

# Synchronization and State Estimation for Discrete-Time Complex Networks With Distributed Delays

Yurong Liu, Zidong Wang, *Senior Member, IEEE*, Jinling Liang, and Xiaohui Liu

**Abstract**—In this paper, a synchronization problem is investigated for an array of coupled complex discrete-time networks with the simultaneous presence of both the discrete and distributed time delays. The complex networks addressed which include neural and social networks as special cases are quite general. Rather than the commonly used Lipschitz-type function, a more general sector-like nonlinear function is employed to describe the nonlinearities existing in the network. The distributed infinite time delays in the discrete-time domain are first defined. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, it is shown that the addressed discrete-time complex network with distributed delays is synchronized if certain linear matrix inequalities (LMIs) are feasible. The state estimation problem is then studied for the same complex network, where the purpose is to design a state estimator to estimate the network states through available output measurements such that, for all admissible discrete and distributed delays, the dynamics of the estimation error is guaranteed to be globally asymptotically stable. Again, an LMI approach is developed for the state estimation problem. Two simulation examples are provided to show the usefulness of the proposed global synchronization and state estimation conditions. It is worth pointing out that our main results are valid even if the nominal subsystems within the network are unstable.

**Index Terms**—Complex networks, discrete time delays, distributed time delays, linear matrix inequality (LMI), Lyapunov–Krasovskii functional, neural networks, state estimation, synchronization.

## I. INTRODUCTION

COMPLEX networks are composed of a large number of highly interconnected dynamical units and therefore exhibit very complicate dynamics. Examples of such complex networks include the Internet, which is a network of routers or

domains, the World Wide Web, which is a network of web sites, the brain, which is a network of neurons, and an organization, which is a network of people. Since the introduction of the small-world network principle by Watts and Strogatz [35], a great deal of research has been focused on the dependence of the asymptotic behavior of interconnected oscillatory agents on the structural properties of complex networks. It has been found out that the general structure of the interaction network may play a crucial role in the emergence of synchronization phenomena in various fields such as physics, technology, and the life sciences. Synchronization is attracting more and more research attention due to its ubiquity in many system models of the natural world, for example, the large-scale and complex networks of chaotic oscillators [15], [19], [26], [32], the coupled systems exhibiting spatiotemporal chaos and autowaves [28], [29], and the array of coupled neural networks [1], [14], [18], [27], [36], [37].

Time delays occur commonly in complex networks because of the network traffic congestions as well as the finite speed of signal transmission over the links [4], [8], [9], [12], [13], [29], [30], [38]. Also, the network couplings often give rise to delays in biological neural networks, gene regulatory networks, communication networks, and electrical power grids [2], [9], [17], [19]. It has been recognized that time delays can cause complex dynamics such as periodic or quasi-periodic motions, Hopf bifurcation, and higher dimensional chaos. In recent years, the synchronization problem for various types of networks with delayed coupling has been extensively studied [2], [9], [17], [19], [23], [37]. For example, the synchronization criteria have been established in [17] for complex dynamical network models with coupling delays for both continuous and discrete-time cases, which have further been improved in [9] by using less conservative delay-dependent techniques. A variational method has been used in [23] to deal with the synchronization problem for an array of linearly coupled identical connected neural networks with delays, whereas the similar problem has been addressed in [37] for an array of coupled nonlinear systems with delay and nonreciprocal time-varying coupling. It is worth mentioning that most of the reported results have addressed the synchronization problem for networks with discrete time delays. Another kind of time delay, namely, continuously distributed delays, has started to gain research attention in the context of synchronization because a complex network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of node sizes and lengths. Very recently, in [22], the synchronization problem has been investigated for coupled networks with both discrete and distributed time delays.

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Y. Liu is with the Department of Mathematics, Yangzhou University, Yangzhou 225002, China (e-mail: liuyurong@gmail.com).

Z. Wang and X. Liu are with the Department of Information Systems and Computing, Brunel University, Uxbridge UB8 3PH, U.K. (e-mail: Zidong.Wang@brunel.ac.uk).

J. Liang was with the Department of Information Systems and Computing, Brunel University, Uxbridge UB8 3PH, U.K. She is now with the Department of Mathematics, Southeast University, Nanjing 210096, China (e-mail: jinliang@gmail.com).

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One of the important yet challenging issues for understanding the interaction topology of complex networks has to do with the discrete nature of network topology [31]. The reason is mainly threefold: 1) the discretization process of a continuous-time network cannot preserve the dynamics of the continuous-time part even for small sampling periods; 2) a discrete-time network is in a better position to model digitally transmitted signals in a dynamical way than its continuous-time analog, and 3) the discrete-time networks have already been applied in a wide range of areas, such as image processing, time series analysis, quadratic optimization problems, and system identification. Recently, the synchronization problem for discrete-time networks has received some initial research interests. For example, the master–slave synchronization has been discussed in [19] where the activation function was assumed to be of the traditional Lipschitz type. In [24] and [25], the synchronization problem has been studied for an array of discrete-time coupled complex networks in a systematic way and a series of results was obtained by using innovative manifold/graph approaches.

Although the synchronization problem for discrete-time complex networks is now drawing increasing research attention, there are still several open problems deserving further investigation. First, despite their importance in modeling the distribution of propagation delays over a period of sampling time, the distributed time delays have not yet been addressed in the synchronization problems for discrete-time complex networks. The main reason is that it is nontrivial to represent the distributed time delays in the discrete-time domain and establish a unified framework to handle both the discrete and distributed time delays. Second, for large-scale complex networks, it is quite common that only partial information about the network nodes (states) is accessible from the network outputs. Therefore, in order to make use of key network nodes in practice, it becomes necessary to estimate the network nodes through available measurements. Note that the state estimation problem for neural networks (a special class of complex networks) was first addressed in [34] and has then drawn particular research interests, see, e.g., [11] and [12], where the networks are deterministic and continuous time. Unfortunately, the state estimation problem for discrete-time complex networks with or without distributed delays has not been researched yet. Therefore, the aim of this paper is to deal with the synchronization and state estimation problems for discrete-time complex networks with distributed delays.

In this paper, we investigate the synchronization problem for an array of coupled complex discrete-time networks with the simultaneous presence of both the discrete and distributed time delays. Rather than the commonly used Lipschitz-type function, a more general sectorlike nonlinear function is employed to describe the nonlinearities existing in the network. We first define the distributed time delays for the complex networks in the discrete-time domain. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, we show that the addressed synchronization problem can be converted into the feasibility problem of a set of linear matrix inequalities (LMIs). We then turn to the state estimation problem for the same complex networks. Through available output measurements, we aim to design a state estimator to estimate the network states such that, for all admissible discrete

and distributed delays, the dynamics of the estimation error is guaranteed to be globally asymptotically stable. Again, an LMI approach is used with the main proof omitted for the state estimation case. Two simulation examples are provided to show the usefulness of the proposed theory. It is worth pointing out that our main results are valid even if the nominal subsystems within the network are unstable.

*Notations:* The notations are quite standard. Throughout this paper,  $\mathbb{R}^n$  and  $\mathbb{R}^{n \times m}$  denote the  $n$ -dimensional Euclidean space and the set of all  $n \times m$  real matrices, respectively. The superscript “T” denotes matrix transposition, and the notation  $X \geq Y$  ( $X > Y$ ), where  $X$  and  $Y$  are symmetric matrices, means that  $X - Y$  is positive semidefinite (positive definite). For vector or matrix  $z$ ,  $z \succeq 0$  means that each entry of  $z$  is non-negative.  $I_n$  is the  $n \times n$  identity matrix.  $\|\cdot\|$  is the Euclidean norm in  $\mathbb{R}^n$ . The Kronecker product of an  $n \times m$  matrix  $X$  and a  $p \times q$  matrix  $Y$  is defined by an  $np \times mq$  matrix  $X \otimes Y$ . If  $A$  is a matrix, denote by  $\lambda_{\max}(A)$  ( $\lambda_{\min}(A)$ ) the largest (smallest) eigenvalue of  $A$ . Matrices, if not explicitly specified, are assumed to have compatible dimensions. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

## II. PROBLEM FORMULATION

Consider the following discrete-time delayed complex network consisting of  $N$  coupled nodes of the form:

$$\begin{aligned} x_i(k+1) = & f(x_i(k)) + g(x_i(k - \tau(k))) \\ & + \sum_{m=1}^{+\infty} \mu_m h(x_i(k-m)) + \sum_{j=1}^N w_{ij} \Gamma x_j(k), \\ & i = 1, 2, \dots, N \end{aligned} \quad (1)$$

where  $x_i(k) = (x_{i1}(k), x_{i2}(k), \dots, x_{in}(k))^T$  is the state vector of the  $i$ th node.  $f(\cdot)$ ,  $g(\cdot)$ , and  $h(\cdot)$  are nonlinear vector-valued functions satisfying certain conditions given later. The positive integer  $\tau(k)$  denotes the discrete time-varying delay satisfying

$$\tau_m \leq \tau(k) \leq \tau_M, \quad k \in \mathbb{N} \quad (2)$$

where  $\tau_m$  and  $\tau_M$  are known positive integers. The constants  $\mu_m \geq 0$  ( $m=1, 2, \dots$ ) satisfy the following convergent conditions:

$$\sum_{m=1}^{+\infty} \mu_m < +\infty \quad \sum_{m=1}^{+\infty} m\mu_m < +\infty. \quad (3)$$

$\Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\} \geq 0$  is a matrix linking the  $j$ th state variable if  $\gamma_j \neq 0$ , and  $W = (w_{ij}) \in \mathbb{R}^{N \times N}$  is the coupled configuration matrix of the network with  $w_{ij} \geq 0$  ( $i \neq j$ ) but not all zero. As usual, the coupling configuration matrix  $W = (w_{ij})$  is symmetric (i.e.,  $W = W^T$ ) and satisfies

$$\sum_{l=1}^N w_{kl} = \sum_{l=1}^N w_{lk} = 0, \quad k = 1, 2, \dots, N. \quad (4)$$

*Remark 1:* The model (1) includes the term of the distributed time delays  $\sum_{m=1}^{+\infty} \mu_m h(x_i(k-m))$  in the discrete-time setting. Such a term is proposed, to the best of the authors’

knowledge, for the first time for complex networks and the model and the model (1) can be interpreted as the discrete analog of the following continuous-time complex network with mixed time delay:

$$\frac{dx_i(t)}{dt} = f(x_i(t)) + g(x_i(t - \tau(t))) + \int_{-\infty}^t k(t-s)h(x_i(s))ds + \sum_{j=1}^N w_{ij}\Gamma x_j(t).$$

As can be seen in the sequel, the inclusion of such a distributed delay term will bring additional difficulty in the analysis and a special inequality will need to be developed.

*Remark 2:* The convergent condition (3) is used to make sure that the term of  $\sum_{m=1}^{+\infty} \mu_m h(x_i(k-m))$  in (1) as well as the Lyapunov functional (to be constructed later) are convergent.

*Remark 3:* The addressed discrete-time delayed complex networks (1) that include many different kinds of networks (e.g., neural and social networks) as special cases are quite general. For example, we consider the following  $n$ -neuron discrete-time neural network with discrete and distributed delays of the form:

$$u_i(k+1) = a_i u(k) + \sum_{j=1}^n b_{ij} \hat{f}_j(u_j(k)) + \sum_{j=1}^n c_{ij} \hat{g}_j(u_j(k-\tau(k))) + \sum_{j=1}^n d_{ij} \sum_{m=1}^{+\infty} \mu_m \hat{h}_j(u_j(k-m)) \quad (5)$$

or, in an equivalent vector form

$$u(k+1) = Au(k) + B\hat{F}(u(k)) + C\hat{G}(u(k-\tau(k))) + D \sum_{m=1}^{+\infty} \mu_m \hat{H}(u(k-m)) \quad (6)$$

where  $u(k) = (u_1(k), u_2(k), \dots, u_n(k))^T$  is the neural state vector,  $A = \text{diag}\{a_1, a_2, \dots, a_n\}$  with  $|a_i| < 1$  is the state feedback coefficient matrix, the  $n \times n$  matrices  $B = [b_{ij}]_{n \times n}$ ,  $C = [c_{ij}]_{n \times n}$ , and  $D = [d_{ij}]_{n \times n}$  are the connection weight matrix, the discretely delayed connection weight matrix, and distributively delayed connection weight matrix, respectively. The positive integer  $\tau(k)$  is the same as in (2). In (6),  $\hat{F}(u(k))$ ,  $\hat{G}(u(k))$ , and  $\hat{H}(u(k))$  denote the neuron activation functions.  $\mu_m$  ( $m = 1, 2, \dots$ ) are scalar constants. It is obvious to see that the neural network model (5) is just a subnetwork of an array of coupled neural networks described by (1).

In the literature concerning the synchronization problem for complex networks, the nonlinearities are usually assumed to satisfy the Lipschitz-type conditions. In the following, we adopt a more general sectorlike nonlinear function to describe the nonlinearities existing in the networks.

*Assumption 1:* The nonlinear vector-valued functions  $f$ ,  $g$ , and  $h$  are continuous and satisfy [20], [33]

$$[f(x) - f(y) - B_1(x-y)]^T [f(x) - f(y) - B_2(x-y)] \leq 0 \quad \forall x, y \in \mathbb{R}^n \quad (7)$$

$$[g(x) - g(y) - D_1(x-y)]^T [g(x) - g(y) - D_2(x-y)] \leq 0 \quad \forall x, y \in \mathbb{R}^n \quad (8)$$

$$[h(x) - h(y) - V_1(x-y)]^T [h(x) - h(y) - V_2(x-y)] \leq 0 \quad \forall x, y \in \mathbb{R}^n \quad (9)$$

where  $B_1, B_2, D_1, D_2, V_1$ , and  $V_2$  are constant matrices.

*Remark 4:* The conditions (7)–(9) are known as sector-like descriptions of the nonlinearities, which are in a more general form than the usual Lipschitz functions. By adopting such a presentation, it would be possible to reduce the conservatism of the main results caused by quantifying the nonlinear functions via an LMI technique.

For notation simplicity, we let

$$x(k) = (x_1^T(k), x_2^T(k), \dots, x_N^T(k))^T$$

$$F(x(k)) = (f^T(x_1(k)), f^T(x_2(k)), \dots, f^T(x_N(k)))^T$$

$$G(x(k)) = (g^T(x_1(k)), g^T(x_2(k)), \dots, g^T(x_N(k)))^T$$

$$H(x(k)) = (h^T(x_1(k)), h^T(x_2(k)), \dots, h^T(x_N(k)))^T.$$

With the matrix Kronecker product, we can rewrite the network (1) in the following compact form:

$$x(k+1) = F(x(k)) + G(x(k-\tau(k))) + \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + (W \otimes \Gamma)x(k). \quad (10)$$

*Definition 1:* The discrete-time complex network (1) or (10) is said to be globally synchronized if, for all addressed discrete and distributed delays, the following holds:

$$\lim_{k \rightarrow +\infty} |x_i(k) - x_j(k)| = 0, \quad 1 \leq i < j \leq N.$$

In the rest of this paper, we shall focus on the synchronization problem and the state estimation problem for the discrete-time complex network (10) with both discrete and distributed time delays. By utilizing new Lyapunov–Krasovskii functionals, we develop an LMI approach to derive sufficient conditions under which the discrete-time complex network (10) is globally synchronized, and then, we further extend the results obtained to design the desired state estimator for the same complex network through available network output.

### III. SYNCHRONIZATION OF DISCRETE-TIME COMPLEX NETWORK

In this section, let us deal with the synchronization problem for the complex network (10). First, we introduce several lemmas to be used in the sequel.

*Lemma 1:* Let  $X$  and  $Y$  be any  $n$ -dimensional real vectors, and let  $P$  be an  $n \times n$  positive semidefinite matrix. Then, the following matrix inequality holds:

$$2X^T P Y \leq X^T P X + Y^T P Y.$$

*Lemma 2:* Let  $M \in \mathbb{R}^{n \times n}$  be a positive semidefinite matrix,  $\mathbf{x}_i \in \mathbb{R}^n$ , and scalar constant  $a_i \geq 0$  ( $i = 1, 2, \dots$ ). If the series concerned is convergent, then the following inequality holds:

$$\left( \sum_{i=1}^{+\infty} a_i \mathbf{x}_i \right)^T M \left( \sum_{i=1}^{+\infty} a_i \mathbf{x}_i \right) \leq \left( \sum_{i=1}^{+\infty} a_i \right) \sum_{i=1}^{+\infty} a_i \mathbf{x}_i^T M \mathbf{x}_i. \quad (11)$$

*Proof:* Letting  $m$  be a positive integer, we have

$$\begin{aligned} \left( \sum_{i=1}^m a_i \mathbf{x}_i \right)^T M \left( \sum_{i=1}^m a_i \mathbf{x}_i \right) &= \left( \sum_{i=1}^m a_i \mathbf{x}_i \right)^T M \left( \sum_{j=1}^m a_j \mathbf{x}_j \right) \\ &= \sum_{i=1}^m \sum_{j=1}^m a_i a_j \mathbf{x}_i^T M \mathbf{x}_j \\ &\leq \sum_{i=1}^m \sum_{j=1}^m \frac{1}{2} a_i a_j (\mathbf{x}_i^T M \mathbf{x}_i + \mathbf{x}_j^T M \mathbf{x}_j) \\ &\quad (\text{By Lemma 1}) \\ &= \left( \sum_{i=1}^m a_i \right) \sum_{i=1}^m a_i \mathbf{x}_i^T M \mathbf{x}_i \end{aligned}$$

and then (11) follows directly by letting  $m \rightarrow +\infty$ , which completes the proof. ■

**Lemma 3:** (Schur Complement) Given constant matrices  $\Omega_1$ ,  $\Omega_2$ , and  $\Omega_3$ , where  $\Omega_1 = \Omega_1^T$  and  $\Omega_2 > 0$ , then

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

if only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0.$$

**Lemma 4:** Let  $\mathcal{U} = (\alpha_{ij})_{N \times N}$ ,  $P \in \mathbb{R}^{n \times n}$ ,  $x = (x_1^T, x_2^T, \dots, x_N^T)^T$ , and  $y = (y_1^T, y_2^T, \dots, y_N^T)^T$  with  $x_k, y_k \in \mathbb{R}^n (k = 1, 2, \dots, N)$ . If  $\mathcal{U} = \mathcal{U}^T$  and each row sum of  $\mathcal{U}$  is zero, then

$$x^T (\mathcal{U} \otimes P) y = - \sum_{1 \leq i < j \leq N} \alpha_{ij} (x_i - x_j)^T P (y_i - y_j).$$

**Lemma 5:** Let  $A = (a_{ij})_{m \times n}$ ,  $B = (b_{ij})_{n \times q}$ , and  $C = (c_{ij})_{m \times q} = AB$ . If the sum of all elements in each column of  $A$  (row of  $B$ ) is zero, then the sum of all elements in each column (row) of  $C$  is zero. Moreover, if  $A$  is a symmetric matrix and the sum of all elements in each row of  $A$  is zero, then, for any positive integer  $n$ , the sum of all elements in each row of  $A^n$  is zero.

For notation simplicity, we denote by  $w_{ij}^{(2)}$  the  $(i, j)$  entry of the matrix  $W^2$ . Now, we give our main result in this paper as follows.

**Theorem 1:** Under Assumption 1, the discrete-time complex network (10) is globally asymptotically synchronized if there

exist three scalar constants  $\delta_1 > 0$ ,  $\delta_2 > 0$ , and  $\delta_3 > 0$  and three positive definite matrices  $P, Q$ , and  $R$  such that the LMIs shown at the bottom of the page hold, where

$$\check{B}_1 = (B_1^T B_2 + B_2^T B_1)/2 \quad \check{B}_2 = (B_1^T + B_2^T)/2 \quad (13)$$

$$\check{D}_1 = (D_1^T D_2 + D_2^T D_1)/2 \quad \check{D}_2 = (D_1^T + D_2^T)/2 \quad (14)$$

$$\check{V}_1 = (V_1^T V_2 + V_2^T V_1)/2 \quad \check{V}_2 = (V_1^T + V_2^T)/2 \quad (15)$$

$$\Xi = (\tau_M - \tau_m + 1)Q - \delta_2 I \quad \bar{\mu} = \sum_{k=1}^{+\infty} \mu_k \quad (16)$$

$$\Theta_{ij} = -P - Nw_{ij}^{(2)} \Gamma P \Gamma - \delta_1 \check{B}_1 - \delta_2 \check{D}_1 - \delta_3 \check{V}_1. \quad (17)$$

*Proof:* Let  $\mathbf{x}_{ij}(k) = x_i(k) - x_j(k)$ ,  $\mathbf{f}_{ij}(k) = f(x_i(k)) - f(x_j(k))$ ,  $\mathbf{g}_{ij}(k) = g(x_i(k)) - g(x_j(k))$ ,  $\mathbf{h}_{ij}(k) = h(x_i(k)) - h(x_j(k))$ , and  $\hat{\mathbf{h}}_{ij}(k) = \sum_{m=1}^{+\infty} \mu_m h(x_i(k-m)) - \sum_{m=1}^{+\infty} \mu_m h(x_j(k-m))$ . From (7), it follows readily that

$$\begin{aligned} &\begin{bmatrix} x_i(k) - x_j(k) \\ f(x_i(k)) - f(x_j(k)) \end{bmatrix}^T \\ &\times \begin{bmatrix} (B_1^T B_2 + B_2^T B_1)/2 & -(B_1^T + B_2^T)/2 \\ -(B_1 + B_2)/2 & I \end{bmatrix} \\ &\times \begin{bmatrix} x_i(k) - x_j(k) \\ f(x_i(k)) - f(x_j(k)) \end{bmatrix} \leq 0 \end{aligned}$$

namely

$$\begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{f}_{ij}(k) \end{bmatrix}^T \begin{bmatrix} \check{B}_1 & -\check{B}_2 \\ -\check{B}_2^T & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{f}_{ij}(k) \end{bmatrix} \leq 0. \quad (18)$$

Similarly, from (8) and (9), we have

$$\begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{g}_{ij}(k) \end{bmatrix}^T \begin{bmatrix} \check{D}_1 & -\check{D}_2 \\ -\check{D}_2^T & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{g}_{ij}(k) \end{bmatrix} \leq 0 \quad (19)$$

$$\begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{h}_{ij}(k) \end{bmatrix}^T \begin{bmatrix} \check{V}_1 & -\check{V}_2 \\ -\check{V}_2^T & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{h}_{ij}(k) \end{bmatrix} \leq 0. \quad (20)$$

To deal with the synchronization of the network (10), we introduce the following Lyapunov–Krasovskii functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) \quad (21)$$

$$\Phi_{ij} = \begin{bmatrix} \Theta_{ij} & -Nw_{ij}\Gamma P + \delta_1 \check{B}_2 & \delta_2 \check{D}_2 & -Nw_{ij}\Gamma P & \delta_3 \check{V}_2 & -Nw_{ij}\Gamma P \\ * & P - \delta_1 I & 0 & P & 0 & P \\ * & * & \Xi & 0 & 0 & 0 \\ * & * & * & P - Q & 0 & P \\ * & * & * & * & \bar{\mu}R - \delta_3 I & 0 \\ * & * & * & * & * & P - \frac{1}{\bar{\mu}}R \end{bmatrix} < 0, \quad 1 \leq i < j \leq N \quad (12)$$

where

$$V_1(k) = x^T(k)(U \otimes P)x(k) \quad (22)$$

$$V_2(k) = \sum_{i=k-\tau(k)}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \quad (23)$$

$$V_3(k) = \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \quad (24)$$

$$V_4(k) = \sum_{i=1}^{+\infty} \mu_i \sum_{j=k-i}^{k-1} H^T(x(j))(U \otimes R)H(x(j)) \quad (25)$$

with

$$U = \begin{bmatrix} N-1 & -1 & \cdots & -1 \\ -1 & N-1 & \cdots & -1 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & N-1 \end{bmatrix}_{N \times N}.$$

Notice that from the condition (3),  $V_4(k)$  is convergent. Calculating the difference of  $V(k)$  along the system (10), we have

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) \quad (26)$$

where

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= \left( F(x(k)) + G(x(k-\tau(k))) \right. \\ &\quad \left. + \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + (W \otimes \Gamma)x(k) \right)^T \\ &\quad \times (U \otimes P) \left( F(x(k)) + G(x(k-\tau(k))) \right. \\ &\quad \left. + \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + (W \otimes \Gamma)x(k) \right) \\ &\quad - x^T(k)(U \otimes P)x(k) \\ &= F^T(x(k))(U \otimes P)F(x(k)) + G^T(x(k-\tau(k))) \\ &\quad \times (U \otimes P)G(x(k-\tau(k))) \\ &\quad + \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right)^T \\ &\quad \times (U \otimes P) \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \\ &\quad + x^T(k)(W \otimes \Gamma)^T(U \otimes P) \\ &\quad \times (W \otimes \Gamma)x(k) + 2F^T(x(k))(U \otimes P) \\ &\quad \times G(x(k-\tau(k))) + 2F^T(x(k))(U \otimes P) \\ &\quad \times \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + 2F^T(x(k))(U \otimes P) \\ &\quad \times (W \otimes \Gamma)x(k) + 2(G(x(k-\tau(k))))^T \\ &\quad \times (U \otimes P) \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \end{aligned}$$

$$\begin{aligned} &+ 2(G(x(k-\tau(k))))^T (U \otimes P)(W \otimes \Gamma)x(k) \\ &+ 2 \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right)^T \\ &\times (U \otimes P)(W \otimes \Gamma)x(k) - x^T(k)(U \otimes P)x(k) \end{aligned} \quad (27)$$

$$\begin{aligned} \Delta V_2(k) &= V_2(k+1) - V_2(k) \\ &= \sum_{i=k+1-\tau(k+1)}^k G^T(x(i))(U \otimes Q)G(x(i)) \\ &\quad - \sum_{i=k-\tau(k)}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &= G^T(x(k))(U \otimes Q)G(x(k)) \\ &\quad - G^T(x(k-\tau(k)))(U \otimes Q)G(x(k-\tau(k))) \\ &\quad + \sum_{i=k-\tau(k+1)+1}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &\quad - \sum_{i=k-\tau(k)+1}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &= G^T(x(k))(U \otimes Q)G(x(k)) \\ &\quad - G^T(x(k-\tau(k)))(U \otimes Q)G(x(k-\tau(k))) \\ &\quad + \sum_{i=k-\tau_m+1}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &\quad + \sum_{i=k-\tau(k+1)+1}^{k-\tau_m} G^T(x(i))(U \otimes Q)G(x(i)) \\ &\quad - \sum_{i=k-\tau(k)+1}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &\leq G^T(x(k))(U \otimes Q)G(x(k)) \\ &\quad - G^T(x(k-\tau(k)))(U \otimes Q)G(x(k-\tau(k))) \\ &\quad + \sum_{i=k-\tau_M+1}^{k-\tau_m} G^T(x(i))(U \otimes Q)G(x(i)) \end{aligned} \quad (28)$$

$$\begin{aligned} \Delta V_3(k) &= V_3(k+1) - V_3(k) \\ &= \sum_{j=k-\tau_M+2}^{k-\tau_m+1} \sum_{i=j}^k G^T(x(i))(U \otimes Q)G(x(i)) \\ &\quad - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &= \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j+1}^k G^T(x(i))(U \otimes Q)G(x(i)) \\ &\quad - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} G^T(x(i))(U \otimes Q)G(x(i)) \\ &= \sum_{j=k-\tau_M+1}^{k-\tau_m} (G^T(x(k))(U \otimes Q)G(x(k)) \\ &\quad - G^T(x(j))(U \otimes Q)G(x(j))) \end{aligned}$$

$$\begin{aligned}
&= (\tau_M - \tau_m) G^T(x(k)) (U \otimes Q) G(x(k)) \\
&\quad - \sum_{i=k-\tau_M+1}^{k-\tau_m} G^T(x(i)) (U \otimes Q) G(x(i)) \quad (29)
\end{aligned}$$

$$\begin{aligned}
\Delta V_4(k) &= V_4(k+1) - V_4(k) \\
&= \sum_{i=1}^{+\infty} \mu_i \sum_{j=k+1-i}^k H^T(x(j)) (U \otimes R) H(x(j)) \\
&\quad - \sum_{i=1}^{+\infty} \mu_i \sum_{j=k-i}^{k-1} H^T(x(j)) (U \otimes R) H(x(j)) \\
&= \sum_{i=1}^{+\infty} \mu_i \left( H^T(x(k)) (U \otimes R) H(x(k)) \right. \\
&\quad \left. - H^T(x(k-i)) (U \otimes R) H(x(k-i)) \right) \\
&= \sum_{i=1}^{+\infty} \mu_i H^T(x(k)) (U \otimes R) H(x(k)) \\
&\quad - \sum_{i=1}^{+\infty} \mu_i H^T(x(k-i)) (U \otimes R) H(x(k-i)) \\
&\quad \text{(By Lemma 2)} \\
&\leq \bar{\mu} H^T(x(k)) (U \otimes R) H(x(k)) \\
&\quad - \frac{1}{\bar{\mu}} \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right)^T (U \otimes R) \\
&\quad \times \sum_{m=1}^{+\infty} \mu_m H(x(k-m)). \quad (30)
\end{aligned}$$

In view of

$$\begin{aligned}
(W \otimes \Gamma)^T (U \otimes P) (W \otimes \Gamma) &= (W^T \otimes \Gamma^T) (U \otimes P) (W \otimes \Gamma) \\
&= (W^T U W) \otimes (\Gamma^T P \Gamma) \\
&= N W^2 \otimes (\Gamma P \Gamma) \\
(U \otimes P) (W \otimes \Gamma) &= (U W) \otimes (P \Gamma) \\
&= N W \otimes (P \Gamma)
\end{aligned}$$

we substitute (27)–(30) into (26) and obtain

$$\begin{aligned}
\Delta V(k) &\leq F^T(x(k)) (U \otimes P) F(x(k)) \\
&\quad + G^T(x(k-\tau(k))) (U \otimes P) G(x(k-\tau(k))) \\
&\quad + \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right)^T (U \otimes P) \\
&\quad \times \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \\
&\quad + x^T(k) (N W^2 \otimes (\Gamma P \Gamma)) x(k) \\
&\quad + 2 F^T(x(k)) (U \otimes P) G(x(k-\tau(k))) \\
&\quad + 2 F^T(x(k)) (U \otimes P)
\end{aligned}$$

$$\begin{aligned}
&\times \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \\
&\quad + 2 F^T(x(k)) (N W \otimes P \Gamma) x(k) \\
&\quad + 2 G^T(x(k-\tau(k))) (U \otimes P) \\
&\quad \times \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \\
&\quad + 2 G^T(x(k-\tau(k))) (N W \otimes P \Gamma) x(k) \\
&\quad + \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right)^T (N W \otimes P \Gamma) x(k) \\
&\quad - x^T(k) (U \otimes P) x(k) \\
&\quad + (1 + \tau_M - \tau_m) G^T(x(k)) (U \otimes Q) G(x(k)) \\
&\quad - G^T(x(k-\tau(k))) (U \otimes Q) G(x(k-\tau(k))) \\
&\quad + \bar{\mu} H^T(x(k)) (U \otimes R) H(x(k)) \\
&\quad - \frac{1}{\bar{\mu}} \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right)^T (U \otimes R) \\
&\quad \times \left( \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \right). \quad (31)
\end{aligned}$$

It follows from Lemma 4, Lemma 5, and (31) that

$$\begin{aligned}
\Delta V(k) &\leq \sum_{1 \leq i < j \leq N} \left\{ -\mathbf{x}_{ij}^T P \mathbf{x}_{ij} - \mathbf{x}_{ij}^T \left( N w_{ij}^{(2)} \Gamma P \Gamma \right) \mathbf{x}_{ij} \right. \\
&\quad + \mathbf{f}_{ij}^T(k) P \mathbf{f}_{ij}(k) + \mathbf{g}_{ij}^T(k-\tau(k)) \\
&\quad \times P \mathbf{g}_{ij}(k-\tau(k)) + \hat{\mathbf{h}}_{ij}^T(k) P \hat{\mathbf{h}}_{ij}(k) \\
&\quad + 2 \mathbf{f}_{ij}^T(k) P \mathbf{g}_{ij}(k-\tau(k)) \\
&\quad + 2 \mathbf{f}_{ij}^T(k) P \hat{\mathbf{h}}_{ij}(k) - 2 \mathbf{f}_{ij}^T(k) \\
&\quad \times (N w_{ij} \Gamma P \Gamma) \mathbf{x}_{ij} + 2 \mathbf{g}_{ij}^T(k-\tau(k)) \\
&\quad \times P \hat{\mathbf{h}}_{ij}(k) - 2 \mathbf{g}_{ij}^T(k-\tau(k)) \\
&\quad \times (N w_{ij} \Gamma P \Gamma) \mathbf{x}_{ij}(k) - \hat{\mathbf{h}}_{ij}^T(k) \\
&\quad \times (N w_{ij} \Gamma P \Gamma) \mathbf{x}_{ij}(k) + (1 + \tau_M - \tau_m) \\
&\quad \times \mathbf{g}_{ij}^T(k) Q \mathbf{g}_{ij}(k) - \mathbf{g}_{ij}^T(k-\tau(k)) \\
&\quad \times Q \mathbf{g}_{ij}(k-\tau(k)) + \bar{\mu} \mathbf{h}_{ij}^T(k) R \mathbf{h}_{ij}(k) \\
&\quad \left. - \frac{1}{\bar{\mu}} \hat{\mathbf{h}}_{ij}^T(k) R \hat{\mathbf{h}}_{ij}(k) \right\} \\
&= \sum_{1 \leq i < j \leq N} \xi_{ij}^T(k) \Phi_{ij}^{(1)} \xi_{ij}(k) \quad (32)
\end{aligned}$$

where the expressions for  $\xi_{ij}(k)$  and  $\Phi_{ij}^{(1)}$  are shown at the bottom of the next page.

Therefore, from (32) along with (18)–(20), we obtain

$$\begin{aligned} \Delta V(k) &\leq \sum_{1 \leq i < j \leq N} \left\{ \xi_{ij}^T(k) \Phi_{ij}^{(1)} \xi_{ij}(k) - \delta_1 \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{f}_{ij}(k) \end{bmatrix}^T \right. \\ &\quad \times \begin{bmatrix} \check{B}_1 & -\check{B}_2 \\ -\check{B}_2^T & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{f}_{ij}(k) \end{bmatrix} \\ &\quad - \delta_2 \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{g}_{ij}(k) \end{bmatrix}^T \begin{bmatrix} \check{D}_1 & -\check{D}_2 \\ -\check{D}_2^T & I \end{bmatrix} \\ &\quad \times \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{g}_{ij}(k) \end{bmatrix} - \delta_3 \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{h}_{ij}(k) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} \check{V}_1 & -\check{V}_2 \\ -\check{V}_2^T & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{h}_{ij}(k) \end{bmatrix} \Big\} \\ &= \sum_{1 \leq i < j \leq N} \xi_{ij}^T(k) \Phi_{ij} \xi_{ij}(k) \\ &\leq \sum_{1 \leq i < j \leq N} \lambda_{\max}(\Phi_{ij}) |\xi_{ij}|^2. \end{aligned} \quad (33)$$

Noticing that  $\lambda_{\max}(\Phi_{ij}) < 0$  and letting  $\lambda_0 = \max_{1 \leq i < j \leq N} \{\lambda_{\max}(\Phi_{ij})\}$ , we have  $\lambda_0 < 0$ , and then, it follows readily from (33) that

$$\Delta V(k) \leq \lambda_0 \sum_{1 \leq i < j \leq N} |\mathbf{x}_{ij}(k)|^2. \quad (34)$$

Letting  $m$  be a positive integer, one has from (34)

$$V(m+1) - V(1) = \sum_{k=1}^m \Delta V(k) = \lambda_0 \sum_{1 \leq i < j \leq N} \sum_{k=1}^m |\mathbf{x}_{ij}(k)|^2$$

which implies that

$$-\lambda_0 \sum_{1 \leq i < j \leq N} \sum_{k=1}^m |\mathbf{x}_{ij}(k)|^2 \leq V(1).$$

By letting  $m \rightarrow +\infty$ , we can deduce that the series  $\sum_{k=1}^{+\infty} |\mathbf{x}_{ij}(k)|^2$  is convergent for  $1 \leq i < j \leq N$ , and therefore,  $|\mathbf{x}_{ij}(k)|^2 \rightarrow 0$ , namely

$$\lim_{k \rightarrow +\infty} |x_i(k) - x_j(k)| = 0$$

which completes the proof of Theorem 1.  $\blacksquare$

The complex network (10) is a quite general model that includes both the discrete and distributed time delays in the

discrete-time domain. For example, if we consider the discrete time delay only, i.e.,  $h = 0$ , then, the complex network (10) reduces to

$$x(k+1) = F(x(t)) + G(x(k-\tau(k))) + (W \otimes \Gamma)x(k). \quad (35)$$

For the complex network (35), it is straightforward to have the following synchronization result from Theorem 1.

*Corollary 1:* Suppose that the conditions (7) and (8) hold in Assumption 1. The discrete complex network (35) is synchronized if there exist two scalar constants  $\delta_1$  and  $\delta_2$  and two positive definite matrices  $P$  and  $Q$  such that the following LMIs hold:

$$\Phi_{ij} = \begin{bmatrix} \Pi_{ij} & -Nw_{ij}\Gamma P + \delta_1 \check{B}_2 & \delta_2 \check{D}_2 & -Nw_{ij}\Gamma P \\ * & P - \delta_1 I & 0 & P \\ * & * & \Xi & 0 \\ * & * & * & P - Q \end{bmatrix} < 0 \quad 1 \leq i < j \leq N \quad (36)$$

where  $\Pi_{ij} = -P - Nw_{ij}^{(2)}\Gamma P\Gamma - \delta_1 \check{B}_1 - \delta_2 \check{D}_1$  and  $\check{B}_1, \check{B}_2, \check{D}_1, \check{D}_2$ , and  $\Xi$  are defined in Theorem 1.

*Remark 5:* In Theorem 1, the synchronization problem is studied for an array of discrete-time neural networks with mixed time delays. Note that the obtained criteria in (12) are dependent not only on the upper and lower bounds of the discrete time delays but also on the distributed delays. Therefore, the criteria are less conservative than the traditional delay-independent ones. Also, the LMI-based criteria can be checked efficiently via the Matlab LMI Toolbox.

#### IV. STATE ESTIMATION OF COMPLEX NETWORKS

As discussed in Section I, for relatively high-order and large-scale complex networks, sometimes, we can only know the partial information about the states of the key network nodes from the network outputs (measurements). Therefore, in order to make use of the networks in practice, it becomes necessary to estimate the node states through available network output.

Suppose that the output from the  $i$ th node of the complex network (10) is of the form

$$y_i(k) = C_i x_i(k), \quad i = 1, 2, \dots, N \quad (37)$$

where  $y_i(k) = (y_{i1}(k), y_{i2}(k), \dots, y_{im}(k)) \in \mathbb{R}^m$  is the measurement output of the  $i$ th node and  $C_i \in \mathbb{R}^{m \times n}$  is a known constant matrix.

$$\begin{aligned} \xi_{ij}(k) &= [\mathbf{x}_{ij}^T(k) \quad \mathbf{f}_{ij}^T(k) \quad \mathbf{g}_{ij}^T(k) \quad \mathbf{g}_{ij}^T(k-\tau(k)) \quad \mathbf{h}_{ij}^T(k) \quad \hat{\mathbf{h}}_{ij}^T(k)]^T \\ \Phi_{ij}^{(1)} &= \begin{bmatrix} -P - Nw_{ij}^{(2)}\Gamma P\Gamma & -Nw_{ij}\Gamma P & 0 & -Nw_{ij}\Gamma P & 0 & -Nw_{ij}\Gamma P \\ * & P & 0 & P & 0 & P \\ * & * & (\tau_M - \tau_m + 1)Q & 0 & 0 & 0 \\ * & * & * & P - Q & 0 & P \\ * & * & * & * & \bar{\mu}R & 0 \\ * & * & * & * & * & P - \frac{1}{\bar{\mu}}R \end{bmatrix} \end{aligned}$$

*Remark 6:* Usually, we have  $m < n$  which means that the network output is a linear combination (although partial) of the information about the network nodes. We wish to design an easy-to-implement estimator/observer to estimate the network states through the available network output.

In order to estimate the states of the complex network (10), we construct the following state estimator:

$$\begin{aligned} \hat{x}(k+1) = & F(\hat{x}(k)) + G(\hat{x}(k-\tau(k))) + \sum_{m=1}^{+\infty} \mu_m H(\hat{x}(k-m)) \\ & + (W \otimes \Gamma)\hat{x}(k) + K[y(k) - C\hat{x}(k)] \end{aligned} \quad (38)$$

where

$$y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_N(k) \end{bmatrix} \quad C = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_N \end{bmatrix}$$

$$K = \begin{bmatrix} K_1 & & & \\ & K_2 & & \\ & & \ddots & \\ & & & K_N \end{bmatrix} \quad (K_i \in \mathbb{R}^{n \times m})$$

with  $K$  being the estimator gain matrix. Our goal hereafter is to choose a suitable  $K_i$  such that  $\hat{x}(k)$  asymptotically approaches  $x(k)$ .

To this end, we let  $\varepsilon(k) = (\varepsilon_1^T(k), \varepsilon_2^T(k), \dots, \varepsilon_N^T(k))^T := \hat{x}(k) - x(k)$  with  $\varepsilon_i(k) = \hat{x}_i(k) - x_i(k)$  being the state estimator error and denote

$$\begin{aligned} \hat{F}(\varepsilon(k)) = & \left[ \hat{f}^T(\varepsilon_1(k)), \hat{f}^T(\varepsilon_2(k)), \dots, \hat{f}^T(\varepsilon_N(k)) \right]^T \\ := & F(\hat{x}(k)) - F(x(k)) \end{aligned} \quad (39)$$

$$\begin{aligned} \hat{G}(\varepsilon(k)) = & \left[ \hat{g}^T(\varepsilon_1(k)), \hat{g}^T(\varepsilon_2(k)), \dots, \hat{g}^T(\varepsilon_N(k)) \right]^T \\ := & G(\hat{x}(k)) - G(x(k)) \end{aligned} \quad (40)$$

$$\begin{aligned} \hat{H}(\varepsilon(k)) = & \left[ \hat{h}^T(\varepsilon_1(k)), \hat{h}^T(\varepsilon_2(k)), \dots, \hat{h}^T(\varepsilon_N(k)) \right]^T \\ := & H(\hat{x}(k)) - G(x(k)). \end{aligned} \quad (41)$$

Then, from (10) and (38), we obtain the following system governing the state error dynamics:

$$\begin{aligned} \varepsilon(k+1) = & -KC\varepsilon(k) + \hat{F}(\varepsilon(k)) + \hat{G}(\varepsilon(k-\tau(k))) \\ & + \sum_{m=1}^{+\infty} \mu_m \hat{H}(\varepsilon(k-m)) + (W \otimes \Gamma)\varepsilon(k). \end{aligned} \quad (42)$$

It is easy to verify that

$$\begin{bmatrix} \varepsilon \\ \hat{F}(\varepsilon) \end{bmatrix}^T \begin{bmatrix} \check{\mathbf{B}}_1 & -\check{\mathbf{B}}_2 \\ -\check{\mathbf{B}}_2^T & I \end{bmatrix} \begin{bmatrix} \varepsilon \\ \hat{F}(\varepsilon) \end{bmatrix} \leq 0 \quad \forall \varepsilon \in \mathbb{R}^{n \times N} \quad (43)$$

$$\begin{bmatrix} \varepsilon \\ \hat{G}(\varepsilon) \end{bmatrix}^T \begin{bmatrix} \check{\mathbf{D}}_1 & -\check{\mathbf{D}}_2 \\ -\check{\mathbf{D}}_2^T & I \end{bmatrix} \begin{bmatrix} \varepsilon \\ \hat{G}(\varepsilon) \end{bmatrix} \leq 0 \quad \forall \varepsilon \in \mathbb{R}^{n \times N} \quad (44)$$

$$\begin{bmatrix} \varepsilon \\ \hat{H}(\varepsilon) \end{bmatrix}^T \begin{bmatrix} \check{\mathbf{V}}_1 & -\check{\mathbf{V}}_2 \\ -\check{\mathbf{V}}_2^T & I \end{bmatrix} \begin{bmatrix} \varepsilon \\ \hat{H}(\varepsilon) \end{bmatrix} \leq 0 \quad \forall \varepsilon \in \mathbb{R}^{n \times N} \quad (45)$$

where

$$\begin{aligned} \check{\mathbf{B}}_1 = & \text{diag}\{\overbrace{\check{B}_1, \check{B}_1, \dots, \check{B}_1}^N\} & \check{\mathbf{B}}_2 = & \text{diag}\{\overbrace{\check{B}_2, \check{B}_2, \dots, \check{B}_2}^N\} \\ \check{\mathbf{D}}_1 = & \text{diag}\{\overbrace{\check{D}_1, \check{D}_1, \dots, \check{D}_1}^N\} & \check{\mathbf{D}}_2 = & \text{diag}\{\overbrace{\check{D}_2, \check{D}_2, \dots, \check{D}_2}^N\} \\ \check{\mathbf{V}}_1 = & \text{diag}\{\overbrace{\check{V}_1, \check{V}_1, \dots, \check{V}_1}^N\} & \check{\mathbf{V}}_2 = & \text{diag}\{\overbrace{\check{V}_2, \check{V}_2, \dots, \check{V}_2}^N\}. \end{aligned}$$

By following a similar line as in the proof of Theorem 1, we can obtain the following result without proof.

*Theorem 2:* Let  $K$  be a given constant matrix. Then, under Assumption 1, the error system (42) is globally asymptotically stable if there exist three scalar constants  $\delta_1 > 0$ ,  $\delta_2 > 0$ , and  $\delta_3 > 0$  and three positive definite diagonal block matrices  $\mathbf{P} = \text{diag}\{P_1, P_2, \dots, P_N\}$ ,  $\mathbf{Q} = \text{diag}\{Q_1, Q_2, \dots, Q_N\}$ , and  $\mathbf{R} = \text{diag}\{R_1, R_2, \dots, R_N\}$  such that the LMI holds

$$\begin{bmatrix} \mathbf{Z}_1 & \mathbf{S}^T \mathbf{P} + \delta_1 \check{\mathbf{B}}_2 & \delta_2 \check{\mathbf{D}}_2 & \mathbf{S}^T \mathbf{P} & \delta_3 \check{\mathbf{V}}_2 & \mathbf{S}^T \mathbf{P} \\ * & \mathbf{P} - \delta_1 I & 0 & \mathbf{P} & 0 & \mathbf{P} \\ * & * & \mathbf{Z}_2 & 0 & 0 & 0 \\ * & * & * & \mathbf{P} - \mathbf{Q} & 0 & \mathbf{P} \\ * & * & * & * & \bar{\mu} \mathbf{R} - \delta_3 I & 0 \\ * & * & * & * & * & \mathbf{P} - \bar{\mu}^{-1} \mathbf{R} \end{bmatrix} < 0$$

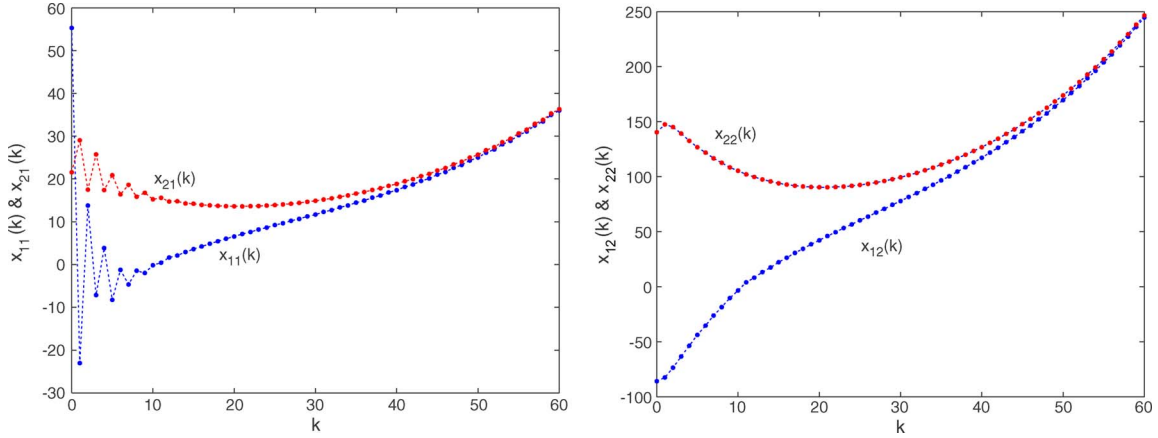
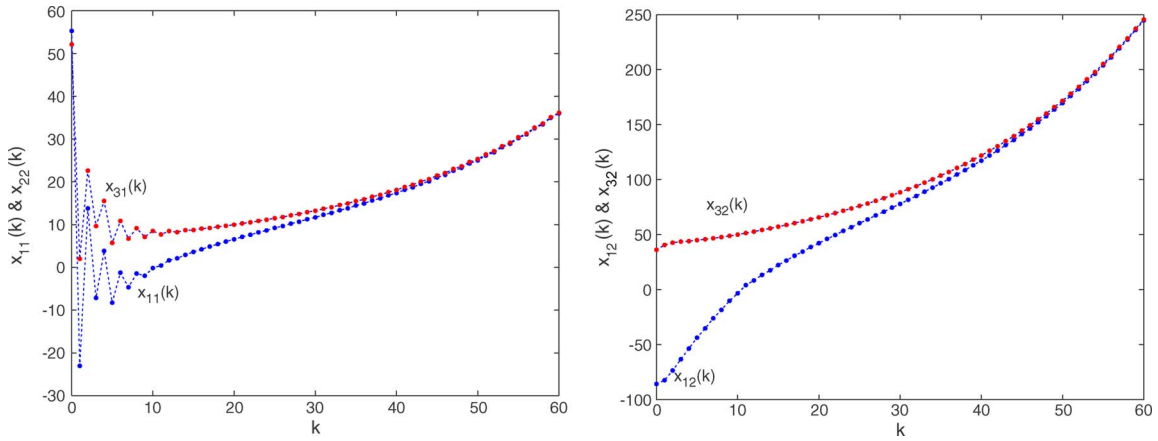
where  $\mathbf{S} = W \otimes \Gamma - KC$ ,  $\mathbf{Z}_1 = \mathbf{S}^T \mathbf{P} \mathbf{S} - \mathbf{P} - \delta_1 \check{\mathbf{B}}_1 - \delta_2 \check{\mathbf{D}}_1 - \delta_3 \check{\mathbf{V}}_1$ , and  $\mathbf{Z}_2 = (\tau_M - \tau_m + 1)\mathbf{Q} - \delta_2 I$ .

Having obtained the analysis results in Theorem 2, we are now ready to consider the design problem of the state estimator (10). From Theorem 2 and Lemma 3, the following result can be derived easily.

*Theorem 3:* Under Assumption 1, the system (38) becomes a state estimator of the discrete-time complex network (10) if there exist three scalar constants  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$  and three positive definite diagonal block matrices  $\mathbf{P} = \text{diag}\{P_1, P_2, \dots, P_N\}$ ,  $\mathbf{Q} = \text{diag}\{Q_1, Q_2, \dots, Q_N\}$ , and  $\mathbf{R} = \text{diag}\{R_1, R_2, \dots, R_N\}$  such that the LMI shown at the bottom of the page holds, where  $\hat{\mathbf{W}} = W \otimes \Gamma$ ,  $\mathbf{Z}_0 = -\mathbf{P} - \delta_1 \check{\mathbf{B}}_1 - \delta_2 \check{\mathbf{D}}_1 - \delta_3 \check{\mathbf{V}}_1$ ,

$$\begin{bmatrix} \mathbf{Z}_0 & \mathbf{Z}_3 & \delta_2 \check{\mathbf{D}}_2 & \hat{\mathbf{W}}^T \mathbf{P} - C^T \mathbf{Y}^T & \delta_3 \check{\mathbf{V}}_2 & \hat{\mathbf{W}}^T \mathbf{P} - C^T \mathbf{Y}^T & \hat{\mathbf{W}}^T \mathbf{P} - C^T \mathbf{Y}^T \\ * & \mathbf{P} - \delta_1 I & 0 & \mathbf{P} & 0 & \mathbf{P} & 0 \\ * & * & \mathbf{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & \mathbf{P} - \mathbf{Q} & 0 & \mathbf{P} & 0 \\ * & * & * & * & \bar{\mu} \mathbf{R} - \delta_3 I & 0 & 0 \\ * & * & * & * & * & \mathbf{P} - \bar{\mu}^{-1} \mathbf{R} & 0 \\ * & * & * & * & * & * & -\mathbf{P} \end{bmatrix} < 0 \quad (46)$$



Fig. 1. Comparison of state trajectories of  $x_1(k)$  and  $x_2(k)$ .Fig. 2. Comparison of state trajectories of  $x_1(k)$  and  $x_3(k)$ .

$\mathbf{Z}_3 = \hat{\mathbf{W}}^T \mathbf{P} - \mathbf{C}^T \mathbf{Y}^T + \delta_1 \check{\mathbf{B}}_2$ , and  $\mathbf{Z}_2$  is defined as in Theorem 2. In this case, the estimator gain matrix  $\mathbf{K}$  can be chosen as  $\mathbf{K} = \mathbf{P}^{-1} \mathbf{Y}$ .

## V. NUMERICAL SIMULATION

In this section, two numerical examples are presented to demonstrate the usefulness of the developed designs on the synchronization as well as state estimation problems for the complex network (10). In order to show the validity of our results, we select an unstable complex network on which the proposed synchronization and estimation schemes still work well.

*Example 1:* For simplicity, let us consider the system (10) of three nodes. Suppose that

$$n = 2 \quad \tau(k) = 3 + (1 + (-1)^k) / 2 \quad \mu_m = 2^{-(m+3)}$$

$$\Gamma = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} \quad w_{ij} = \begin{cases} 0.1, & i \neq j \\ -0.2, & i = j. \end{cases}$$

Let the nonlinear vector-valued functions be given by

$$f(x_i(k)) = (-0.5x_{i1}(k) + \tanh(0.2x_{i1}(k)) \\ + 0.2x_{i2}(k), 0.95x_{i2}(k) \\ - \tanh(0.75x_{i2}(k)))^T, \quad i = 1, 2, 3$$

$$g(x_i(k)) = h(x_i(k)) \\ = (0.2x_{i1}(k) - \tanh(0.1x_{i1}(k)), 0.1x_{i2}(k))^T \\ i = 1, 2, 3.$$

Then, it can be verified that  $\tau_m = 3$ ,  $\tau_M = 4$ ,  $\bar{\mu} = 1/8$ , and

$$B_1 = \begin{pmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{pmatrix} \quad B_2 = \begin{pmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{pmatrix}$$

$$D_1 = V_1 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix} \quad D_2 = V_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}.$$

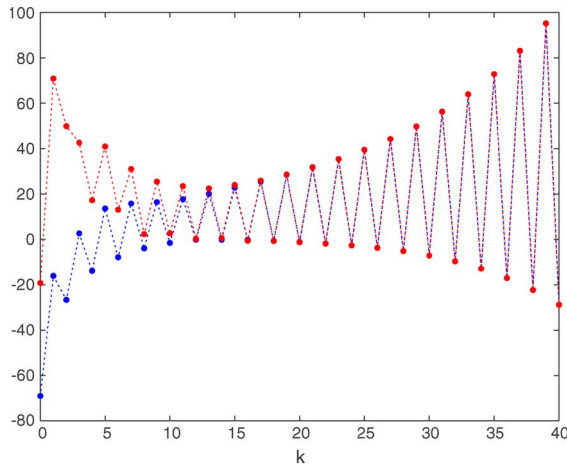
By using the Matlab LMI Toolbox, we solve LMI (12) and obtain a feasible solution as follows:

$$P = \begin{pmatrix} 1.5314 & -0.2020 \\ -0.2020 & 2.9295 \end{pmatrix} \quad Q = \begin{pmatrix} 6.1702 & -0.7353 \\ -0.7353 & 13.2959 \end{pmatrix}$$

$$R = \begin{pmatrix} 5.5543 & -0.4468 \\ -0.4468 & 9.8905 \end{pmatrix}$$

$$\delta_1 = 8.6589 \quad \delta_2 = 73.7843 \quad \delta_3 = 16.4907.$$

Then, it follows from Theorem 1 that the system (10) with given parameters reaches synchronization, which is further verified by the simulation result shown in Figs. 1 and 2. Fig. 1 shows that the state of  $x_2(k)$  asymptotically approaches that of  $x_1(k)$ , whereas Fig. 2 shows that the state of  $x_3(k)$  asymptotically tends to that of  $x_1(k)$ .


 Fig. 3. State trajectories of  $x_{11}$  and  $\hat{x}_{11}$ .

**Example 2:** Consider the system (10) of three nodes. Suppose that

$$n = 3 \quad \tau(k) = 3 + (1 + (-1)^k) / 2 \quad \mu_m = 2^{-(m+1)}$$

$$\Gamma = I_3 \quad w_{ij} = \begin{cases} 0.1, & i \neq j \\ -0.2, & i = j. \end{cases} \quad C_i = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Let the nonlinear vector-valued functions be given by

$$f(x_i(k)) = \begin{pmatrix} -0.8x_{i1}(k) + \tanh(0.5x_{i1}(k)) + 0.4x_{i2}(k), \\ 0.9x_{i2}(k) - \tanh(0.6x_{i2}(k)), \\ 0.6x_{i3} - \tanh(0.4x_{i3}(k)) \end{pmatrix}^T, \quad i = 1, 2, 3$$

$$g(x_i(k)) = h(x_i(k)) = \begin{pmatrix} 0.2x_{i1}(k) - \tanh(0.1x_{i1}(k)), \\ 0.3x_{i2}(k) - \tanh(0.2x_{i2}(k)), \\ 0.3x_{i3}(k) - \tanh(0.2x_{i3}(k)) \end{pmatrix}^T, \quad i = 1, 2, 3.$$

It can also be checked that  $\tau_m = 3$ ,  $\tau_M = 4$ ,  $\bar{\mu} = 1/2$ , and

$$B_1 = \begin{pmatrix} -0.8 & 0.4 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.6 \end{pmatrix} \quad B_2 = \begin{pmatrix} -0.3 & 0.4 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{pmatrix}$$

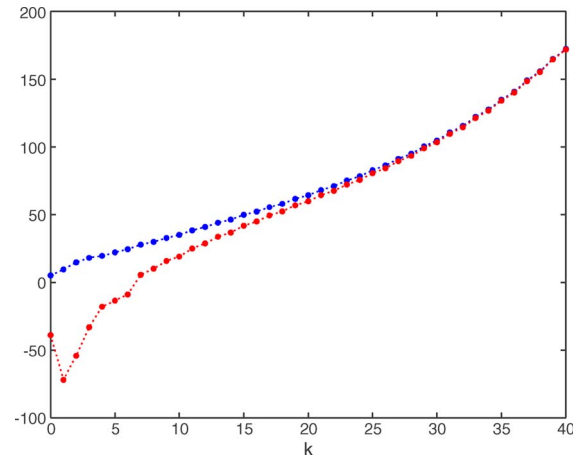
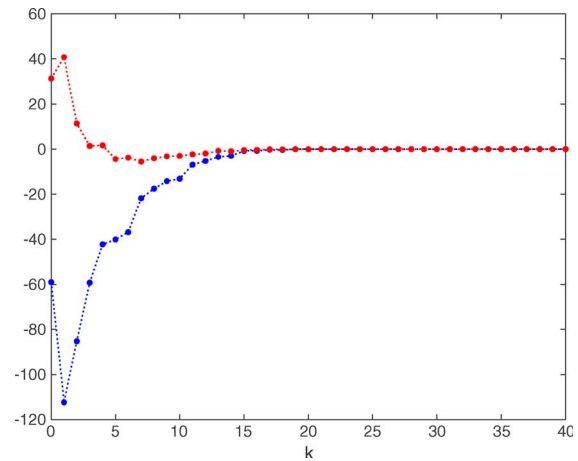
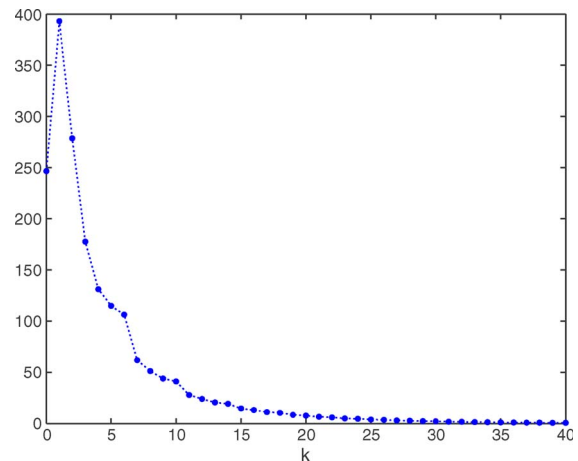
$$D_1 = V_1 = \text{diag}\{0.2, 0.3, 0.3\}$$

$$D_2 = V_2 = \text{diag}\{0.1, 0.1, 0.1\}.$$

By using the Matlab LMI Toolbox, the LMI (46) can be solved and the estimator gain matrices are given by

$$K_i = \begin{pmatrix} -0.4958 & 0.4003 \\ 0.0001 & 0.3405 \\ 0.1114 & 0.0002 \end{pmatrix}, \quad (i = 1, 2, 3).$$

Then, according to Theorem 3, the system (38) becomes a state estimator of the discrete-time complex network (10). That is, the state of the system (38) asymptotically approaches that of (10). The numerical simulation perfectly supports the theoretical results. Specifically, in Figs. 3–6, we show the evolution of the state  $x_1(k)$  and its estimator  $\hat{x}_1(k)$  of node 1, as well as the magnitude  $|\varepsilon(k)|$  of the estimate error for the whole complex network. It is noticed from Fig. 6 that the magnitude  $|\varepsilon(k)|$  of the error between the states of the whole network and its estimator approaches zero asymptotically.


 Fig. 4. State trajectories of  $x_{12}$  and  $\hat{x}_{12}$ .

 Fig. 5. State trajectories of  $x_{13}$  and  $\hat{x}_{13}$ 

 Fig. 6. Magnitude  $|\varepsilon(k)|$  of estimate error.

**Remark 7:** It is trivial to consider the synchronization between the nodes for a stable network. To validate our analysis results, in Example 1, the given network is unstable which is of more significance to be used to test the theoretical results. Similarly, in Example 2, we again employ an unstable network to test the performance of our designed state estimator. In other words, both the two examples are nontrivial in evaluating the designed synchronizer and estimator.

## VI. CONCLUSION

In this paper, we have investigated the synchronization problem for an array of coupled complex discrete-time networks with the simultaneous presence of both the discrete and distributed time delays. Rather than the commonly used Lipschitz-type function, a more general sectorlike nonlinear function has been employed to describe the nonlinearities existing in the network. We have first defined the distributed time delays for the complex networks in the discrete-time domain. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, we have shown that the addressed synchronization problem can be converted into the feasibility problem of a set of LMIs. We have then tackled the state estimation problem for the same complex networks. Through available output measurements, we have developed an LMI approach to design a state estimator in order to estimate the network states such that the dynamics of the estimation error is guaranteed to be globally asymptotically stable. Two simulation examples have been provided to show the effectiveness of the proposed approach. It has been confirmed through simulation that our main results are valid even if the nominal subsystems within the network are unstable.

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**Yurong Liu** received the B.Sc. degree in mathematics from Suzhou University, Suzhou, China, in 1986, the M.Sc. degree in applied mathematics from the Nanjing University of Science and Technology, Nanjing, China, in 1989, and the Ph.D. degree in applied mathematics from Suzhou University in 2000.

He is currently a Professor with the Department of Mathematics, Yangzhou University, Yangzhou, China. His current interests include neural networks, nonlinear dynamics, time-delay systems, and chaotic dynamics.

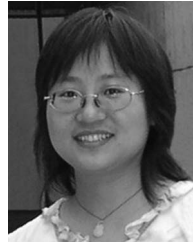


**Zidong Wang** (M'03–SM'03) was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics from Suzhou University, Suzhou, China, in 1986, and the M.Sc. degree in applied mathematics and the Ph.D. degree in electrical and computer engineering from the Nanjing University of Science and Technology, Nanjing, China, in 1990 and 1994, respectively.

He is currently a Professor of dynamical systems and computing with Brunel University, Uxbridge, U.K. He was appointed as a Lecturer in 1990 and

Associate Professor in 1994 with the Nanjing University of Science and Technology. From January 1997 to December 1998, he was an Alexander von Humboldt Research Fellow with the Control Engineering Laboratory, Ruhr-University Bochum, Bochum, Germany. From January 1999 to February 2001, he was a Lecturer with the Department of Mathematics, University of Kaiserslautern, Kaiserslautern, Germany. From March 2001 to July 2002, he was a University Senior Research Fellow with the School of Mathematical and Information Sciences, Coventry University, U.K. In August 2002, he was with the Department of Information Systems and Computing, Brunel University, U.K., as a Lecturer, and was promoted to Reader in September 2003 and to Professor in July 2007. He is also holding an Adjunct TePin Professorship at Donghua University, Shanghai, China, a Visiting Professorship at Fuzhou University, Fuzhou, China, and a Guest Professorship at Nanjing Normal University, Nanjing. His research interests include dynamical systems, signal processing, bioinformatics, control theory, and applications. He has published more than 80 papers in refereed international journals.

Dr. Wang is a fellow of the Royal Statistical Society, a member of program committees for many international conferences, and a very active Reviewer for many international journals. He was nominated an Appreciated Reviewer for the IEEE TRANSACTIONS ON SIGNAL PROCESSING in 2006 and 2007, an Outstanding Reviewer for the IEEE TRANSACTIONS ON AUTOMATIC CONTROL in 2004 and for the journal *Automatica* in 2000. He received the Standing Membership of the Technical Committee on Control of the International Association of Science and Technology for Development in 2000. From 2001 to 2007, he served as a member of technical program committees for 35 international conferences in the general area of systems theory and computing. He was awarded the Humboldt research fellowship in 1996 from the Alexander von Humboldt Foundation, the JSPS Research Fellowship in 1998 from the Japan Society for the Promotion of Science, and the William Mong Visiting Research Fellowship in 2002 from the University of Hong Kong. He was a recipient of the Outstanding Science and Technology Development Awards, once in 2005 and twice in 1997 from the National Education Committee of China, once in 1997 from the Military Industry General Company of China, once in 1997 from Jiangsu Province Government of China, and once in 1999 from the National Education Ministry of China. He is currently serving as an Associate Editor for the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, IEEE TRANSACTIONS ON NEURAL NETWORKS, IEEE TRANSACTIONS ON SIGNAL PROCESSING, IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART C, IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, and *Circuits, Systems and Signal Processing*, an Action Editor for *Neural Networks*, an Editorial Board Member for the *International Journal of Systems Science*, *Neurocomputing*, *International Journal of Computer Mathematics*, *International Journal of General Systems*, and an Associate Editor on the Conference Editorial Board for the IEEE Control Systems Society.



**Jinling Liang** received the B.Sc. and M.Sc. degrees in mathematics from Northwest University, Xi'an, China, in 1997 and 1999, respectively, and the Ph.D. degree in applied mathematics from Southeast University, Nanjing, China, in 2006.

She is currently an Associate Professor with the Department of Mathematics, Southeast University, Nanjing. From January 2004 to March 2004, she was a Research Assistant with the University of Hong Kong. From March to April 2004, she was a Research Assistant with the City University of

Hong Kong. From April 2007 to March 2008, she was a Postdoctoral Research Fellow with the Department of Information Systems and Computing, Brunel University, Uxbridge, U.K., sponsored by the Royal Society Sino-British Fellowship Trust Award of the U.K. Her research interests include neural networks, complex networks, nonlinear systems, and bioinformatics. She has published around 20 papers in refereed international journals.

Dr. Liang is a member of program committee for several international conferences and serves as a very active Reviewer for many international journals.



**Xiaohui Liu** received the B.Eng. degree in computing from Hohai University, Nanjing, China, in 1982 and the Ph.D. degree in computer science from Heriot-Watt University, Edinburgh, U.K., in 1988.

He is currently a Professor of computing with Brunel University, Uxbridge, U.K. He leads the Intelligent Data Analysis (IDA) Group, performing interdisciplinary research involving artificial intelligence, dynamic systems, image and signal processing, and statistics, particularly for applications in biology, engineering, and medicine.

Prof. Liu serves on editorial boards of four computing journals, founded the biennial international conference series on IDA in 1995, and has given numerous invited talks in bioinformatics, data mining, and statistics conferences.