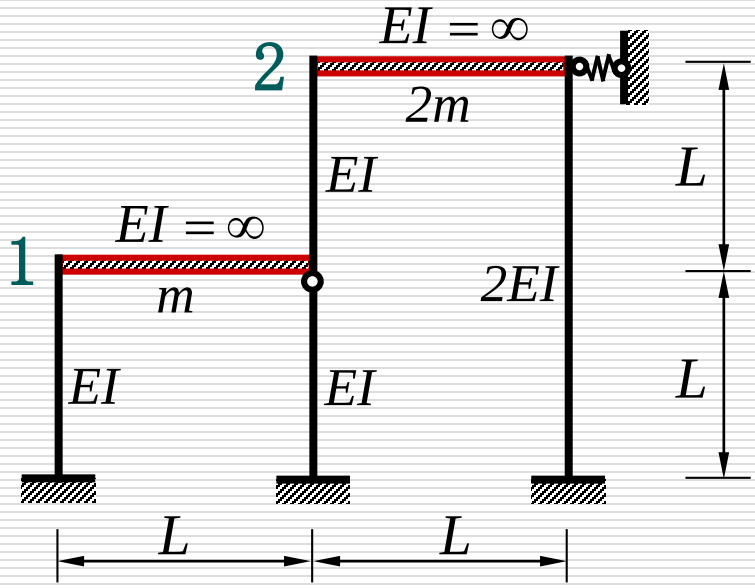
习题课

[习题1] 求图示结构的自振频率和主振型，并验证主振型的正交性。已知弹簧刚度为EI/L^3，不计杆件的轴向变形。



解:用刚度法求解

(1) 求刚度系数

$$k_{11}=\frac{12EI}{L^3}\times 2+\frac{3EI}{L^3}=\frac{27EI}{L^3}$$

$$k_{12}=k_{21}=-\frac{12EI}{L^3}$$

$$k_{22}=\frac{12EI}{L^3}+\frac{12(2EI)}{(2L)^3}+\frac{EI}{L^3}=\frac{16EI}{L^3}$$

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(2) 求自振频率

$$\omega_{1,2}^2=\frac{1}{2}\left(\frac{k_{11}}{m_1}+\frac{k_{22}}{m_2}\right)\pm\sqrt{\left(\frac{k_{11}}{m_1}+\frac{k_{22}}{m_2}\right)^2-\frac{4\left(k_{11}k_{22}-k_{12}k_{21}\right)}{m_1m_2}}$$

将质量参数和刚度系数代入上式，得：

$$\omega_1=2.182\sqrt{\frac{EI}{mL^3}}\qquad\omega_2=5.499\sqrt{\frac{EI}{mL^3}}$$

(3) 求主振型

$$\frac{Y_1^{(1)}}{Y_2^{(1)}}=-\frac{k_{12}}{k_{11}-\omega_1^2m_1}$$

设：$Y_1^{(1)}=1.000$

得：$Y_2^{(1)}=1.853$

$$\left\{Y^{(1)}\right\}=\begin{Bmatrix}1.000\\1.853\end{Bmatrix}$$

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$$\frac{Y_1^{(2)}}{Y_2^{(2)}} = -\frac{k_{12}}{k_{11} - \omega_2^2 m_1}$$

设：$Y_1^{(2)} = 1.000$

得：$Y_2^{(2)} = -0.270$

$$\left\{Y^{(2)}\right\} = \begin{Bmatrix} 1.000 \\ -0.270 \end{Bmatrix}$$

(4) 验证主振型的正交性

$$\left\{Y^{(1)}\right\}^T [M] \left\{Y^{(2)}\right\}$$

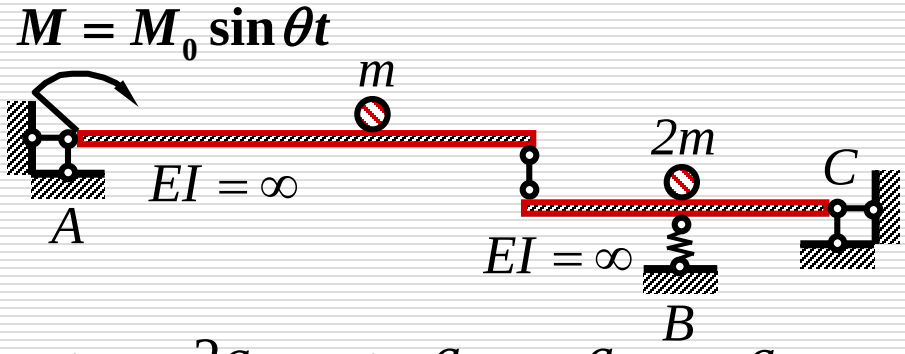
$$= \begin{bmatrix} 1 & 1.853 \end{bmatrix} \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \begin{Bmatrix} 1 \\ -0.270 \end{Bmatrix} = -6.2 \times 10^{-4} m \approx 0$$

$$\left\{Y^{(1)}\right\}^T [K] \left\{Y^{(2)}\right\}$$

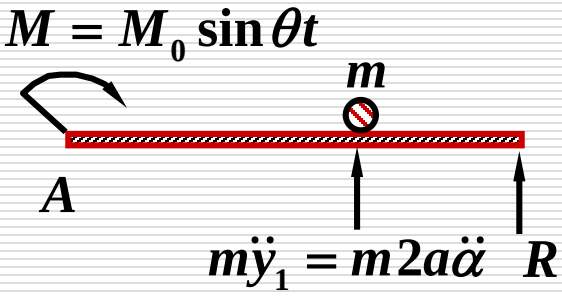
$$= \begin{bmatrix} 1 & 1.853 \end{bmatrix} \frac{EI}{L^3} \begin{bmatrix} 27 & -12 \\ -12 & 16 \end{bmatrix} \begin{Bmatrix} 1 \\ -0.270 \end{Bmatrix} = -9.6 \times 10^{-4} \frac{EI}{L^3} \approx 0$$

习题课

[习题2] 图示结构在杆端A处作用力矩$M=M_0\sin\theta t$，弹性支座的刚度为k，各刚性杆的质量可以不计，求各质量的最大动位移。

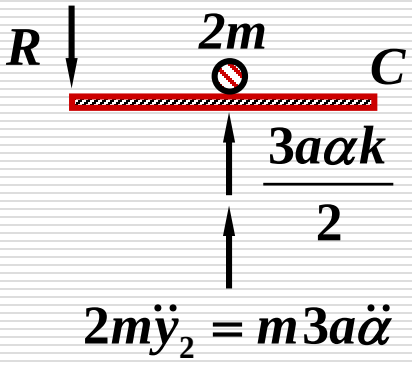


解:这是个单自由度体系，先建立运动方程，再求解。



$$\sum M_A = 0$$

$$3Ra + 4ma^2\ddot{\alpha} - M(t) = 0$$



$$\sum M_C = 0$$

$$2Ra - 3ma^2\ddot{\alpha} - \frac{3ka^2\alpha}{2} = 0$$

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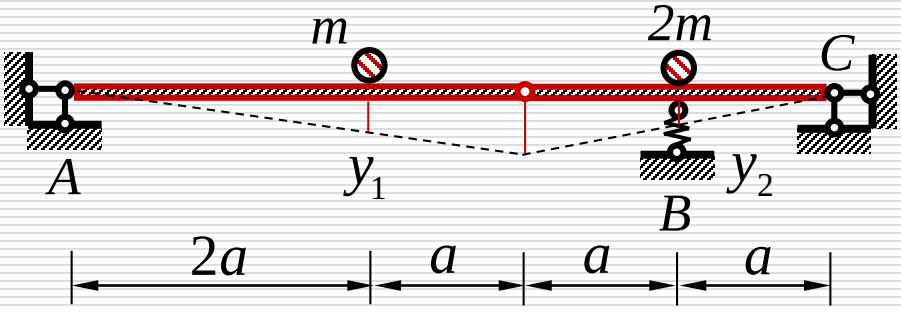
消去两部分的互相作用力R,求得:

$$34ma^2\ddot{\alpha}+9ka^2\alpha=4M_0\sin\theta t\Rightarrow\ddot{\alpha}+\frac{9ka^2}{34ma^2}\alpha=\frac{4M_0\sin\theta t}{34ma^2}$$

$$\Rightarrow\alpha_{\max}=\frac{4M_0}{34ma^2\left(\dfrac{9k}{34m}-\theta^2\right)}=\frac{4M_0}{a^2\left(9k-34m\theta^2\right)}$$

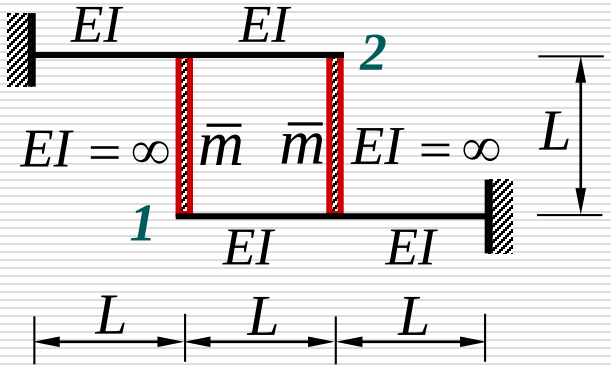


$$y_{1\max}=2a\alpha_{\max}=\frac{8M_0}{a\left(9k-34m\theta^2\right)}$$

$$y_{2\max}=\frac{3a}{2}\alpha_{\max}=\frac{6M_0}{a\left(9k-34m\theta^2\right)}$$

习题课

[习题3] 求图示结构的自振频率，其中刚性杆每单位长度具有均布质量，而弹性杆的质量可以不计。



解:用刚度法求解

(1) 求刚度系数

$$k_{11}=\frac{12EI}{L^3}\times 3=\frac{36EI}{L^3}=k_{22}$$

$$k_{12}=-\frac{12EI}{L^3}\times 2=-\frac{24EI}{L^3}=k_{21}$$

(2) 求自振频率

$$\omega_{1,2}^2=\frac{1}{2}\left(\frac{k_{11}}{m_1}+\frac{k_{22}}{m_2}\right)\pm\sqrt{\left(\frac{k_{11}}{m_1}+\frac{k_{22}}{m_2}\right)^2-\frac{4\left(k_{11}k_{22}-k_{12}k_{21}\right)}{m_1m_2}}$$

$$\omega_1=\sqrt{\frac{12EI}{\bar{m}L^4}}$$

$$\omega_2=\sqrt{\frac{60EI}{\bar{m}L^4}}$$

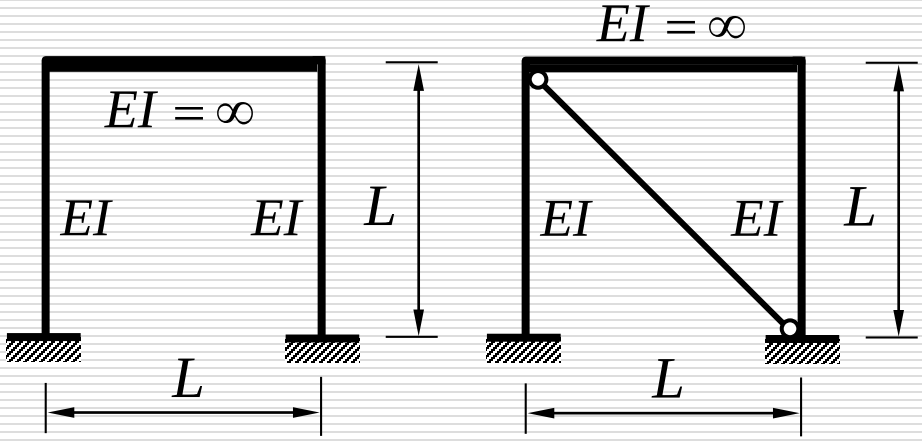
习题课

[习题4] 图示刚架横梁为无穷刚性，自由振动测得自振周期为0.65s，经过10个周期后，振幅衰减至原来的5%；后加设斜支撑进行加固，已知斜支撑的提供的侧向刚度为10kN/m，加固后自由振动测得自振周期为0.5s，L=3m。求：加固前刚架的阻尼比；EI和m的具体数值。

$$\xi=\frac{1}{2\pi n}\ln\left(\frac{y_k}{y_{k+n}}\right)$$

$$\xi=\frac{1}{2\pi\times 10}\ln\left(\frac{y_k}{0.05\times y_k}\right)$$

$$=\frac{\ln 20}{20\pi}=0.0478$$



加固前　　加固后

习题课

加固前后周期比：$\dfrac{T_1}{T_2}=\dfrac{\omega_2}{\omega_1}=\sqrt{\dfrac{k_2}{k_1}}=\sqrt{\dfrac{k+k'}{k}}$

已知：$k'=10.00kN/m$

求得：$k=14.49kN/m$

因为：$k=\dfrac{24EI}{h^3}$

所以：$EI=16.3kN\cdot m^2$

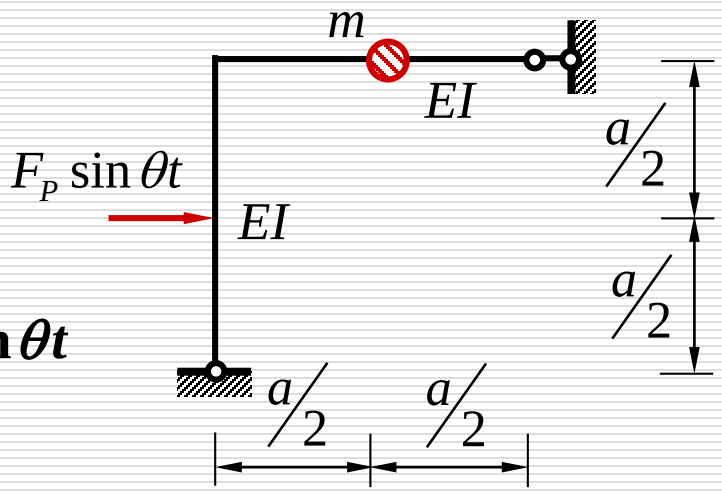
因为：$T=\dfrac{2\pi}{\omega}=2\pi\sqrt{\dfrac{m}{k}}$

所以：$m=\dfrac{kT^2}{4\pi^2}=0.155kg$

习题课

[习题6] 计算图示结构的最大动弯矩图。已知柔度系数：

$$\delta_{11}=\frac{a^3}{8EI}\quad \delta_{22}=\frac{a^3}{48EI}\quad \delta_{12}=\delta_{21}=-\frac{a^3}{32EI}\quad \theta^2=12EI/ma^3$$



解：质点的竖向运动方程：

$$y(t)=\delta_{11}(-m\ddot{y})+\delta_{12}F_p(t)$$

$$\ddot{y}+\frac{1}{m\delta_{11}}y=\frac{\delta_{12}}{\delta_{11}}\frac{F_p}{m}\sin\theta t=-0.25\frac{F_P}{m}\sin\theta t$$

$$y(t)=-0.25\frac{F_P}{m\omega^2(1-\dfrac{\theta^2}{\omega^2})}\sin\theta t$$

$$-m\ddot{y}=-0.25\frac{F_P\theta^2}{\omega^2(1-\dfrac{\theta^2}{\omega^2})}\sin\theta t$$

$$\omega=\sqrt{\frac{1}{m\delta_{11}}}=\sqrt{\frac{8EI}{ma^3}}$$

习题课

把惯性力的最大值：$0.75F_P$、动荷载的最大值同时作用在结构上画出最大动弯矩图：