty)$ and $y \in (-\infty,\infty)$ if and only if (A,C) is detectable.

Proof: (Necessity) It is trivial by letting $x=1$ and $y=0$.

(Sufficiency) Because (A,C) is detectable, there exists a matrix F such that $A+FC$ is Hurwitz, i.e., there exists a matrix $P>0$ such that

$$(A+FC)^{\mathrm{T}}P+P(A+FC)<0.$$

Let $PF=Y$. Then, the aforesaid inequality becomes

$$A^{\mathrm{T}}P+PA+YC+C^{\mathrm{T}}Y^{\mathrm{T}}<0.$$

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By Finsler's lemma [14], there exists a matrix Y satisfying the aforementioned inequality if and only if there exists a scalar $\tau > 0$ such that

$$A^{\mathrm{T}}P + PA - \tau C^{\mathrm{T}}C < 0. \tag{19}$$

Because matrix P is to be determined, let $\tau = 2$ without loss of generality. Then

$$A^{\mathrm{T}}P + PA - 2C^{\mathrm{T}}C < 0. \tag{20}$$

Obviously, when (20) holds, for any $\tau \geq 2$, (19) holds. Take $Y = -C^{\mathrm{T}}$, i.e.,$F = -P^{-1}C^{\mathrm{T}}$. By the previous inequalities, $A + xFC$ is Hurwitz for all $x \in [1, \infty)$. Thus, one has

$$\begin{aligned}(A + (x + yi)FC)^{*}P &+ P(A + (x + yi)FC) \\ &= (A + (x - yi)FC)^{\mathrm{T}}P + P(A + (x + yi)FC) \\ &= A^{\mathrm{T}}P + PA - 2xC^{\mathrm{T}}C < 0\end{aligned}$$

for all $x \in [1, \infty)$ and $y \in (-\infty, \infty)$. ■

Theorem 1 and the aforementioned proposition together lead to the following result.

Theorem 3: For the multiagent network (4) with $\mathcal{G}$ containing a directed spanning tree, there exists a distributed protocol in the form of (3) that solves the consensus problem and meanwhile yields an unbounded consensus region $\mathcal{S} \triangleq [1, \infty) \times (-\infty, \infty)$ if and only if (A, B, C) is stabilizable and detectable.

A multistep consensus protocol design procedure based on the consensus region notion is now presented.

Algorithm 1: Given (A, B, C) that is stabilizable and detectable and $\mathcal{G}$ containing a directed spanning tree, a protocol in the form of (3) solving the consensus problem can be constructed according to the following steps.

1) Choose matrix K such that $A + BK$ is Hurwitz.
2) Solve the linear matrix inequality (LMI)(20) to get one solution $P > 0$. Then, choose the feedback gain matrix $F = -P^{-1}C^{\mathrm{T}}$.
3) Select a coupling strength c that is larger than the threshold value c_{th}, which is given by

$$c_{\mathrm{th}} = \frac{1}{\min\limits_{i=2,\ldots,N} \mathrm{Re}(\lambda_i)} \tag{21}$$

where λ_i, $i = 2, \ldots, N$, denotes the nonzero eigenvalues of $\mathcal{L}$.

Remark 5: One distinct feature of Algorithm 1 is that it decouples the effects of the agent and protocol dynamics on the consensus stability from that of the communication topology. More specifically, steps 1) and 2) deal only with the agent dynamics and feedback gain matrices of the consensus protocol, leaving the communication topology of the multiagent network to be handled in step 3) by manipulating the coupling strength. One favorable consequence of this decoupling property is that the protocol so designed to achieve consensus for one given communication graph can be used directly to solve the consensus problem for any other graphs containing a spanning tree, with the only different task of appropriately selecting the coupling strength as in step 3). This feature will be more desirable for the case when agent number N is large, for which the eigenvalues of the corresponding Laplacian matrix are hard to determine or even troublesome to estimate. Here, in this case, one only needs to choose the coupling strength to be large enough.

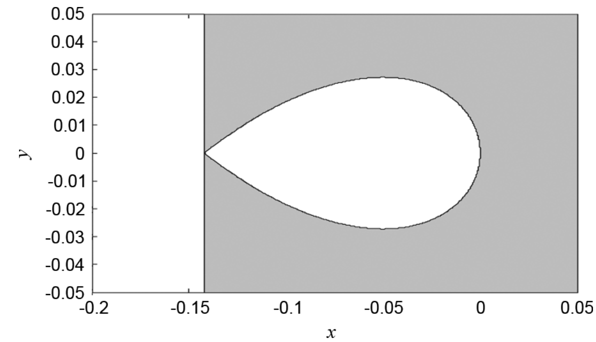

Fig. 5. Unbounded consensus region.

Now, Example 1 in the previous section is revisited.

Example 3: The agent dynamics and feedback gain matrix K of protocol (3) remain the same as in Example 1, while matrix F will be redesigned via Algorithm 1. By solving LMI (20), a feasible solution F is obtained as $F = \begin{bmatrix} -7.0314 \\ 8.4153 \end{bmatrix}$. Differing from Example 1, an unbounded consensus region in the form of $[1, \infty) \times (-\infty, \infty)$ can be obtained here. This can be verified in another way by noticing that the characteristic polynomial of $A + \sigma FC$ becomes

$$\begin{aligned}&\det(sI - (A + \sigma FC)) \\ &\qquad = s^2 + (1 + 7.0314(x + iy))\, s - 23.862(x + iy).\end{aligned}$$

Thus,$A + (x + iy)FC$ is Hurwitz if and only if

$$\begin{aligned}&1 + 7.0314x > 0 \\ &(7.0314 + 49.4406x)y^2 + x(1 + 7.0314x)^2 + 23.862y^2 > 0.\end{aligned}$$

The consensus region $\mathcal{S}$ in this case is the right-half plane by the vertical line $x = -1/7.0314$, except the white area shown in Fig. 5, which obviously contains the region $[1, \infty) \times (-\infty, \infty)$. Protocol (3) with feedback gains F and K (the same as that in the previous) and any $c > 0$ will solve the consensus problem for any communication graph containing a spanning tree.

C. Relative-State Consensus Protocol

In this section, a special case when the relative states between neighboring agents are available is considered. For this case, a distributed static protocol is proposed as

$$u_i = cL \sum_{j=1}^{N} a_{ij}(x_i - x_j) \tag{22}$$

where $c > 0$ and a_{ij} are the same as that defined in (2), and $L \in \mathbf{R}^{p \times n}$ is the feedback gain matrix. With (22), system (1) becomes

$$\dot{x}_i = Ax_i + BL \sum_{j=1}^{N} c\mathcal{L}_{ij}x_i, \qquad i = 1, 2, \ldots, N \tag{23}$$

where $\mathcal{L}_{ij}$ is defined in (4).

Corollary 1: For a directed network of N agents with communication topology $\mathcal{G}$ that has a directed spanning tree, pro-

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tocol (22) solves the consensus problem if and only if all matrices $A + c\lambda_i BL$,$i = 2, \ldots, N$, are Hurwitz.

Corollary 2: Consider the agent network (23) whose communication topology $\mathcal{G}$ has a directed spanning tree. If protocol (22) satisfies Corollary 1, then the states $x_i \to \varpi(t)$, as $t \to \infty$, for $i = 1, 2, \ldots, N$, where $\varpi(t)$ is defined in (13).

It is observed by comparing Theorem 2 and Corollary 2 that even if the consensus protocol takes the dynamic form (3) or the static form (22), the final consensus value reached by the agents will be the same, which relies only on the communication topology, the initial states, and the agent dynamics.

Similar to that in previous section, the consensus region of protocol(22) corresponds to the stability region of system $\dot{\tilde{\varsigma}} = (A + \sigma BL)\tilde{\varsigma}$ with respect to $\sigma \in \mathbf{C}$, where $\tilde{\varsigma} \in \mathbf{R}^n$.

The dual of Proposition 1 is presented as follows.

Proposition 2: Given the agent dynamics(1), there exists a matrix L such that $A+(x+iy)BL$ is Hurwitz for all $x \in [1, \infty)$ and $y \in (-\infty, \infty)$ if and only if (A, B) is stabilizable.

Algorithm 2: Given (A, B) that is stabilizable, a protocol in the form of(22) solving the consensus problem can be constructed according to the following steps.

1) Solve the following LMI:

$$AP + PA^{\mathrm{T}} - 2BB^{\mathrm{T}} < 0 \tag{24}$$

to get one solution $P > 0$. Then, choose the feedback gain matrix $L = -B^{\mathrm{T}}P^{-1}$.

2) Select the coupling strength $c > c_{\mathrm{th}}$, with c_{th} being given in (21).

D. Application to Spacecraft Formation Flying

In this section, a possible application of the consensus algorithm developed in the previous sections to spacecraft formation flying in the low Earth orbit is considered. A pertinent work is[33], which addresses the formation keeping problem of deep-space spacecraft whose translational dynamics are modeled as double integrators. In order to simplify the analysis, assume that a virtual reference spacecraft is moving in a circular orbit of radius R_0. The relative dynamics of the other spacecraft with respect to the virtual spacecraft will be linearized in the following coordinate system, where the origin is on the mass center of the virtual spacecraft, the x-axis is along the velocity vector, the y-axis is aligned with the position vector, and the$z$-axis completes the right-hand coordinate system.

The linearized equations of the relative dynamics of the ith satellite with respect to the virtual satellite are given by Hill's equations, which are

$$\begin{aligned} \ddot{\tilde{x}}_i - 2\omega_0\dot{\tilde{y}}_i &= u_{x_i} \\ \ddot{\tilde{y}}_i + 2\omega_0\dot{\tilde{x}}_i - 3\omega_0^2 y_i &= u_{y_i} \\ \ddot{\tilde{z}}_i + \omega_0^2\tilde{z}_i &= u_{z_i} \end{aligned} \tag{25}$$

where $\tilde{x}_i$,$\tilde{y}_i$, and $\tilde{z}_i$ are the position components of the ith satellite in the rotating coordinate; u_{x_i}, u_{y_i}, u_{z_i} are the control inputs; and ω_0 denotes the angular rate of the virtual satellite. The main assumption inherent in Hill's equations is that the distance between the ith and virtual satellites is very small in comparison to orbital radius R_0.

Denote the position vector by$r_i = [\tilde{x}_i, \tilde{y}_i, \tilde{z}_i]^{\mathrm{T}}$ and the control vector by $u_i = [u_{x_i}, u_{y_i}, u_{z_i}]^{\mathrm{T}}$. Then, (25) can be rewritten as

$$\begin{bmatrix} \dot{r}_i \\ \ddot{r}_i \end{bmatrix} = \begin{bmatrix} 0 & I_3 \\ A_1 & A_2 \end{bmatrix} \begin{bmatrix} r_i \\ \dot{r}_i \end{bmatrix} + \begin{bmatrix} 0 \\ I_3 \end{bmatrix} u_i \tag{26}$$

where

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3\omega_0^2 & 0 \\ 0 & 0 & -\omega_0^2 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 2\omega_0 & 0 \\ -2\omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Satellites are said to achieve formation flying if their velocity vectors converge to the same value and their positions maintain a prescribed separation, i.e.,$r_i - h_i \to r_j - h_j$, $\dot{r}_i \to \dot{r}_j$, as $t \to \infty$, where $h_i - h_j \in \mathbf{R}^3$ denotes the desired constant separation between satellites i and j.

Represent the communication topology among the N satellites by a directed graph $\mathcal{G}$. Assume that measurements of both relative positions and relative velocities between neighboring satellites are available. The control input to satellite i is proposed here as

$$u_i = -A_1 h_i + c\sum_{j=1}^{N} a_{ij} \left[F_1(r_i - h_i - r_j + h_j) + F_2(\dot{r}_i - \dot{r}_j)\right] \tag{27}$$

where $c > 0$; F_1 and $F_2 \in \mathbf{R}^{3\times 3}$ are the constant feedback gain matrices to be determined; and $a_{ii} = 0$ and $a_{ij} = 1$ if satellite i can obtain information from satellite j but 0 otherwise. If satellite k receives no information from any other satellite, then the term $A_1 h_k$ is set to zero. With (27), (26) can be reformulated as

$$\begin{aligned} \begin{bmatrix} \dot{r}_i \\ \ddot{r}_i \end{bmatrix} &= \begin{bmatrix} 0 & I_3 \\ A_1 & A_2 \end{bmatrix} \begin{bmatrix} r_i - h_i \\ \dot{r}_i \end{bmatrix} \\ &\quad + c\sum_{j=1}^{N} a_{ij} \begin{bmatrix} 0 & 0 \\ F_1 & F_2 \end{bmatrix} \begin{bmatrix} r_i - h_i - r_j + h_j \\ \dot{r}_i - \dot{r}_j \end{bmatrix}. \end{aligned} \tag{28}$$

The following result is a direct consequence of Corollary 1.

Corollary 3: Assume that graph $\mathcal{G}$ has a directed spanning tree. Then, protocol (27) solves the formation flying problem if and only if the matrices $\begin{bmatrix} 0 & I_3 \\ A_1 & A_2 \end{bmatrix} + c\lambda_i \begin{bmatrix} 0 & 0 \\ F_1 & F_2 \end{bmatrix}$ are Hurwitz for $i = 2, \ldots, N$, where λ_i, $i = 2, \ldots, N$, denote the nonzero eigenvalues of the Laplacian matrix of $\mathcal{G}$.

The feedback gain matrices F_1 and F_2 satisfying Corollary 3 can be designed by following Algorithm 2. Because system (26) is stabilizable, they always exist.

Example 4: Consider the formation flying of four satellites with respect to a virtual satellite that moves in a circular orbit at rate $\omega_0 = 0.001$. Assume that the communication topology is given by Fig. 6(a), from which it is observed that satellite 1 plays the leader's role. The nonzero eigenvalues of the Laplacian matrix in this case are 1, 1, and 2. Select c in (27) to be 1 for simplicity. One can solve the formation flying problem in the following steps: 1) Select properly the initial state of leading satellite 1 such that it moves in a spatial circular relative orbit with respect to the virtual satellite; 2) design the consensus protocol in the form of(27) such that the four satellites together maintain a desired formation flying situation.

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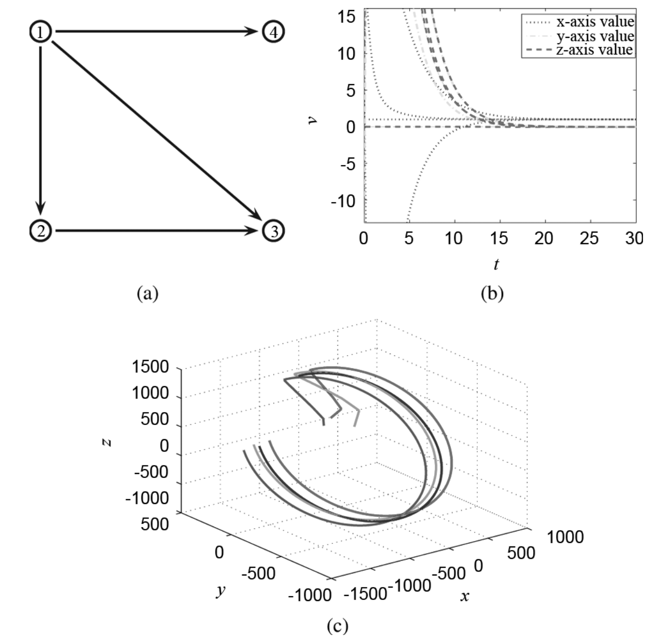

Fig. 6. (a) Communication topology. (b) Relative velocity. (c) Relative positions($t \in [0, 5000]\, s$).

Select the initial state of satellite 1 as $x_0 = 0, y_0 = 500, z_0 = 866, \dot{x}_0 = 1, \dot{y}_0 = 0, \dot{z}_0 = 0$. In such a case, satellite 1 maintains a circular relative orbit with respect to the virtual satellite, of radius $r = 1000$ m, and with the tangent angle to the orbital place $\alpha = 30^\circ$. Suppose that the four satellites will maintain a square shape with a separation of 500 m in a plane tangent to the orbit of the virtual satellite by an angle $\theta = 60^\circ$. Let $h_1 = (100, 100, 0)$, $h_2 = (-100, 100, 0)$, $h_3 = (100, 0, 173.21)$, and $h_4 = (-100, 10, 173.21)$. By Algorithm 2, the feedback gain matrices solving the formation flying problem can be obtained by using the LMI Toolbox [11] as

$$F_1 = \begin{bmatrix} 0.6596 & -0.0013 & 0 \\ 0.0013 & 0.6596 & 0 \\ 0 & 0 & 0.6596 \end{bmatrix}$$
$$F_2 = \begin{bmatrix} 1.9789 & 0 & 0 \\ 0 & 1.9789 & 0 \\ 0 & 0 & 1.9789 \end{bmatrix}.$$

The simulation result is shown in Fig. 6.

III. Consensus With Respect to a Reference State

In the previous section, the consensus problem of multiagent systems with consensus protocols (3) and (22) has been studied. It is worth noting that the final consensus value, which depends on the initial values and agent dynamics, might be unknown *a priori*. However, in many practical cases, it is desirable that the agents' states asymptotically approach a reference state, which can also be time varying. This is called a model-reference consensus problem in [31] and[32], where only integrator dynamics are discussed. If the reference model is taken as a virtual leader, the model-reference consensus problem is actually the leader–follower consensus problem, as studied in [12], [13], and [15]. The model-reference consensus problem is, in some sense, related to the target acquisition problem concerned in [10]. In this section, the problem is extended to the general framework proposed in the last section.

The agent's dynamics are still described by (1). The reference trajectory $x_r \in \mathbf{R}^n$, which the agents' states x_i will follow, satisfies

$$\dot{x}_r = Ax_r + Bu_r \quad \text{and} \quad y_r = Cx_r \tag{29}$$

where $u_r \in \mathbf{R}^p$, $y_r \in \mathbf{R}^q$, and matrices A, B, and C are the same as those defined in (1). It is assumed that only a subset of agents have access to the output variable of the reference model (29), whereas all the agents in the network have access to the reference input. In this case, the information available to agent i is given by

$$\hat{\zeta}_i = c\left(\sum_{j=1}^{N} a_{ij}(y_i - y_j) + d_i(y_i - y_r)\right) \tag{30}$$

where $c > 0$, a_{ij} is the same as that defined in (3), and $d_i > 0$ if agent i has access to the reference model (29) but zero otherwise. A distributed protocol based on(30) is proposed as

$$\begin{aligned} \dot{v}_i &= (A + BK)v_i \\ &\quad + F\left(c\sum_{j=1}^{N} a_{ij}C(v_i - v_j) + cd_iC(v_i - v_r) - \hat{\zeta}_i\right) \\ u_i &= Kv_i + u_r \end{aligned} \tag{31}$$

where $v_i \in \mathbf{R^n}$, $F \in \mathbf{R^{q\times n}}$, and $K \in \mathbf{R^{p\times n}}$ are the feedback gain matrices to be determined and $v_r \in \mathbf{R^n}$ is the state of system $\dot{v}_r = (A + BK)v_r$.

With protocol (31), the model-reference consensus problem is said to be solved if

$$x_i(t) \to x_r(t) \quad v_i(t) \to 0 \qquad \forall\, i = 1, 2, \ldots, N, \text{ as } t \to \infty.$$

Let $\widehat{D} = \text{diag}(d_1, d_2, \ldots, d_N)$, $x_{e,i} = x_i - x_r$, $v_{e,i} = v_i - v_r$, $\vartheta_i = [x_{e,i}^{\mathrm{T}}, v_{e,i}^{\mathrm{T}}]^{\mathrm{T}}$, $\vartheta = [\vartheta_1^{\mathrm{T}}, \ldots, \vartheta_N^{\mathrm{T}}]^{\mathrm{T}}$. Then, the closed-loop network dynamics can be written in the following form, as is the case in the last section:

$$\dot{\vartheta} = (I_N \otimes \mathcal{A} + c\hat{\mathcal{L}} \otimes \mathcal{H})\vartheta \tag{32}$$

where $\mathcal{A}$ and $\mathcal{H}$ are defined in (4), $\widehat{\mathcal{L}} = \mathcal{L} + \widehat{D}$, and $\mathcal{L}$ is the Laplacian matrix of communication topology $\mathcal{G}$ among the N agents.

It is easy to see that the model-reference consensus problem is solved if and only if all the states of (32) converge to zero.

Lemma 5: Suppose that the directed communication graph $\mathcal{G}$ has a spanning tree and that the root agent of such a tree has access to the reference model. Then, all the eigenvalues of the matrix $\widehat{\mathcal{L}}$ defined in (32) have positive real parts.

Proof: Without loss of generality, one can rearrange if necessary the order of the nodes in the network such that Laplacian matrix $\mathcal{L}$ takes the Frobenius normal form. Because the root agent has access to the reference model, in light of Lemma 3, all the submatrices along the diagonal of $\widehat{\mathcal{L}}$ are irreducible, and thus, it has at least one row with positive row sum. Then, by following the steps in proving Lemma 1 in [6], one can easily show that all the eigenvalues of $\widehat{\mathcal{L}}$ have positive real parts. ■

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Corollary 4: Assume that communication topology $\mathcal{G}$ has a directed spanning tree and that the root agent of such a tree has access to the reference model. Then, protocol (31) solves the model-reference consensus problem if and only if all matrices $A+BK$, $A+c\hat{\lambda}_i FC$,$i=1,\ldots,N$, are Hurwitz, where $\hat{\lambda}_i$, $i=1,\ldots,N$, are the eigenvalues of $\hat{\mathcal{L}}$.

Proof: It follows from the proof of Theorem 1 that the stability of $N+1$ matrices is equivalent to the stability of (32), which implies that $\vartheta_i\to 0$, $i=1,\ldots,N$, as $t\to\infty$. Therefore,$x_i\to x_r$,$v_i\to v_r\to 0$, $i=1,\ldots,N$, as $t\to\infty$, i.e., the model-reference consensus problem is solved. ■

Remark 6: Similar to Theorem 1, this corollary casts the consensus problem with respect to a reference state equivalently into the stability of$N+1$ matrices that all have the same dimension (equal to that of a single agent). A distinct feature of this corollary is that matrix A here is allowed to be unstable for the model-reference consensus problem, as opposed to Theorem 1 where matrix A is required to have no eigenvalues with positive real parts. According to Lemma 5, the reference consensus can possibly be reached by using protocol (31), even when only the root agent has access to the output variable of the reference model.

Similar toSection III, both the consensus region analysis and consensus protocol design can be discussed correspondingly with very little modification, so they are omitted here for brevity.

IV. Consensus With H_∞ Performance Specification

Practically, an agent itself may be subjected to external disturbances. It is interesting to study the robustness of consensus protocols to external disturbances, which is formulated as an issue of additional H_∞ performance specification. A related work is [16], where the H_∞ gain of multivehicle formations with a double-graph strategy was investigated. For simplicity, the connections among agents are assumed to be undirected, keeping symmetric Laplacian matrices with real eigenvalues.

The agent dynamics perturbed by external disturbances are described by

$$\begin{aligned}\dot{x}_i &= Ax_i+Bu_i+B_2\omega_i\\ y_i &= Cx_i+D_2\omega_i, \qquad i=1,2,\ldots,N\end{aligned}\tag{33}$$

where $x_i\in\mathbf{R}^n$ is the state of agent i, $u_i\in\mathbf{R}^p$ is the control input, $\omega_i\in\mathcal{L}_2^{m_1}[0,\infty]$ is the external disturbance, and $y_i\in\mathbf{R}^q$ is the measured output.

First, consider the robustness of protocol(31) to external disturbances. Because the agent states are desired to converge to the reference state x_r satisfying (29), it is natural to define the performance variables $z_i\in\mathbf{R}^{m_2}$ as

$$z_i=C_2(x_i-x_r), \qquad i=1,2,\ldots,N.\tag{34}$$

Let $z=[z_1^{\mathrm{T}},\ldots,z_N^{\mathrm{T}}]^{\mathrm{T}}$ and $\omega=[\omega_1^{\mathrm{T}},\ldots,\omega_N^{\mathrm{T}}]^{\mathrm{T}}$. Then, the closed-loop network dynamics resulting from (31), (33), and(34) are written as

$$\begin{aligned}\dot{\vartheta} &= (I_N\otimes\mathcal{A}+c\hat{\mathcal{L}}\otimes\mathcal{H})\vartheta+(I_N\otimes\mathcal{B}_2+c\hat{\mathcal{L}}\otimes\mathcal{D}_2)\omega\\ z &= (I_N\otimes\mathcal{C}_2)\vartheta\end{aligned}\tag{35}$$

where ϑ, $\mathcal{A}$,$\mathcal{H}$, and $\hat{\mathcal{L}}$ are the same as that in(32) and

$$\mathcal{B}_2=\begin{bmatrix}B_2\\0\end{bmatrix}\quad \mathcal{C}_2=[\,C_2\quad 0\,]\quad \mathcal{D}_2=\begin{bmatrix}0\\-D_2\end{bmatrix}.$$

Denote by T_{wz} the transfer function matrix from ω to z of system (35).

The model-reference consensus with the desired H_∞ performance specification is stated as follows. For a given $\gamma>0$, find an appropriate protocol in the form of(31) such that the following are achieved: 1) The model-reference consensus is solved with $\omega=0$, i.e., the state of (35) with $\omega=0$ converges to zero, and 2) $\|T_{wz}\|_\infty<\gamma$, where $\|T_{wz}\|_\infty$ is the H_∞ norm of T_{wz}, defined by $\|T_{wz}(s)\|_\infty=\sup_{w\in\mathbf{R}}\bar{\sigma}(T_{wz}(iw))$ [45].

Theorem 4: For an undirected network of agents described by (33), with at least one agent having access to the reference model, the model-reference consensus is solved along with $\|T_{wz}\|_\infty<\gamma$ if and only if there exists a protocol (31) such that the following N systems are asymptotically stable and, moreover, the H_∞ norms of their transfer function matrices are all less than γ:

$$\begin{aligned}\dot{\hat{\vartheta}}_i &= (\mathcal{A}+c\hat{\lambda}_i\mathcal{H})\hat{\vartheta}_i+(\mathcal{B}_2+c\hat{\lambda}_i\mathcal{D}_2)\hat{\omega}_i\\ \hat{z}_i &= \mathcal{C}_2\hat{\vartheta}_i, \qquad i=1,2,\ldots,N\end{aligned}\tag{36}$$

where $0<\hat{\lambda}_1\le\hat{\lambda}_2\le\cdots\le\hat{\lambda}_N$ denotes the eigenvalues of $\hat{\mathcal{L}}$.

Proof: Because graph $\mathcal{G}$ is assumed to be undirected and at least one agent has access to the reference model, it follows from Lemma 5 that matrix $\hat{\mathcal{L}}$ is positive definite. Let U be such a unitary matrix in that $U^{-1}\hat{\mathcal{L}}U=\Lambda=\mathrm{diag}(\hat{\lambda}_1,\ldots,\hat{\lambda}_N)$. Introduce the following sate transformations: $\vartheta=(U\otimes I_n)\hat{\vartheta}$, with $\hat{\vartheta}=[\hat{\vartheta}_1^{\mathrm{T}},\ldots,\hat{\vartheta}_N^{\mathrm{T}}]^{\mathrm{T}}$. Then, network (35) can be rewritten as

$$\begin{aligned}\dot{\hat{\vartheta}} &= (I_N\otimes\mathcal{A}+c\Lambda\otimes\mathcal{H})\hat{\vartheta}+(U^{-1}\otimes\mathcal{B}_2+cU^{-1}\hat{\mathcal{L}}\otimes\mathcal{D}_2)\omega\\ z &= (U\otimes\mathcal{C}_2)\hat{\vartheta}.\end{aligned}\tag{37}$$

Furthermore, reformulate disturbance variable ω and performance variable z via

$$\omega=(U\otimes I_{m_1})\hat{\omega}\quad z=(U\otimes I_{m_2})\hat{z}.\tag{38}$$

where $\hat{\omega}=[\hat{\omega}_1^{\mathrm{T}},\ldots,\hat{\omega}_N^{\mathrm{T}}]^{\mathrm{T}}$ and $\hat{z}=[\hat{z}_1^{\mathrm{T}},\ldots,\hat{z}_N^{\mathrm{T}}]^{\mathrm{T}}$. Then, substituting (38) into (37) gives

$$\begin{aligned}\dot{\hat{\vartheta}} &= (I_N\otimes\mathcal{A}+c\Lambda\otimes\mathcal{H})\hat{\vartheta}+(I_N\otimes\mathcal{B}_2+c\Lambda\otimes\mathcal{D}_2)\hat{\omega}\\ \hat{z} &= (I_N\otimes\mathcal{C}_2)\hat{\vartheta}.\end{aligned}\tag{39}$$

It is worth noting that (39) is composed of N independent systems as in (36). Denote by $T_{\hat{w}\hat{z}}$ and $T_{\hat{w}_i\hat{z}_i}$ the transfer function matrices of systems (39) and (36), respectively. Then, it follows from (36)–(39) that

$$\begin{aligned}T_{\hat{w}\hat{z}} &= \mathrm{diag}(T_{\hat{\omega}_1\hat{z}_1},\ldots,T_{\hat{\omega}_N\hat{z}_N})\\ &= (U^{-1}\otimes I_{m_2})T_{\omega z}(U\otimes I_{m_1}).\end{aligned}\tag{40}$$

By observing that both $U\otimes I_{m_1}$ and $U\otimes I_{m_2}$ are unitary matrices, from (40), one obtains the relationships between the H_∞ norms of T_{wz}, $T_{\hat{w}\hat{z}}$, and $T_{\hat{w}_i\hat{z}_i}$ as follows:

$$\|T_{\omega z}\|_\infty=\|T_{\hat{w}\hat{z}}\|_\infty=\max_{i=1,\ldots,N}\|T_{\hat{\omega}_i\hat{z}_i}\|_\infty.\tag{41}$$

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Therefore, the model-reference consensus problem with $\|T_{\omega z}\|_\infty < \gamma$ is reduced to designing a common controller in the form of (31) such that all the N systems in (36) are simultaneously asymptotically stable, with $\|T_{\hat{\omega}_i \hat{z}_i}\|_\infty < \gamma$. ■

Remark 7: Similar to Theorem 1, the previous theorem converts the model-reference consensus problem with prescribed H_∞ performance specification into the H_∞ control problem of a set of independent systems, each of which has a dimension that is equal to that of a single agent. Here, the communication graph is assumed to be undirected and connected, which is more stringent than the assumptions in Theorem 1. It is worth noting that the key to this theorem relies heavily on the input and output transformations in addition to the state transformation.

Next, consider the robustness of protocol(3) to external disturbances. In this case, define the performance variable as $\tilde{z}_i = C_2 x_i$, $i = 1, \ldots, N$. The closed-loop network dynamics can be written as

$$\begin{aligned} \dot{\xi} &= (I_N \otimes \mathcal{A} + c\mathcal{L} \otimes \mathcal{H})\xi + (I_N \otimes \mathcal{B}_2 + c\mathcal{L} \otimes \mathcal{D}_2)\omega \\ \tilde{z} &= (I_N \otimes \mathcal{C}_2)\xi \end{aligned} \tag{42}$$

where $\tilde{z} = [\tilde{z}_1^{\mathrm{T}}, \ldots, \tilde{z}_N^{\mathrm{T}}]^{\mathrm{T}}$, $\xi = [\xi_1^{\mathrm{T}}, \ldots, \xi_N^{\mathrm{T}}]^{\mathrm{T}}$, ξ_i, and $\mathcal{L}$ are defined in (4). Denote by $\widetilde{T}_{w\tilde{z}}$ the transfer function matrix from ω to $\tilde{z}$ of system (42).

Corollary 5: For an undirected network of N agents described by (33), protocol (3) solves the consensus problem with $\|\widetilde{T}_{wz}\|_\infty < \gamma$ if and only if the following N systems are asymptotically stable and, moreover, the H_∞ norms of their transfer function matrices are all less than γ:

$$\begin{aligned} \dot{\bar{\xi}}_i &= (\mathcal{A} + c\lambda_i \mathcal{H})\bar{\xi}_i + (\mathcal{B}_2 + c\lambda_i \mathcal{D}_2)\bar{\omega}_i \\ \bar{z}_i &= \mathcal{C}_2 \bar{\xi}_i, \qquad i = 1, 2, \ldots, N \end{aligned} \tag{43}$$

where $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ are the eigenvalues of $\mathcal{L}$.

Remark 8: Because of the singularity of Laplacian matrix $\mathcal{L}$, the state matrix A of system (1) is required to be Hurwitz so as to guarantee the existence of the H_∞ norm from the disturbance to performance variables of the resulting network. This gives rise to the following conflict: Matrix A is desired to have eigenvalues on the $i\omega$-axis in order to reach nonzero consensus values, whereas A is not allowed to have eigenvalues on the $i\omega$-axis in order to validate the H_∞ performance specification. It should be noted that the transfer function of the system in (43) corresponding to $\lambda_1 = 0$ is the same as that of an isolated agent from w_i to $\tilde{z}_i$. Therefore, an interesting consequence is that theH_∞ performance of the agent network(42) (if it exists) will never be better than that of an isolated agent, implying that protocol(3) cannot render the agent network any better disturbance rejection level as compared to that of an isolated agent. On the contrary, it is possible for the model-reference consensus protocol(31) to be so (see Theorem 4).

V. Conclusion

This paper has studied the consensus problem of multiagent systems under a time-invariant communication topology, with each agent having a general form of linear dynamics. An observer-type consensus protocol based on relative output measurements between neighboring agents has been proposed and analyzed. A novel framework has been introduced, which can describe in a unified way both the consensus of multiagent systems and the synchronization of complex dynamic networks. The notion of consensus region has been introduced and analyzed by using tools from the stability of matrix pencils, on which a multistep consensus protocol design procedure has also been presented. Finally, two important issues of consensus with respect to a time-varying state and the robustness of consensus protocols to external disturbances have been addressed. Generalizing the results of this paper to the case where the communication topology is dynamically evolving or has time delays is an important yet challenging topic for future research.

Acknowledgment

The authors would like to thank the Associate Editor and all the anonymous reviewers for their careful reading of this paper and their constructive suggestions, which have led to the improvement of the presentation of this paper.

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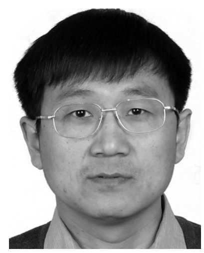
Zhongkui Li received the B.S. degree in aerospace engineering from the National University of Defense Technology, Changsha, China, in 2005. He is currently working toward the Ph.D. degree in the State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing, China.

From November 2008 to February 2009, he was a Research Assistant with the City University of Hong Kong, Kowloon, Hong Kong. His research interests include nonlinear control, complex networks, and cooperative control of multi-agent systems with applications to aerospace engineering.



Zhisheng Duan received the M.S. degree in mathematics from Inner Mongolia University, Hohhot, China, and the Ph.D. degree in control theory from Peking University, Beijing, China, in 1997 and 2000, respectively.

From 2000 to 2002, he was a Postdoctor with Peking University. Since 2008, he has been a Professor with the Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, where he is also currently with the State Key Laboratory for Turbulence and Complex Systems. His research interests include robust control, stability of interconnected systems, flight control, and analysis and control of complex dynamical networks.

Dr. Duan was the recipient of the 2001 Chinese Control Conference Guan Zhao-Zhi Award.

Guanrong Chen (M'87–SM'92–F'97) received the M.Sc. degree in computer science from Zhongshan (Sun Yat-Sen) University, Guangzhou, Taiwan, in 1981 and the Ph.D. degree in applied mathematics from Texas A&M University, College Station, in 1987.

He was a tenured Full Professor with the University of Houston, Houston, TX. He is currently a Chair Professor with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong, where he has been the founding Director of the Centre for Chaos and Complex Networks since January 2000. He is also with the State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing, China. His research interests include chaotic dynamics, complex networks, and nonlinear control.

Lin Huang received the B.S. and M.S. degrees in mathematics and mechanics from Peking University, Beijing, China, in 1957 and 1961, respectively.

In 1961, he joined the Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, where he is currently a Professor of control theory and is with the State Key Laboratory for Turbulence and Complex Systems. His research interests include the stability of dynamical systems, robust control, and nonlinear systems. He has authored three books and coauthored more than 150 papers in the fields of stability theory and control systems.

Prof. Huang is a member of the Chinese Academy of Sciences.

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