Nonlinear Dynamics manuscript No.
(will be inserted by the editor)

Mean-square exponential synchronization of stochastic complex dynamical networks with switching topology by impulsive control

Xuefei Wu

Received: date / Accepted: date

Abstract This paper investigates the mean-square exponential synchronization issues of delayed stochastic complex dynamical networks with switching topology and impulsive control. By using the Lyapunov functional method, impulsive control theory and linear matrix inequality (LMI) approaches, some sufficient conditions are derived to guarantee the mean-square exponential synchronization of delay complex dynamical network with switch topology, which are independent of the network size and switch topology. Numerical simulations are given to illustrate the effectiveness of the obtained results in the end.

Keywords mean-square exponential synchronization · stochastic complex dynamical networks · switching topology · impulsive control

1 Introduction

Complex networks are everywhere in nature and our daily life, such as the Internet, Ecosystems, Social Networks, World Wide Web and Neural Networks, and so on. A complex network can be described by a set of nodes and edges interconnecting these nodes together. During the last two decades, complex networks have been focused on by scientists from various fields, such as mathematical, engineering, social and economic science. There are many literatures concerning the collective behaviors of complex networks [1]-[11]. Among them, synchronization is the most interesting phenomenon [3]-[11], because the synchronization is a kind of typical collective behaviors exhibited in many natural systems.

X. Wu
School of Computer Engineering,
Shenzhen Polytechnic,
Shenzhen 518055, China
E-mail: wuxuefei@szpt.edu.cn

Recently, the synchronization of complex network with switching topology has been studied in [12]-[17]. Because many complex systems may experience abrupt changes in their connection caused by some phenomena such as link failures, component failures or repairs, changing subsystem interconnections, and abrupt environmental disturbance, etc. In [12], Yao et. al. studied the synchronization of a general complex dynamical network with switching topology and the time-varying coupling is unknown but bounded. Wang [13] investigated the synchronization issues of complex dynamical networks with switching topology. Yu [14] et. al. explored the synchronization of switched linearly coupled neural networks with delay. Some sufficient conditions were given to guarantee the global synchronization. In [15], Liu et. al. studied the local and global exponential synchronization of complex dynamical network with switching topology and time-varying coupling delays. In [16], some sufficient conditions were given to guarantee the synchronization of leader following issues with switching connective network and coupling delay. In [17], authors investigates the consensus problem in mean square for uncertain multiagent systems with stochastic measurement noises and symmetric or asymmetric time-varying delays.

Uncertainties commonly exist in the real world, such as stochastic forces on the physical systems and noisy measurements caused by environmental uncertainties. Thus, a stochastic behavior should be produced instead of a deterministic one [18]. In fact, signals transmitted between nodes of complex networks are unavoidably subject to stochastic perturbations from environment, which may cause information contained in these signals to be lost [19]. Therefore, stochastic perturbations should be considered [18][19] [20][21][22][23]. In [19]-[21], stochastic perturbations are all one-dimensional, which means that the signal transmitted by nodes is influenced by the same noise. In [18] and [22], the authors considered stochastic synchronization of coupled neural networks, in which noise perturbations are vector forms. Vector-form perturbation means that different nodes is influenced by different noise, which is more practical in the real world. In [23], authors investigates the mean-square exponential synchronization of stochastic complex networks with Markovian switching and time-varying delays by using the pinning control method.

In many systems, the impulsive effects are common phenomena due to instantaneous perturbations at certain moments [24]-[28]. In the past several years, impulsive control strategies have been widely used to stabilize and synchronize coupled complex dynamical system, such as signal processing system, computer networks, automatic control systems and telecommunications, etc. In [24], Cai et. al. investigated the robust impulsive synchronization of complex delayed dynamical networks. Yang and Cao [25] studied the exponential synchronization of complex dynamical network with a coupling delay and impulsive. Zhu et. al. gave some global impulsive exponential synchronization criteria of time-delayed coupled chaotic systems [27]. In [28], some sufficient conditions were given to guarantee the consensus of nonlinear multi-agent systems with switching topology.

Based on the above discussion, studying the synchronization problem of the complex dynamical network with switch topology and impulsive effective is very useful and meaningful. It should be pointed out that exponential synchronization of the coupling delay complex dynamical networks with switch topology and impulsive effects has received very little research attention.

In this paper, we investigate the problem of exponential synchronization of coupling delay complex dynamical network with switching topology and impulsive effects, basing on the Lyapunov theory and impulsive control theory and by assuming that there exist finite connective topology of the complex dynamical network, and the connective topology may be switched (or jumped) from one to another at different moments. For controlling, the synchronization state can be any weighting average of the network states. It means that each node of the dynamical network can contribute to synchronization of the network in its weight. Finally, some sufficient conditions are given to guarantee the synchronization of the complex dynamical network, which are independent of the network size and switch topology.

2 Model and Preliminaries

The switching complex dynamical networks investigated on this paper consist of N nodes, whose state is described as:

$$\begin{cases} dx_i(t) = \Big\{ f(t, x_i(t), x_i(t-\tau(t))) + \sum_{j=1}^{N} a_{ij}(r(t)) \Sigma x_j(t) + \sum_{j=1}^{N} b_{ij}(r(t)) \Sigma x_j(t-\tau_c(t)) \Big\} dt \\ \qquad + \sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r(t)) dw_i(t), & t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = \epsilon_{ik} x_i(t_k^-), & t = t_k, \\ k \in Z^+, i = 1, 2, \cdots, N \end{cases} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node of the network, $f(t, x_i(t), x_i(t-\tau(t))) = [f_1(t, x_i(t), x_i(t-\tau(t))), f_2(t, x_i(t), x_i(t-\tau(t))), \ldots, f_n(t, x_i(t), x_i(t-\tau(t)))]^T$ is a continuous vector-valued function, $\Sigma = \mathrm{diag}(\varrho_1, \varrho_2, \ldots, \varrho_n)$ is an inner coupling of the networks that satisfies $\varrho_j > 0$, $j = 1, 2, \ldots, n$, and $r(t) : [0, \infty) \to \aleph = \{1, 2, \cdots, M\}$ is a switching signal, which is a piecewise constant function. Here, $A(r(t)) = [a_{ij}(r(t))] \in \mathbb{R}^{n \times n}$ and $B(r(t)) = [b_{ij}(r(t))] \in \mathbb{R}^{n \times n}$ are the outer coupling matrices of the network at time t at nodes $r(t)$, $t - \tau_c(t)$ and $r(t)$ respectively such that $a_{ij}(r(t)) \geq 0$ for $i \neq j$, $a_{ii}(r(t)) = -\sum_{j=i, j\neq i}^{N} a_{ij}(r(t))$, $b_{ij}(r(t)) \geq 0$ for $i \neq j$ and $b_{ii}(r(t)) = -\sum_{j=i, j\neq i}^{N} b_{ij}(r(t))$. $\tau(t)$ is the inner time-varying delay satisfying $\tau \geq \tau(t) \geq 0$ and $\tau_c(t)$ is the coupling time-varying delay satisfying $\tau_c \geq \tau_c(t) \geq 0$. Finally, $\sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r(t)) = \sigma_i(t, x_1(t), \ldots, x_n(t), x_1(t-\tau(t)), \ldots, x_n(t-\tau(t)), x_1(t-\tau_c(t)), \ldots, x_n(t-\tau_c(t)), r(t)) \in \mathbb{R}^{n \times n}$ and $w_i(t) =$

$(w_{i1}(t), w_{i2}(t), \ldots, w_{in}(t))^T \in \mathbb{R}^n$ is a bounded vector-form Weiner process, satisfying

$$\mathbb{E}w_{ij}(t) = 0, \quad \mathbb{E}w_{ij}^2(t) = 1, \quad \mathbb{E}w_{ij}(t)w_{ij}(s) = 0(s \neq t).$$

ϵ_{ik} is the ith node impulsive gain at $t = t_k$. The discrete set $\{t_k\}$ satisfies $t_k = t_{k-1} + T$, and $t_0 \geq 0$, $t_k \to +\infty$ as $k \to +\infty$, note $x(t_k^-) = \lim_{t \to t_k^-} x(t)$, and $x(t_k^+) = \lim_{t \to t_k^+} x(t) = x(t_k)$. In this paper, $A(r(t))$ is assumed to be irreducible in the sense that there are no isolated nodes.

The initial conditions associated with (1) are

$$x_i(s) = \xi_i(s), \quad -\check{\tau} \leq s \leq 0, \quad i = 1, 2, \ldots, N,$$

where $\check{\tau} = \max\{\tau(t), \tau_c(t)\}$, $\xi_i \in C^b_{\mathcal{F}_0}([-\check{\tau}, 0], \mathbb{R}^n)$ with the norm $||\xi_i||^2 = \sup_{-\check{\tau} \leq s \leq 0} \xi_i(s)^T \xi_i(s)$ and our objective is to control system (1) so that it stays in the trajectory $s(t) \in \mathbb{R}^n$ of the system

$$ds(t) = f(t, s(t), s(t - \tau(t)))dt \tag{2}$$

Remark 1 *In general, the synchronization state $s(t)$ may be an equilibrium point, a periodic orbit, or a chaotic attractor.*

The following assumptions will be used throughout this paper for establishing the synchronization conditions.

H1 $\tau(t)$ and $\tau_c(t)$ are bounded and continuously differentiable functions such that $0 < \tau(t) \leq \tau$, $\dot{\tau}(t) < \bar{\tau} < 1$, $0 < \tau_c(t) \leq \tau_c$ and $\dot{\tau}_c(t) < \bar{\tau}_c < 1$. Let $\check{\tau} = \max\{\tau, \tau_c\}$ and $\check{\bar{\tau}} = \max\{\bar{\tau}, \bar{\tau}_c\}$.

H2 Let $\sigma(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r) = \sigma(t, e_1(t), \ldots, e_N(t), e_1(t-\tau(t)), \ldots, e_N(t-\tau(t)), e_1(t-\tau_c(t)), \ldots, e_N(t-\tau_c(t)), r)$. Then there exist positive definite constant matrices Υ^r_{i1}, Υ^r_{i2} and Υ^r_{i3} for $i = 1, 2, \ldots, N$ and $r = 1, 2, \ldots, M$ such that

$$\begin{aligned} &\mathrm{Tr}\Big[\sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r)^T \sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r)\Big] \\ &\leq \sum_{j=1}^{N} e_j(t)^T \Upsilon^r_{i1} e_j(t) + \sum_{j=1}^{N} e_j(t-\tau(t))^T \Upsilon^r_{i2} e_j(t-\tau(t)) + \sum_{j=1}^{N} e_j(t-\tau_c(t))^T \Upsilon^r_{i3} e_j(t-\tau_c(t)). \end{aligned} \tag{3}$$

H3 $\sigma_i(t, s(t), \ldots, s(t), s(t-\tau(t)), \ldots, s(t-\tau(t)), s(t-\tau_c(t)), \ldots, s(t-\tau_c(t)), r(t)) = 0$

Define error state $e_i(t) = x_i(t) - s(t)$,

$$\begin{cases} de_i(t) = \Big\{ f(t, x_i(t), x_i(t-\tau(t))) - f(t, s(t), s(t-\tau(t))) \\ \qquad + \sum_{j=1}^{N} a_{ij}(r(t)) \Sigma e_j(t) + \sum_{j=1}^{N} b_{ij}(r(t)) \Sigma e_j(t-\tau_c(t)) + u_i(t) \Big\} dt & k \in Z^+, i = 1, 2, \cdots, N. \\ \qquad + \sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r(t)) dw_i(t), & t \neq t_k \\ \Delta e_i(t_k) = e_i(t_k^+) - e_i(t_k^-) = \epsilon_{ik} e_i(t_k^-), & t = t_k, \end{cases}$$

(4)

Definition 1 *The complex network (1) is said to be* exponentially synchronized in mean square *if the trivial solution of system (4) is such that*

$$\sum_{i=1}^{N}\mathbb{E}||e_i(t,t_0,\xi_i)||^2 \leq Ke^{-\kappa t},$$

for some $K>0$ *and some* $\kappa>0$ *for any initial data* $\xi_i \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$.

Definition 2 *A continuous function* $f(t,x,y):[0,+\infty]\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$ *is said to belong to the function class QUAD, denoted by* $f\in QUAD(P,\Delta,\eta,\theta)$ *for some given matrix* $\Sigma=diag\{\varrho_1,\varrho_2,\ldots,\varrho_n\}$ *if there exists a positive definite diagonal matrix* $P=diag\{p_1,p_2,\ldots,p_n\}$, *a diagonal matrix* $\Delta=diag\{\delta_1,\delta_2,\ldots,\delta_n\}$ *and constants* $\eta>0$,$\theta>0$ *such that* $f(\cdot)$ *satisfies the condition*

$$(x-y)^TP\big((f(t,x,z)-f(t,y,w))-\Delta\Sigma(x-y)\big)\leq-\eta(x-y)^T(x-y)+\theta(z-w)^T(z-w) \tag{5}$$

for all $x,y,z,w\in\mathbb{R}^n$.

Remark 2 *The function class QUAD includes almost all the well-known chaotic systems with or without delays such as the Lorenz system, the Rössler system, the Chen system, the delayed Chua's circuit, the logistic delayed differential system, the delayed Hopfield neural network and the delayed CNNs. We shall simply write*

$$\check{p}=\max\{p_1,p_2,\ldots,p_n\},\quad \hat{p}=\min\{p_1,p_2,\ldots,p_n\},\quad \check{\delta}=\max\{\delta_1,\delta_2,\ldots,\delta_n\}.$$

In order to derive the main results, it is necessary to propose the following Lemmas.

Lemma 1 *[16] The following linear matrix inequality:*

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

where $Q=Q^T$,$R=R^T$, *is equivalent to one of the following conditions:*

(i) $Q>0, R-S^TQ^{-1}S>0$.
(ii) $R>0, Q-S^TR^{-1}S>0$.

3 Main Result

In this section, we investigate the exponential stability condition of the error system (4) Some new criteria are presented for the exponential synchronization of network (1) based on Lyapunov functional method, linear matrix inequality approach and impulsive control theory.

Theorem 1 *Let assumptions **H1** and **H2** be true and let $f \in QUAD(P, \Delta, \eta, \theta)$. If there exist positive constants α_r and β_r such that*

$$\begin{bmatrix} A(r)^s + \check{\delta} I_N - \alpha_r I_N & B(r)/2 \\ B(r)^T/2 & -\beta_r I_N \end{bmatrix} \leq 0, \quad for \quad r = 1, 2, \ldots, M, \tag{6}$$

and

$$(\gamma + \check{a} + \frac{b}{1-\bar{\tau}} e^{\gamma\tau} + \frac{\check{c}}{1-\bar{\tau}_c} e^{\gamma\tau_c})(1 + T\check{\tau})/T + 2\ln|1+\epsilon|/T - \gamma \leq -\eta \tag{7}$$

where

$$\begin{aligned}
a_r &= \frac{\lambda_{\min}(-2\eta I_n + \check{p}\sum_{i=1}^N \Upsilon_{i1}^r + 2\alpha_r P\Sigma)}{\check{p}}, & \check{a} &= \max_{r\in S} a_r, \\
b_r &= \frac{\lambda_{\max}(\sum_{i=1}^N P\Upsilon_{i2}^r + 2\theta I_N)}{\hat{p}}, & \check{b} &= \max_{r\in S} b_r, \\
c_r &= \frac{\lambda_{\max}(\sum_{i=1}^N P\Upsilon_{i3}^r + 2\beta_r P\Sigma)}{\hat{p}}, & \check{c} &= \max_{r\in S} c_r, \\
|1+\epsilon| &= \max_{i=1,2,\cdots,N, k\in Z^+} |1+\epsilon_{ik}|.
\end{aligned}$$

Then the solutions $x_1(t)$, $x_1(t)$, ... and $x_N(t)$ of system (4) are globally and exponentially stable.

Proof Define the Lyapunov-Krasovskii function

$$V(t, e(t)) = \frac{1}{2}\sum_{i=1}^N e_i(t)^T P e_i(t)$$

and let $\tilde{e}^k(t) = (e_{1k}(t), e_{2k}(t), ..., e_{Nk}(t))^T$, $k = 1, 2, ..., n$. For $r(t) = r$, we have

$$\begin{aligned}
\mathcal{L}V(t, e(t), r) = &\sum_{i=1}^N e_i(t)^T P\Big\{ f(t, x_i(t), x_i(t-\tau(t))) - f(t, s(t), s(t-\tau(t))) \\
&+ \sum_{j=1}^N a_{ij}(r)\Sigma e_j(t) + \sum_{j=1}^N b_{ij}(r)\Sigma e_j(t-\tau_c(t))\Big\} \\
&+ \frac{1}{2}\sum_{i=1}^N \mathrm{Tr}\Big\{\sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r)^T P \sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r)\Big\}
\end{aligned}$$

$$
\begin{aligned}
&=\sum_{i=1}^{N} e_i(t)^T P\Big\{f(t,x_i(t),x_i(t-\tau(t)))-f(t,s(t),s(t-\tau(t)))-\Delta\Sigma e_i(t)\Big\}\\
&\quad+\sum_{i=1}^{N} e_i(t)^T P\Delta\Sigma e_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(r)e_i(t)^T P\Sigma e_j(t)+\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}(r)e_i(t)^T P\Sigma e_j(t-\tau_c(t))\\
&\quad+\frac{1}{2}\sum_{i=1}^{N}\mathrm{Tr}\Big\{\sigma_i(t,x(t),x(t-\tau(t)),x(t-\tau_c(t)),r)^T P\sigma_i(t,x(t),x(t-\tau(t)),x(t-\tau_c(t)),r)\Big\}\\
&\le\Big\{-\eta\sum_{i=1}^{N} e_i(t)^T e_i(t)+\theta\sum_{i=1}^{N} e_i(t-\tau(t))^T e_i(t-\tau(t))+\sum_{k=1}^{n} p_k\varrho_k\delta_k\tilde{e}^k(t)^T\tilde{e}^k(t)\\
&\quad+\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t)^T A(r)\tilde{e}^k(t)+\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t)^T B(r)\tilde{e}^k(t-\tau_c(t))\\
&\quad+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Big[\sum_{i=1}^{N} e_i(t)^T\Upsilon_{j1}^r e_i(t)+\sum_{i=1}^{N} e_i(t-\tau(t))^T\Upsilon_{j2}^r e_i(t-\tau(t))+\sum_{i=1}^{N} e_i(t-\tau_c(t))^T\Upsilon_{j3}^r e_i(t-\tau_c(t))\Big]\Big\}\\
&=\Big\{-\eta\sum_{i=1}^{N} e_i(t)^T e_i(t)+\theta\sum_{i=1}^{N} e_i(t-\tau(t))^T e_i(t-\tau(t))+\frac{1}{2}\check{p}\sum_{i=1}^{N}\sum_{j=1}^{N} e_i(t-\tau_c(t))^T\Upsilon_{j3}^r e_i(t-\tau_c(t))\Big\}\\
&\quad+\Big\{\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t)^T A(r)\tilde{e}^k(t)+\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t)^T B(r)\tilde{e}^k(t-\tau_c(t))\\
&\quad+\frac{1}{2}\check{p}\Big[\sum_{i=1}^{N}\sum_{j=1}^{N} e_i(t)^T\Upsilon_{j1}^r e_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N} e_i(t-\tau(t))^T\Upsilon_{j2}^r e_i(t-\tau(t))\Big]+\sum_{k=1}^{n} p_k\varrho_k\delta_k\tilde{e}^k(t)^T\tilde{e}^k(t)\Big\}\\
&=\Big\{\sum_{i=1}^{N} e_i(t)^T(-\eta I_N+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^r+\alpha_r P\Sigma)e_i(t)+\sum_{i=1}^{N} e_i(t-\tau(t))^T(\theta+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j2}^r)e_i(t-\tau(t))\\
&\quad+\sum_{i=1}^{N} e_i(t-\tau_c(t))^T(\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j3}^r+\beta_r P\Sigma)e_i(t-\tau_c(t))\Big\}\\
&\quad+\Big\{\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t)^T[A(r)+(\check{\delta}-\alpha_r)I_N]\tilde{e}^k(t)\\
&\quad+\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t)^T B(r)\tilde{e}^k(t-\tau_c(t))\\
&\quad-\sum_{k=1}^{n} p_k\varrho_k\tilde{e}^k(t-\tau_c(t))^T\beta_r\tilde{e}^k(t-\tau_c(t))\Big\}\\
&\le\Big\{\sum_{i=1}^{N} e_i(t)^T(-\eta I_N+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^r+\alpha_r P\Sigma)e_i(t)
\end{aligned}
$$

$$+\sum_{i=1}^{N} e_i(t-\tau(t))^T(\theta I_N+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j2}^r)e_i(t-\tau(t))$$
$$+\sum_{i=1}^{N} e_i(t-\tau_c(t))^T(\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j3}^r+\beta_r P\Sigma)e_i(t-\tau_c(t))\Big\}.$$

Then we have

$$\mathcal{L}V(t,e(t)) \leq a_r V(t)+b_r V(t-\tau(t))+c_r V(t-\tau_c(t))$$

So we have

$$\mathcal{L}V(t) \leq \check{a}V(t)+\check{b}V(t-\tau(t))+\check{c}V(t-\tau_c(t)). \tag{8}$$

Define

$$W(t)=e^{\gamma t}V(t)$$

and use equation (8) to compute the operator

$$\begin{aligned}\mathcal{L}W(t) =& e^{\gamma t}[\gamma V(t)+\mathcal{L}V(t)]\\ \leq& e^{\gamma t}[\gamma V(t)+\check{a}V(t)+\check{b}V(t-\tau(t))+\check{c}V(t-\tau_c(t))],\end{aligned}$$

which after applying the generalized Itô's formula, gives

$$e^{\gamma t}\mathbb{E}V(t)=e^{\gamma t_0}\mathbb{E}V(t_0)+\mathbb{E}\int_{t_0}^{t}\mathcal{L}W(s)ds \tag{9}$$

for any $t_k > t > t_0 > t_{k-1} \geq 0$. Hence we have

$$\begin{aligned}e^{\gamma t}\mathbb{E}V(t) \leq& e^{\gamma t_0}\mathbb{E}V(t_0)+\int_{t_0}^{t} e^{\gamma s}[\gamma V(s)+\check{a}V(s)+\check{b}V(s-\tau(s))+\check{c}V(s-\tau_c(s))]ds\\ \leq& e^{\gamma t_0}\mathbb{E}V(t_0)+(\gamma+\check{a})\int_{t_0}^{t} e^{\gamma s}\mathbb{E}V(s)ds+\check{b}e^{\gamma\tau}\int_{t_0}^{t} e^{\gamma(s-\tau(s))}\mathbb{E}V(s-\tau(s))ds\\ &+\check{c}e^{\gamma\tau_c}\int_{t_0}^{t} e^{\gamma(s-\tau_c(s))}\mathbb{E}V(s-\tau_c(s))ds.\end{aligned} \tag{10}$$

By changing variable $s-\tau(s)=u$, we have

$$\begin{aligned}\int_{t_0}^{t} e^{\gamma(s-\tau(s))}\mathbb{E}V(s-\tau(s))ds =& \int_{t_0-\tau(t_0)}^{t-\tau(t)} e^{\gamma u}\mathbb{E}V(u)\frac{du}{1-\dot{\tau}(t)}\\ \leq& \int_{t_0-\tau}^{t} e^{\gamma u}\mathbb{E}V(u)\frac{du}{1-\bar{\tau}}\\ \leq& \frac{1}{1-\bar{\tau}}\int_{t_0-\tau}^{t} e^{\gamma u}\mathbb{E}V(u)du\end{aligned} \tag{11}$$

Similarly, we have

$$\int_{t_0}^{t} e^{\gamma(s-\tau_c(s))}\mathbb{E}V(s-\tau(s))ds \leq \frac{1}{1-\bar{\tau}_c}\int_{t_0-\tau_c}^{t} e^{\gamma u}\mathbb{E}V(u)du \tag{12}$$

Substituting equation (11) and (12) into equation (10), we get

$$e^{\gamma t}\mathbb{E}V(t) \leq e^{\gamma t_0}\mathbb{E}V(t_0) + (\gamma + \check{a} + \frac{\check{b}e^{\gamma\tau}}{1-\bar{\tau}} + \frac{\check{c}e^{\gamma\tau_c}}{1-\bar{\tau}_c})\int_{t_0-\check{\tau}}^{t} e^{\gamma u}\mathbb{E}V(u)du.$$

By using Gronwall inequality, we get

$$e^{\gamma t}\mathbb{E}V(t) \leq e^{\gamma t_0}\mathbb{E}V(t_0)e^{(\gamma+\check{a}+\frac{\check{b}}{1-\bar{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\bar{\tau}_c}e^{\gamma\tau_c})(t-t_0+\check{\tau})}.$$

On the other hand, from the construction of $V(t)$, we have

$$V(t_k) \leq (1+\epsilon_k)^2 V(t_k^-) \tag{13}$$

where $|1+\epsilon_k| = \max_{i=1,2,\cdots,N}|1+\epsilon_{ik}|$.

According to Eqs.(5)-(6), for any $t \in [t_{k-1}, t_k)$, we get

$$\begin{aligned} e^{\gamma t}\mathbb{E}V(t) \leq & e^{\gamma t_{k-1}}\mathbb{E}V(t_{k-1})e^{(\gamma+\check{a}+\frac{\check{b}}{1-\bar{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\bar{\tau}_c}e^{\gamma\tau_c})(t-t_{k-1}+\check{\tau})} \\ \leq & e^{\gamma t_{k-1}}\mathbb{E}V(t_{k-1}^-)e^{(\gamma+\check{a}+\frac{\check{b}}{1-\bar{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\bar{\tau}_c}e^{\gamma\tau_c})(t-t_{k-1}+\check{\tau})+2\ln|1+\epsilon_{k-1}|} \\ \leq & \cdots \leq e^{\gamma t_0}\mathbb{E}V(t_0)e^{(\gamma+\check{a}+\frac{\check{b}}{1-\bar{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\bar{\tau}_c}e^{\gamma\tau_c})(t-t_0+k\check{\tau})+\sum_{v=1}^{k-1}2\ln|1+\epsilon_v|} \end{aligned}$$

Let $|1+\epsilon| = \max_{v\in Z^+}|1+\epsilon_v|$. Because of $k = \lfloor\frac{t-t_0}{T}\rfloor$, we have

$$\begin{aligned} \mathbb{E}V(t) \leq & \mathbb{E}V(t_0)e^{(\gamma+\check{a}+\frac{\check{b}}{1-\bar{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\bar{\tau}_c}e^{\gamma\tau_c})(t-t_0+k\check{\tau})+2(k-1)\ln|1+\epsilon_v|-(t-t_0)\gamma} \\ \leq & \mathbb{E}V(t_0)e^{(\gamma+\check{a}+\frac{\check{b}}{1-\bar{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\bar{\tau}_c}e^{\gamma\tau_c})(1+T)\check{\tau}/T(t-t_0)+(2(t-t_0)/T-4)\ln|1+\epsilon_v|-(t-t_0)\gamma} \end{aligned}$$

Using Condition 15 of Theorem 1, we get $\mathbb{E}V(t) \leq \mathbb{E}V(t_0)e^{-\eta(t-t_0)}$. Hence, $\mathbb{E}||e_i(t)|| \leq (\frac{V(t_0)}{\check{p}})^{\frac{1}{2}}e^{-\frac{\eta}{2}(t-t_0)}$. The proof of Theorem 1 is completed.

Remark 3 *Theorem* 1 *provides a sufficient condition for exponential synchronization of coupling delay switched stochastic dynamical networks with impulsive effects. If the time T is sufficiently small and the impulsive gains ϵ_{ik}, then exponential synchronization of the network* (1) *could be achieved.*

If the switching signal $\sigma(t) \equiv 1$, then the network (1) has only one coupling matrix G. Suppose G is irreducible and $\xi^T = (\xi_1, \xi_2, \cdots, \xi_N)$ is the left eigenvector of coupling matrix G corresponding to eigenvalue 0. Let impulsive gains $\epsilon_{ik} = b_k$. By the proof of Theorem 1, we can derive the exponential synchronization criteria of the network (1) with only one topology, which is given as follows.

Corollary 1 *Let assumptions **H1** and **H2** be true and let $f \in QUAD(P, \Delta, \eta, \theta)$. If there exist positive constants α and β such that*

$$\begin{bmatrix} A^s + \check{\delta} I_N - \alpha I_N & B/2 \\ B^T/2 & -\beta I_N \end{bmatrix} \leq 0, \tag{14}$$

and

$$(\gamma + a + \frac{b}{1-\bar{\tau}} e^{\gamma\tau} + \frac{c}{1-\bar{\tau}_c} e^{\gamma\tau_c})(1 + T\check{\tau})/T + 2\ln|1+\epsilon|/T - \gamma \leq -\eta \tag{15}$$

where

$$a = \frac{\lambda_{\min}(-2\eta I_n + \check{p}\sum_{i=1}^{N} \Upsilon_{i1} + 2\alpha P\Sigma)}{\check{p}},$$
$$b = \frac{\lambda_{\max}(\sum_{i=1}^{N} P\Upsilon_{i2} + 2\theta I_N)}{\hat{p}},$$
$$c = \frac{\lambda_{\max}(\sum_{i=1}^{N} P\Upsilon_{i3} + 2\beta P\Sigma)}{\hat{p}},$$
$$|1+\epsilon| = \max_{i=1,2,\cdots,N, k\in Z^+} |1+\epsilon_{ik}|.$$

Then the solutions $x_1(t)$, $x_1(t)$, ... and $x_N(t)$ of system (4) are globally and exponentially stable.

Let impulsive gains $\epsilon_{ik} = b$, and choose the synchronization state $s(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$. By the proof of Theorem 2, we can derive the exponential synchronization criteria of the network (1) with the fixed impulsive gain, which is given as follows.

4 Numerical Simulation

In this section, we give two numerical simulations to illustrate the feasibility and effectiveness of the theoretical results presented in the previous sections.

Consider a three-order Chua's circuit described as follows:

$$\dot{x}(t) = f(x(t))$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$ and function $f(x(t))$ was chosen below:

$$f(x(t)) = \begin{bmatrix} m[x_2 - h(x_1) \\ x_1 - x_2 + x_3 \\ -nx_2 \end{bmatrix}$$

where $h(x_1) = \frac{2}{7}x_1 - \frac{3}{14}[|x_1+1| - |x_1-1|]$, $m = 9$ and $n = 14\frac{2}{7}$.

Consider a network model consists of 5nodes and 2 connective topology. Each node in the network is a three-order Chua's circuit described by

$$\begin{cases} \dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{5} g_{ij}^{\sigma(t)} \Gamma x_j(t-\tau) + \sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r(t)) dw_i(t), \quad t \neq t_k \\ \Delta x_i(t_k) = x_i(t_k) - x_i(t_k^-) = \epsilon_{ik} x_i(t_k^-), \quad t = t_k, i = 1, 2, \cdots, 5, \end{cases} \tag{16}$$

where $\tau = 1$ and $\Gamma = I_3$ and $\sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t))) = 0.1 \times \text{diag}\{x_{i1}(t) - x_{i+1,1}(t), x_{i2}(t) - x_{i+1,2}(t)\}$.

The coupling matrices are as follows:

$$G^1 = \begin{bmatrix} -3 & 2 & 1 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 1 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & -2 & 2 \\ 2 & 0 & 0 & 0 & -3 \end{bmatrix}, G^2 = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 0 & -2 & 1 \\ 1 & 1 & 0 & 0 & -2 \end{bmatrix}.$$

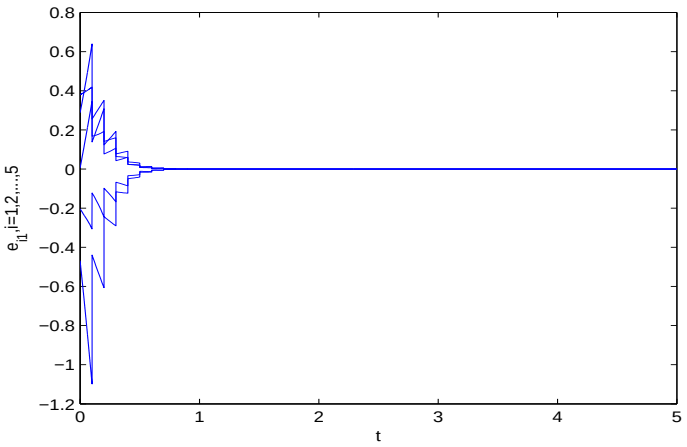

Fig. 1 Time evolution of synchronization errors $e_{i1}(t), (i = 1, 2, \ldots, 5)$ of the asymmetric coupled network (10) with random initial values.

If we choose $P = I_3$ and $\Delta = 10I_3$, then function $f(x)$ satisfies the condition of function class QUAD(Δ, P), where $\alpha = 0.6218$. Let $\beta = 23.7$, $Q = 4.1I_3$, $\epsilon_{ik} = -0.8$, $T = 0.1$, $\eta = -3.488$, $a = 4.1$. $a_1 = 80.1500$, $a_2 = 62.3271$, $b = 6.5300$, $c_1 = 0.0307$, $c_2 = 80.0300$, $\alpha_1 = 40.0000$, $\beta_1 = 0.0002$, $\alpha_2 = 15.6568$, $\beta_2 = 20.0000$ and the synchronization state is $s(t) = 0.2x_1 + 0.3x_2 + 0.1x_3 + 0.3x_4 + 0.1x_5$, then all the conditions in Theorem 1 are satisfied (By using the Matlab LMI toolbox). The switch time is $t = 0.2s$. The simulation results are given in Fig.1-Fig.3. It can be seen clearly from Fig.2,

Fig.3 and Fig.4 that all states of the asymmetric coupled network (10) tend to the synchronization state $s(t)$.

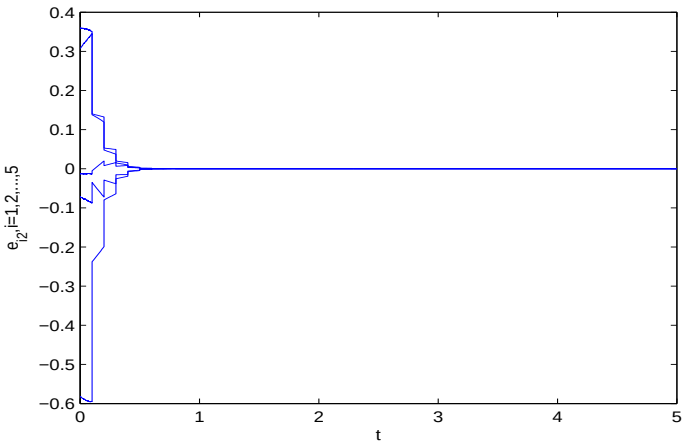

Fig. 2 Time evolution of synchronization errors $e_{i2}(t), (i = 1, 2, \ldots, 5)$ of the asymmetric coupled network (10) with random initial values.

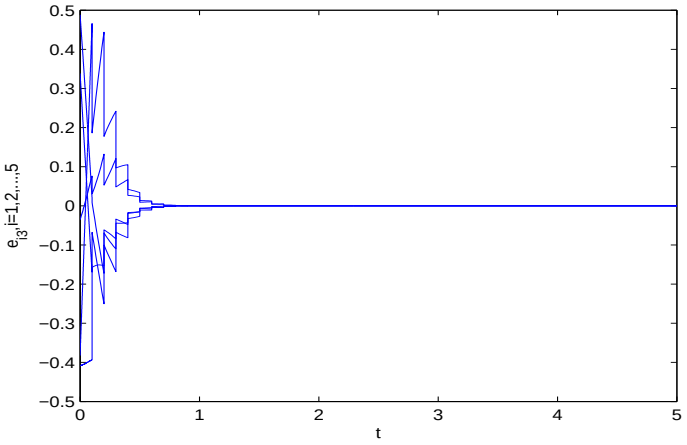

Fig. 3 Time evolution of synchronization errors $e_{i3}(t), (i = 1, 2, \ldots, 5)$ of the asymmetric coupled network (10) with random initial values.

5 Conclusion

In this paper, the exponential synchronization of the coupling delay stochastic complex networks with switch topology and impulsive effects has been investigated. Based on Lyapunov stability theory, LMI and impulsive control theory, some simple yet generic criteria for exponential synchronization have been derived. It has shown that criteria can provide an effective control scheme to synchronize for a given coupled delay, the network size and switch topology. Furthermore,the effectiveness of the presented method have been verified by numerical simulations.

Acknowledgement

This work was supported by the National Science Foundation of China under Grant No.61070087, the Guangdong Education University Industry Cooperation Projects (2009B090300355), the Shenzhen Basic Research Project (JC200903120040A, JC201006010743A) and the Shenzhen Polytechnic Youth Innovation Project (2210k3010020). The authors are very grateful to the reviewers and editors for their valuable comments and suggestions that improved the presentation of this paper.

References

1. S. H. Strogatz, Exploring complex networks, Nature, vol. 410, no. 6825, pp. 268-276, 2001.
2. S. Boccaletti, V. Latora, Y. Moreno, M. Chevez, and D. U. Hwqng, Complex networks: structure and dynamics, Physics Reports, vol. 424, no. 4-5, pp. 175-308, 2006.
3. A. Arenas, A. Guilera, J. Kurths, Y. Morenob, C. S. Zhoug, Synchronization in complex networks, Physics Reports, vol. 469, no. 3, pp. 93-153, 2008.
4. JH. L*ü*, GR. Chen, A time-varying complex dynamical network mode and its controlled synchronization criteria, IEEE Transactions on Automatic Control, vol. 50, no. 6, pp. 841-846, 2005.
5. JH. L*ü*, XH. Yu, GR. Chen, Chaos synchronization of general complex dynamical networks, Physic A, vol. 334, no. 1-2, pp. 281-302, 2004.
6. CG. Li, GR. Chen, Synchronization in general complex dynamical networks with coupling delays, Physic A, vol. 343, pp. 263-278, 2004.
7. W. Wu, T. Chen, Global synchronization criteria of linearly coupled neural network systems with time-varying coupling, IEEE Transactions on Neural Networks, vol. 19, no. 2, pp. 319-332, 2008.
8. J. Zhao, J. Lu, X. Wu, Pinning control of general complex dynamical networks with optimization, Science China Information Sciences, vol. 53, no. 4, pp. 813-822, 2010.
9. C. Wu, Synchronization in arrays of coupled nonlinear systems with delay and nonreciprocal time-varying coupling, IEEE Transactions on Circuits Systems, vol. 52, no. 5, pp. 282-286, 2005.
10. W. Lu, T. Chen, Synchronization of coupling connected neural networks with delays, IEEE Transactions on Circuits and Systems. vol. 51, no. 12, pp. 2491-2503, 2004.
11. X. Liu, T. Chen, Exponential synchronzition of nonlinear coupled dynamical networks with a delayed coupling, Physic A, vol. 381, pp. 82-92, 2007.
12. J. Yao, David J. Hill, Z. Guan, H. Wang, Synchronizaion of complex dynamical networks with switching topology via adaptic control, Proceedings of the 45th IEEE Conference on Decision and Control, pp. 2819-2824, 2006

13. L. Wang, Q. Wang, Synchronization in complex networks with switching topology, Physics Letters A, vol. 375, no. 34, pp. 3070-3074, 2011.
14. W. Yu, J. Cao, Synchornization conrol of switched linearly coupled neural networks with delay, Physics Letters A, vol. 375, no. 4-6, pp. 3070-3074, 2010.
15. T. Liu, J. Zhao, David J. Hill, Exponential synchronization of complex delayed dynamical networks with switching topology, IEEE Transactions on Circuits and Systems, vol. 57, no. 11, pp. 2967-2980, 2010.
16. Q. Jia, W. Tang, Wolfgang A. Halang, Leader following of nonlinear agents with switching connective network and coupling delay, IEEE Transactions on Circuits and Systems, vol. 58, no. 10, pp. 2508-2519, 2011
17. Y.Sun, Mean Square Consensus for Uncertain Multiagent Systems with Noises and Delays, Abstract and Applied Analysis, doi:10.1155/2012/621060
18. X. S. Yang, J. D. Cao, Adaptive Pinning Synchronization of Complex Networks with Stochastic Perturbations,Discrete Dynamics in Nature and Society, Volume 2010, Article ID 416182, 21 pages
19. J. Cao, Z.Wang, Y. Sun, Synchronization in an array of linearly stochastically coupled networks with time delays, Physica A 385 (2007) 718-728
20. J. Liang, Z. Wang, Y. Liu, X. Liu, Global synchronization control of general delayed discretetime networks with stochastic coupling and disturbances, IEEE Transactions on Systems, Man, and Cybernetics, Part B 38 (2008) 1073-1083
21. A. Pototsky and N. Janson, Synchronization of a large number of continuous one-dimensional stochastic elements with time-delayed mean-field coupling, Physica D 238 (2009) 175-183
22. X. Yang, J. Cao, Stochastic synchronization of coupled neural networks with intermittent control, Physics Letters A 373 (2009) 3259-3272
23. J.Wang, C.Xu, J.Feng, Mean-Square Exponential Synchronization of Markovian Switching Stochastic Complex Networks with Time-Varying Delays by Pinning Control, Abstract And Applied Analysis, doi: 10.1155/2012/298095
24. S. Cai, J. Zhao, L. Xiang, Z. Liu, Robust impulsive synchronization of complex delayed dynamical networks, Physics Letters A, vol. 372, no. 30, pp. 4990-4995, 2008.
25. Y. Yang, J. Cao, Exponential synchronization of the complex dynamical networks with a coupling delay and impulsive effects, Nonlinear Analysis: Real World Applications, vol. 11, no. 3, pp. 1650-1659, 2010
26. J. Zhou, L. Xiang, Z. Liu, Synchronization in complex delayed dynamical networks with impulsive effects, Physics A, vol. 384, no. 2, pp. 684-692, 2007.
27. W. Zhu, D. Xu, Y. Huang, Global impulsive exponential synchronization of time-delayed coupled chaotic systems, Chaos, Solitons, Fractals, vol. 35, no. 5, pp. 904-912, 2008.
28. H. Jiang, Q. Bi, S. Zheng, Impulsive consensus in directed networks of identical nonlinear oscillators with switching topologies, Commun Nonlinear Sci Numer Simulat, vol. 17, no. 1, pp. 378-387, 2012.
29. W. Xia, J. cao, Pinning synchronization of delayed dynamical networks via periodically intermittent control, Chaos, vol. 19, no. 1, 2009.