

谢树艺 《矢量分析与场论》 习题 1.5

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题目. 设 $\mathbf{a}(t)$ 三阶可导, 证明

$$\frac{d}{dt} \left[\mathbf{a} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^2\mathbf{a}}{dt^2} \right) \right] = \mathbf{a} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^3\mathbf{a}}{dt^3} \right).$$

证明.

$$\begin{aligned} \frac{d}{dt} \left[\mathbf{a} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^2\mathbf{a}}{dt^2} \right) \right] &= \frac{d\mathbf{a}}{dt} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^2\mathbf{a}}{dt^2} \right) + \mathbf{a} \cdot \frac{d}{dt} \left(\frac{d\mathbf{a}}{dt} \times \frac{d^2\mathbf{a}}{dt^2} \right) \\ &= \frac{d\mathbf{a}}{dt} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^2\mathbf{a}}{dt^2} \right) + \mathbf{a} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^3\mathbf{a}}{dt^3} \right) \\ &= \mathbf{a} \cdot \left(\frac{d\mathbf{a}}{dt} \times \frac{d^3\mathbf{a}}{dt^3} \right). \end{aligned}$$

□

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