2 3	Nonlinear dynamics of North Atlantic Oscillation events: A multi-scale interaction view
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Abstract

This paper first reviews some important advances of observational and theoretical studies 2 of North Atlantic Oscillation (NAO) events. On this basis, the planetary- and synoptic-scale 3 features during the evolution of NAO events are examined. It is found that the time-space 4 evolution of synoptic-scale eddy fields depends strongly on the planetary-scale field of the 5 NAO event for the different phase. There is a positive (negative) correlation between the 6 synoptic-scale eddy kinetic energy (EKE) and NAO strength for the negative (positive) phase 7 of NAO. 8 A nonlinear multi-scale interaction (NMI) model is used to examine the dynamical process 9 of NAO events. It is shown that the synoptic EKE is intensified (weakened) as the NAO-10 (NAO⁺) amplitude increases during the NAO life cycle. The nonlinear feedback of the 11 planetary-scale NAO field on synoptic eddies is opposite between two phases of the NAO, 12 which is able to increase (decrease) the eddy vorticity flux divergence due to the enhanced 13 (reduced) cyclonic wave breaking as the NAO amplitude intensifies. The enhanced (reduced) 14 eddy vorticity flux divergence can (cannot) counteract the dissipation of NAO⁻ (NAO⁺) 15 events so that the life times of NAO⁻ (NAO⁺) events are prolonged (shortened). As a result, 16 NAO⁻ (NAO⁺) events may be more (less) persistent, as observed. A comparison of the model 17 results with observations reveals that the NMI model is an efficient tool for investigating 18 internal dynamical processes of NAO events. 19

20

1 **1. Introduction**

The North Atlantic Oscillation is a very important low-frequency dipole mode confined in the Atlantic basin. This topic has attracted a great interest of scientists because it plays a significant role in cold and warm winters over Europe and its adjacent regions (Hurrell 1995; Scaife et al. 2008; Sillmann et al. 2010; Luo et al. 2014a).

In recent years, the dynamics of the NAO event has been an important research topic 6 7 (Benedict et al. 2004; Franzke et al. 2004; Vallis et al. 2004; Jin et al 2006; Luo et al. 2007a-c; Woollings et al. 2008). As suggested by Benedict et al. (2004), Franzke et al. (2004) and 8 Woollings et al. (2008), the presence of cyclonic (anticyclonic) wave breaking or CWB (AWB) 9 hereafter is responsible for the occurrence of NAO⁻ (NAO⁺) events. However, no analytical 10 models can describe how the CWB (AWB) excites NAO⁻ (NAO⁺) events. Woollings et al. 11 (2008) suggested that NAO⁻ (NAO⁺) events correspond to the presence (absence) of CWB. 12 Luo et al. (2007a,c) developed a nonlinear multi-scale interaction (NMI) model to describe how 13 synoptic-scale eddies drive NAO events with 1-2 week timescales. Based upon this model they 14 noted that the presence of the CWB (AWB) is a concomitant phenomenon of the NAO-15 (NAO⁺) event occurrence. 16

It needs to be pointed out that our NMI model is different from previous models (Vallis et al. 2004; Jin et al 2006). In the models of Vallis et al. (2004) and Jin et al (2006), the eddy energy transfer from the synoptic-scale eddies to the NAO flow (planetary-scale) can be described by separating the atmospheric flow into time-mean and transient eddy parts, defined as $\bar{\psi}$ and ψ' respectively, where $\bar{\psi}$ is the time-mean stream function and ψ' the transient eddy stream

1 function. Then the time-mean PV equation as a description of a large-scale NAO flow can be obtained as $\frac{\partial \overline{q}}{\partial t} + J(\overline{\psi}, \overline{q}) = -\nabla \cdot (\overline{\mathbf{v}'q'}) - D$, where the overbar denotes an average over a long 2 time period compared with a transient eddy lifetime, $\mathbf{v}' = (-\partial \psi' / \partial y, \partial \psi' / \partial x), q'$ is the eddy 3 PV and D is the time-mean dissipation as a PV sink. It is clear that because $-\nabla \cdot \overline{(\mathbf{v}'q')}$ is a 4 time-mean variable, this model can only describe the time-mean state of the NAO flow. On the 5 other hand, in previous models $-\nabla \cdot \overline{(\mathbf{v}'q')}$ is parameterized as a stochastic forcing (Vallis et al. 6 7 2004) and a large-scale term proportional to the NAO anomaly (Jin et al. 2006). Naturally, these models cannot describe the mutual relationship between planetary and synoptic scales 8 during the NAO life process. In the NMI model of blocking proposed by Luo (2005) 9 $-J(\psi',q')_p = -\nabla \cdot (\mathbf{v}'q')_p$ is used instead of $-\overline{J(\psi',q')} = -\nabla \cdot (\overline{\mathbf{v}'q'})$, where "P" denotes the 10 zonal wavenumber of $-\nabla \cdot (\mathbf{v}'q')_p$ close to that of blocking. The most merit of this model is 11 able to describe the life cycle (onset, maintenance and decay) of a blocking flow because 12 $-\nabla \cdot (\mathbf{v}'q')_p$ is a time-dependent forcing term. By extending this blocking model, Luo et al. 13 (2007a,b, c) developed the NMI model of the NAO event to clarify the basic physics of the 14 NAO dynamics. Results from this model can implicate that the NAO event is a nonlinear initial 15 16 value problem (Benedict et al. 2004; Franzke et al. 2004). The most advantage of this model is 17 able to better identify the mutual relationships between planetary and synoptic scale fields and 18 between the phase of NAO and the frequency of European blocking events during the NAO life cycle (Luo et al. 2007a). Thus, in this paper the NMI model will be used to examine our 19 problems. 20



previous studies (Benedict et al. 2004; Franzke et al. 2004; Vallis et al. 2004; Jin et al. 2006; Luo et al. 2007a-c), the causal relationship between the NAO anomaly and synoptic-scale eddies during the NAO life cycle is unclear so far. In this paper, we will reveal from both observational and theoretical aspects that the spatial structure of synoptic-scale eddies prior to the NAO onset and the feedback of subsequent NAO anomaly are important for the mutual relationship between the NAO anomaly and synoptic-scale eddy variations as well as the asymmetry of the NAO persistence between two phases.

8 This paper is organized as follows: The reanalysis data and method used in this paper are 9 introduced in section 2. The main characters of observed NAO events are presented in section 3. 10 In section 4, we briefly describe the NMI model of NAO events. This NMI model is used to 11 examine the mutual relationship between planetary- and synoptic-scale fields during the NAO 12 life process and free mode characteristics of NAO events in sections 5 and 6, respectively. In 13 section 7, with the NMI model we examine if the wave breaking can affect the persistence of 14 NAO events. Conclusion and discussions are summarized in the final section.

15 **2. Data and methodology**

In this paper, we used the $2.5^{\circ} \times 2.5^{\circ}$ gridded daily data of 300-hPa geopotential height from the National Centers for Environmental Prediction-National Center for Atmospheric Research (NCEP- NCAR) reanalysis data. The daily NAO indices used here are available from the National Oceanic and Atmospheric Administration (NOAA) and the Climate Prediction Center (CPC; <u>http://www.cpc.noaa.gov/</u>). The daily NAO index is defined as the principal component time series of the leading rotated empirical orthogonal function (REOF) of the 500-hPa geopotential height. The seasonal cycle has been subtracted from the fields at each
 grid point.

A NAO event is defined to have taken place if the daily NAO index has one standard 3 deviation persisting for at least three consecutive days. In this paper, our study is focused on the 4 winter from December to February (DJF). Similar to the definition of Luo et al. (2012a), 5 individual NAO events are defined to be events that are not followed by opposite phase events, 6 7 whereas a NAO regime transition event must include both positive and negative phase NAO (NAO⁺ and NAO⁻) events. The time interval of a NAO transition event from the beginning of 8 a NAO⁺ (NAO⁻) event to the end of a NAO⁻ (NAO⁺) event is defined to not exceed 45 9 10 days.

3. Characteristics of observed NAO events

In terms of the definitions of individual NAO and NAO regime transition events, we calculate the event numbers of NAO⁺, NAO⁻, NAO⁺ to NAO⁻ and NAO⁻ to NAO⁺ transition events in winter during 1950-2013 and show their results in Fig. 1. It is clear that the individual NAO⁺ (NAO⁻) events are more frequent during 1970-2013 (1950-1970). In particular, during the period 1950-2013 the total number of individual NAO⁺ (NAO⁺ to NAO⁻) events is larger than that of individual NAO⁻ (NAO⁻ to NAO⁺) events. Based upon these events, we can conveniently show the features of observed NAO events.

a) Asymmetry of the NAO persistence between positive and negative phases.

Because individual NAO events are much more frequent than NAO transition events during
1950-2013, here we only consider the case of individual NAO events. We show the composite

daily NAO indices of individual NAO⁺ and NAO⁻ events in Fig. 2. It is found that the e-fold
time of individual NAO⁻ events is longer than that of individual NAO⁺ events. In other words,
there is an asymmetry of the NAO persistence between two phases. Barnes and Hartmann
(2010) attributed the asymmetry of the NAO persistence to the stronger eddy forcing during the
NAO⁻ episodes. In following sections, we will provide a new explanation for why NAO⁻
events are more persistent than NAO⁺ events.

7 b) Two-way relationship between planetary- and synoptic-scale structures of NAO events

As can be seen from the reanalysis data, the unfiltered fields of observed NAO⁻ (NAO⁺) 8 events exhibit the weakening (strengthening) of the zonal westerly wind or the presence 9 (absence) of the meandering midlatitude westerly flow (Berggren et al. 1949; Luo et al. 2007a, 10 their Fig.6). To see clearly the mutual relationship between planetary- and synoptic-scale 11 structures of the NAO event, we consider the composite 300-hpa geopotential height and 12 synoptic-scale (2.5-7 days) eddy kinetic energy (EKE) anomaly fields for two phases of all 13 NAO events as planetary- (ψ_p) and synoptic- (ψ_p) scale fields of NAO events. The composite 14 geopotential height and EKE anomaly fields for all individual NAO⁻ and NAO⁺ events 15 during 1950-2013 are shown in Figs. 3 and 4, respectively. It is seen that the composite 16 NAO - field exhibits a life process (growth, maintenance and decay) of an Omega-type 17 blocking (Fig. 3a), whereas the composite NAO⁺ field shows the life cycle of an intensified 18 zonal flow in the Atlantic basin (Fig. 3b). In an anomaly field, the life cycle of the NAO-19 (NAO⁺) pattern can be characterized by the growth and decay of a positive-over-negative 20 (negative-over-positive) dipole anomaly (Figs. 3c-d). On Lag 0 day, the NAO amplitude is 21

largest for both phases. A comparison of Fig. 3c with Fig.3d shows that the amplitude of the 1 NAO⁻ dipole is stronger than that of the NAO⁺ dipole. Such an asymmetry may be related to 2 the stronger downstream energy dispersion of the NAO⁺ pattern (Luo et al. 2007a). Moreover, 3 we can see that the growth and decay of the NAO⁺ dipole is more rapid than those of the 4 NAO⁻dipole. This hints that the NAO⁻ pattern is more persistent than the NAO⁺ pattern. 5 Another interesting result is that the NAO⁻ (NAO⁺) dipole pattern on intraseasonal time 6 7 scales undergoes a westward (eastward) shift. Such an intreasesonal westward (eastward) displacement of the NAO⁻ (NAO⁺) pattern was detected by Luo et al. (2012b). In fact, the 8 intraseasonal westward shift of the NAO⁻ dipole pattern was also noted by Feldstein (2003) 9 and Sung et al. (2011), while the interannual eastward shift of the NAO⁺ pattern was widely 10 observed by Hilmer and Jung (2000), Jung et al. (2003) and Peterson et al. (2003) as well as its 11 physical cause was investigated by Luo and Gong (2006), Dong et al. (2011) and Davini et al. 12 (2012). 13

It is further seen from the composite EKE field that during the NAO-life process the 14 Atlantic storm track denoted by the maximum EKE region splits into two branches around the 15 north and south sides of the NAO⁻ region, which is more evident during the period from Lag -5 16 day to Lag +3 day (Fig. 4a). For the positive phase, the Atlantic storm track exhibits a single 17 branch structure, whose strength undergoes an opposite variation with the NAO⁺ amplitude 18 (Fig. 4b). To clearly see how the composite EKE strength changes during the NAO life process, 19 we show the time-longitude evolution of the composite EKE anomaly averaged over the region 20 from $90^{\circ}W$ to $20^{\circ}E$ in Fig. 5 for individual NAO and transition events. It is seen that for the 21

1	composite of individual NAO ⁻ events the EKE strength is much stronger over the north side of
2	the NAO-region than its south side, which is statistically significant at the 95% confidence
3	level for a two-sided student's t- test. The strengthening of EKE is more notable during the
4	growing stage of the NAO ⁻ pattern than during its decaying stage. The EKE strength increases
5	as the NAO ⁻ anomaly strengthens, whose amplitude reaches a maximum at lag -1 day (Fig. 5a).
6	For individual NAO ⁺ events, the composite EKE strength exhibits an opposite variation with
7	the NAO ⁺ amplitude. In particular, the EKE is weaker during the period from lag -2 day to lag
8	+3 day (Fig. 5b) because the NAO ⁺ anomaly is stronger during that period (Fig. 3d).
9	To clearly see if there is a strong correlation between the EKE and the NAO strength, we
10	define the meridional average values of the composite daily EKEs in Figs. 5a-b over the
11	regions $40^{\circ}N - 65^{\circ}N$ and $65^{\circ}N - 90^{\circ}N$ as the daily EKE strengths during the NAO ⁻ and
12	NAO+life processes, respectively. The average region choice of the negative phase is based
13	upon the fact that for the negative phase the composite EKE is much stronger in the north side
14	of the NAO ⁻ region than that in its south side. Moreover, to reflect the strength of the NAO
15	anomaly, the absolute value of the difference between the composite daily geoptential height
16	anomalies averaged over the northern ($60^{\circ}N - 75^{\circ}N$, $80^{\circ}W - 0^{\circ}$) and southern
17	$(30^{\circ}N - 45^{\circ}N, 60^{\circ}W - 20^{\circ}E)$ regions is defined as the daily NAO strength for two phases of
18	NAO. As shown in Figs. 5c-d, the EKE strength exhibits a positive correlation of 0.86 with the
19	NAO ^{$-$} strength, and a negative correlation of -0.68 with the NAO ^{$+$} strength. Thus, the above
20	results indicate that the synoptic eddy kinetic energy tends to become stronger (weaker) as the
21	NAO- (NAO+) amplitude strengthens.

For the NAO⁻ to NAO⁺ (NAO⁺ to NAO⁻) transition events, the variation of the composite EKE strength can be reflected by a combination of the EKE variations during both NAO⁻ and NAO⁺ (NAO⁺ and NAO⁻) episodes (not shown). Totally, we can obtain a conclusion that when the NAO- (NAO+) amplitude strengthens, the synoptic-scale EKE tends to be stronger (weaker) and exhibit a double (single) branch structure. The reason of why the synoptic-scale EKE can exhibit such a behavior during the NAO life cycle will be examined below.

8 c) Free mode characteristics

In the diagnostic study of NAO events, Luo et al. (2007b) computed the relationship between 9 the potential vorticity (PV) q and streamfunction ψ . They found that there is a linear relation 10 between the planetary-scale PV q_p and streamfunction ψ_p for two phases of the NAO pattern 11 (Fig. 11 of Luo et al. 2007b). Such a linear functional relationship of $q_P = F(\psi_P)$ implies that 12 the NAO pattern is a free mode for its two phases during the whole life process even though 13 forced by synoptic-scale eddies (Benedict et al. 2004; Franzke et al. 2004). If the PV and 14 streamfunction include synoptic scale eddies, $q = F(\psi)$ does not strictly satisfy a linear 15 relationship (Fig. 13 of Luo et al. 2007b). A similar calculation can be made here and a 16 similar result can be obtained (not shown). 17

18 4. Nonlinear multi-scale interaction model of NAO events

a) Nonlinear multi-scale interaction equation

As noted by Benedict et al. (2004) and Franzke et al. (2004), the NAO process is a nonlinear initial value problem. In this subsection, we will propose a schematic picture to show the two phases of NAO patterns are driven by synoptic-scale eddies prior to the NAO onset. As demonstrated by previous studies (Vallis et al. 2004; Jin et al. 2006), the barotropic model can capture the essential dynamics of NAO events. Thus, here a non-dimensional equivalent barotropic PV equation will be used to examine our problem.

5 The non-dimensional equivalent barotropic PV equation with topography, forcing and 6 dissipation in a beta-plane channel can be written as

7
$$\frac{\partial}{\partial t} (\nabla^2 \Psi_T - F \Psi_T) + J(\Psi_T, \nabla^2 \Psi_T + h) + \beta \frac{\partial \Psi_T}{\partial x} = F + A_H \nabla^2 q_T, \qquad (1)$$

8 where Ψ_T is the non-dimensional total streamfunction, β is the nondimensional meridional 9 gradient of the Coriolis parameter, $q_T = \nabla^2 \Psi_T - F \psi_T + f$ is the PV, $F = (L/R_d)^2$, *h* is 10 characteristic horizontal scale and R_d is the radius of Rossby deformation, *h* is the 11 non-dimensional topographic variable, *F* is the forcing term, A_H is the horizontal 12 dissipation coefficient and the other notation is often used in the meteorological references.

As demonstrated by many investigators (Lau 1988; Feldstein 2003), the occurrence of the NAO pattern is a complicated nonlinear multi-scale interaction (NMI) process. In recent years, Luo and his coauthors have developed a NMI model to describe the life processes of individual NAO events under the synoptic eddy forcing (Luo et al. 2007a-c, Luo and Cha 2012) based upon the blocking model of Luo (2005). In this paper, the NMI model will be used to explain the mutual relationship between planetary- and synoptic-scale structures of the NAO event during its whole process.

As revealed by Luo et al. (2007c), the mean zonal wind in the Atlantic basin is almost the same and approximately uniform in horizontal directions for two phases of the NAO event

before the NAO event occurs. In this case, we may assume that the mean zonal wind prior to 1 the NAO onset is a constant. 2

As a result, if we separate the total streamfunction (Ψ_T) into $\Psi_T = -u_0 y + \psi + \psi'$ (where 3 u_0 is a constant zonal wind prior to the NAO onset, and ψ (ψ') represents planetary-4 (synoptic-) scale anomaly), under the scale separation assumption the equations of the 5 interaction between planetary and synoptic scales in a uniform basic flow u_0 can be obtained 6

as (Luo et al. 2007a-c) 7

8
$$(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}) (\nabla^2 \psi - F \psi) + J(\psi, \nabla^2 \psi + h) + (\beta + F u_0) \frac{\partial \psi}{\partial x} + u_0 \frac{\partial h}{\partial x}$$

9
$$= -J(\psi', \nabla^2 \psi')_P + A_H \nabla^2 q ,$$
 (2a)

10
$$(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}) (\nabla^2 \psi' - F \psi') + (\beta + F u_0) \frac{\partial \psi'}{\partial x} = -J(\psi', \nabla^2 \psi + h) - J(\psi, \nabla^2 \psi')$$

11
$$+ \nabla^2 \psi_s^* + A_\mu \nabla^2 q',$$
(2b)

where $q = \nabla^2 \psi - F \psi + f$, $q' = \nabla^2 \psi' - F \psi'$ and $F = \nabla^2 \psi_s^*$ has been assumed in Eq. (2) and 12 denotes the synoptic-scale vorticity source introduced to maintain synoptic-scale eddies in the 13 west of the Atlantic basin. 14

It should be noted that $-J(\psi', \nabla^2 \psi')_p = -\nabla \cdot (\mathbf{v}' q')_p$, where $\mathbf{v}' = (-\partial \psi' / \partial y, \partial \psi' / \partial x)$, 15 denotes the planetary-scale component of the eddy vorticity flux divergence, which tend to 16 induce the variation of the planetary-scale anomaly (Luo 2005; Luo et al. 2007a, c). The term 17 $-\nabla \cdot (\mathbf{v}'q')_p$ is referred to as the eddy vorticity forcing (EVF), hereafter. Thus, Eq. (2a) reflects 18 the contribution of synoptic-scale eddies to the growth and decay of the planetary-scale 19 anomaly. In Eq. (2b), $J(\psi', \nabla^2 \psi + h)$ and $J(\psi, \nabla^2 \psi')$ are the interaction terms between 20 planetary and synoptic scales, which describe that the feedback of the planetary-scale anomaly 21

on synoptic-scale eddies can induce the deformation of eddies once the planetary- scale
 anomaly is reinforced by eddies. Thus, Eqs.(2a-b) can describe the nonlinear multi-scale
 interaction associated with NAO events during their evolution processes.

4 b) Driving mechanism of the NAO growth

5 Eq. (2a) can be rewritten as

$$6 \qquad \frac{\partial}{\partial t} (\nabla^2 \psi - F \psi) + J(\psi - u_0 y, \nabla^2 \psi - F \psi + \beta y + h) = -\nabla \cdot (\mathbf{v}' q')_p + A_H \nabla^2 q . \tag{3}$$

Previous diagnostic study shows that during the life process of the NAO event there is a linear relationship between the planetary-scale PV and streamfunction although forced by synoptic-scale eddies (Luo et al. 2007b). As a result, we may assume that there exists $J(\psi - u_0 y, \nabla^2 \psi - F \psi + \beta y + h) \approx 0$. In this case, (3) and (2b) reduce to

11
$$\frac{\partial q}{\partial t} \approx -\nabla \cdot (\mathbf{v}' q')_P + A_H \nabla^2 q, \qquad (4a)$$

12
$$(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x})q' + (\beta + Fu_0) \frac{\partial \psi'}{\partial x} = -J(\psi', \nabla^2 \psi + h) - J(\psi, \nabla^2 \psi') + \nabla^2 \psi_s^* + A_H \nabla^2 q'.$$
(4b)

As in Luo (2005) and Luo et al. (2007a), synoptic-scale eddies are assumed to be comprised of two parts: synoptic eddies ψ'_1 prior to the NAO onset and deformed eddies ψ'_2 due to the presence of the NAO anomaly. In $\psi' = \psi'_1 + \psi'_2$, ψ'_1 does not include the role of the NAO anomaly and then is referred to as the preexisting synoptic eddies" hereafter, while $\psi'_2 = \psi'_2(\psi'_1, \psi)$ denotes the deformed eddies due to interaction between the NAO anomaly and preexisting eddies. The term ψ'_2 may be referred to as the NAO feedback term hereafter.

In Eq. (4), if $t = \tau$ is defined to be a short time from the initial time t = 0 of the NAO event, we may assume $\psi'_2 \approx 0$ and $A_H \nabla^2 q \approx 0$ during the time range of $0 \le t < \tau$ because of the NAO amplitude being extremely small. In this case, one obtains from Eq. (4a)

$$\frac{\partial q}{\partial t} \approx -\nabla \cdot (\mathbf{v}_1' q_1')_P, \qquad (5)$$

It is clear that when the NAO anomaly has a high-over-low dipole structure (negative phase), 2 $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ must have a negative-over-positive dipole pattern to reinforce the NAO⁻ anomaly 3 at the initial time. In contrast, for the positive phase $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ must have a positive-4 over-negative dipole pattern to reinforce the NAO⁺ anomaly with a low-over-high meridional 5 dipole. Such a mechanism may be referred to as the eddy-NAO matching (ENM) mechanism, 6 7 which is parallel to the eddy-blocking matching (EBM) mechanism proposed by Luo et al. (2014b). The schematic picture of the ENM mechanism of the NAO occurrence can be 8 described in Fig. 6. Thus, the spatial pattern $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ due to the preexisting synoptic 9 eddies is crucial for whether a NAO⁻ or NAO⁺ event is produced. 10

11 c) What causes synoptic wave breaking?

12 Substituting $\psi' = \psi'_1 + \psi'_2$ into Eq. (4b) and using the assumption as made in Luo (2005)

13
$$(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}) (\nabla^2 \psi_1' - F \psi_1') + (\beta + F u_0) \frac{\partial \psi_1'}{\partial x} = \nabla^2 \psi_s^* + A_H \nabla^2 q_1'$$

14 Eq. (4b) reduces to

1

15
$$(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x})q_2' + (\beta + Fu_0)\frac{\partial \psi_2'}{\partial x} = -J(\psi_1', \nabla^2 \psi + h) - J(\psi, \nabla^2 \psi_1') + A_H \nabla^2 q_2',$$
(6)

where $q'_j = \nabla^2 \psi'_j - F \psi'_j$ (j = 1, 2), and $\nabla^2 \psi^*_s$ is the synoptic-scale vorticity source designed to maintain preexisting synoptic- scale eddies ψ'_1 .

It is seen from Eq. (6) that the feedback of the NAO anomaly will induce deformed eddies once the NAO anomaly is significantly intensified during the period for $t > \tau$. That is to say, eddies ψ'_2 are induced by the feedback of the NAO anomaly due to the presence of both terms $J(\psi'_1, \nabla^2 \psi + h)$ and $J(\psi, \nabla^2 \psi'_1)$. In particular, we will find that the feedback of the amplified NAO- anomaly on eddies propagating in the intensified NAO- anomaly region will lead to the CWB. As we will further demonstrate below with the analytical solutions of a NAO event, eddies ψ'_2 can reflect the presence (absence) of cyclonic wave breaking (CWB) during the NAO- (NAO+) life processes, as observed by Benedict et al. (2004) and Woollings et al. (2008).

6 In the next subsection, the dissipation terms in Eqs.(2a-b) are neglected to obtain the 7 analytical solutions of the model to highlight the contribution of synoptic eddies to the NAO 8 growth and decay. This assumption may be more valid for initial growth of the NAO event and 9 its decay.

10 d) Analytical solutions of the NMI model

Here, we still use the analytical solutions of the NMI model obtained by Luo et al. (2007c) 11 and Luo and Cha (2012) to examine our problem. For a large-scale topography that is an 12 approximation of the land-sea configuration in mid-high latitudes around the Northern 13 Hemisphere, we assume $h = h_0 \exp[ik(x + x_T)]\sin(\frac{m}{2}y) + cc$, where $i = \sqrt{-1}$, x_T is the zonal 14 position of the topographic trough, h_0 is the topographic height, $k = 2k_0$ 15 $(k_0 = 1/(6.371\cos\varphi_0))$ is the zonal wavenumber of the two-wave topography, $m = \pm 2\pi/L_v$, 16 L_{y} is the width of the beta plane channel and cc denotes the complex conjugate of its 17 preceding term. 18

Generally speaking, the travelling synoptic-scale perturbations have a monopole meridional
 structure in the NH mid-high latitudes (Frederisken 1982). In this case, we may assume that the

21 preexisting synoptic-scale eddies are of
$$\psi'_1 = \sum_{j=1}^N \alpha_j f_j(x) \exp[i(\tilde{k}_j x - \tilde{\omega}_j t)] \sin(\frac{m}{2} y) + cc$$
, where

1 $\tilde{\omega}_j$ is the frequency of synoptic-scale eddy with zonal wavenumber \tilde{k}_j , α_j is a constant, 2 $f_j(x)$ is the slowly varying function of x and $N \ge 2$. As a result, $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ has a dipole 3 meridional structure. When the zonal wavenumber of $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ is close to that of the NAO 4 pattern and only when its meridional structure matches the NAO dipole anomaly, the eddy 5 vorticity forcing $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ can reinforce and amplify the NAO pattern. Similar to Luo et al. 6 (2007c), Luo and Cha (2012) and Luo et al. (2014), we consider the simplest case of $\alpha_1 = 1$, 7 $\alpha_2 = \alpha$, $f_1(x) = f_2(x) = f(x)$ and N = 2.

According to Luo et al. (2007c) and Luo and Cha (2012), the analytical solution of the
atmospheric streamfunction
$$(\Psi_T)$$
 of an NAO event with both zonal wavenumber 2 and dipole
meridional structure obtained from the NMI model in a uniform westerly wind u_0 can be
expressed as

12
$$\Psi_T = -u_0 y + \psi + \psi' = \psi_P + \psi',$$
 (7a)

13
$$\psi_P = -u_0 y + \psi \approx -u_0 y + \psi_{NAO} + \psi_C + \psi_m, \qquad (7b)$$

14
$$\psi_{NAO} = B \sqrt{\frac{2}{L_y}} \exp(ikx) \sin(my) + cc , \qquad (7c)$$

15
$$\psi_c = h_A h_0 \exp[ik(x + x_T)]\sin(\frac{m}{2}y) + cc$$
, (7d)

16
$$\psi_m = \psi_{m1} + \psi_{m2}, \qquad (7e)$$

17
$$\psi_{m1} = -|B|^2 \sum_{n=1}^{\infty} q_n g_n \cos(n+1/2)my$$
, (7f)

18
$$\psi_{m2} = -h_0 h_A \sqrt{\frac{2}{L_y}} (Be^{-ikx_T} + B^* e^{ikx_T}) \sum_{n=1}^{\infty} \tilde{q}_n (3a_n - b_n) \cos(nmy), \qquad (7g)$$

19
$$\psi' \approx \varepsilon^{3/2} (\widetilde{\psi}'_0 + \varepsilon \widetilde{\psi}'_1) = \psi'_1 + \psi'_2,$$
 (7h)

20
$$\psi_1' = \varepsilon^{3/2} \widetilde{\psi}_0' = f_0(x) \{ \exp[i(\widetilde{k}_1 x - \widetilde{\omega}_1 t)] + \alpha \exp[i(\widetilde{k}_2 x - \widetilde{\omega}_2 t)] \} \sin(\frac{m}{2} y) + cc,$$
(7i)

1
$$\psi'_{2} = -\frac{m}{4} \sqrt{\frac{2}{L_{y}}} Bf_{0} \sum_{j=1}^{2} Q_{j} \alpha_{j} \exp\{i(\tilde{k}_{j}+k)x - \tilde{\omega}_{j}t\} [p_{j} \sin(\frac{3m}{2}y) + r_{j} \sin(\frac{m}{2}y)]$$

2
$$+\frac{m}{4}\sqrt{\frac{2}{L_{y}}}B^{*}f_{0}\sum_{j=1}^{2}Q_{j}\alpha_{j}\exp\{i(\tilde{k}_{j}-k)x-\tilde{\omega}_{j}t\}[s_{j}\sin(\frac{3m}{2}y)+h_{j}\sin(\frac{m}{2}y)]$$

3
$$-\frac{m}{4}f_0h_0\sum_{j=1}^2\pi_j\alpha_j\exp\{i(\tilde{k}_j+k)x+kx_T-\tilde{\omega}_jt\}\}\sin(2my)$$

4
$$-\frac{m}{4}f_{0}h_{0}\sum_{j=1}^{2}\sigma_{j}\alpha_{j}\exp\{i(\tilde{k}_{j}-k)x-kx_{T}-\tilde{\omega}_{j}t\}\sin(2my)+cc,$$
 (7j)

where $u_0 = u_C + \Delta u$, $u_C = \beta / (k^2 + m^2)$, $|\Delta u| << u_C$, $h_A = -1/[\beta / u_C - (k^2 + m^2 / 4)]$, 5 $\tilde{\omega}_j = u_c \tilde{k}_j - (\beta + F u_c) \tilde{k}_j / (\tilde{k}_j^2 + m^2 / 4 + F)$, $|B|^2 = BB^*$ and $\alpha = \pm 1$. Note that ψ_{NAO} 6 represents the NAO anomaly with the amplitude B whose complex conjugate is B^* and ψ_c 7 denotes the topographically induced stationary monopole wave, which is an approximation of 8 the observed climatological stationary wave (CSW) anomaly in the NH (Luo et al. 2007c and 9 Luo and Cha 2012). Moreover, $f_0(x) = a_0 \exp[-\mu \varepsilon^2 (x + x_0)^2]$ is the spatial distribution of the 10 eddy amplitude for both $0 < \varepsilon \ll 1.0$ and $\mu > 0$, where a_0 is the maximum eddy strength in 11 the western side of the Atlantic basin for $x = -x_0$. The other coefficients and notation can be 12 found in Luo et al. (2007b) and Luo and Cha (2012), except that u_0 in these coefficients 13 should be replaced by u_c . In Eq. (7), $|\Delta u| \ll u_c$ implies that the planetary-scale anomaly 14 prior to the NAO onset is nearly stationary although it is eddy-forced. The choice of $\Delta u > 0$ 15 $(\Delta u < 0)$ is used to represent a strong (weak) background westerly wind. 16

In solution (7), $m = 2\pi/L_y$ and $\alpha = 1$ ($m = -2\pi/L_y$ and $\alpha = -1$) represents the positive (negative) phase of the NAO event. Moreover, $h_0 < 0$ ($h_0 > 0$) is required for the positive (negative) phase (Luo et al. 2007b; Luo and Cha 2012). In winter, the action center of the positive CSW anomaly is generally located near $10^{\circ}W$ and NAO⁻ (NAO⁺) events originates from the Northern Europe (Greenland region) (Feldstein 2003; Sung et al. 2011; Luo et al. 2012b). Thus, at the initial stage of the NAO event $x_T \sim 0$ ($x_T < 0$) should be chosen for its negative (positive) phase so that the center of the initial NAO⁻ (NAO⁺) anomaly is in the same position (upstream side of) the positive CSW anomaly. The spatiotemporal evolution of an initial planetary-scale anomaly into a NAO event with the amplitude B(x,t) can be described by a forced nonlinear Schr \ddot{o} dinger equation of the form

8
$$i(\frac{\partial B}{\partial t} + C_g \frac{\partial B}{\partial x}) + \lambda \frac{\partial^2 B}{\partial x^2} + \delta |B|^2 B + \tilde{\alpha} h_0^2 (B + B^* e^{i2kx_T}) + \Delta u \Gamma B$$

$$+Gf_0^2 \exp[-i(\Delta kx + \Delta \omega t)] = 0, \qquad (8)$$

10 where $\Gamma = -\frac{k(k^2 + m^2)}{(k^2 + m^2 + F)}$, λ and δ , and other coefficients can be found in Luo et al.

11 (2007b) and Luo and Cha (2012).

It is easy to get planetary- and synoptic-scale solutions of a NAO event once the solution to Eq. (8) is obtained for an initial condition. The finite difference scheme used to solve Eq. (8) is the same as that used in Luo (2005). The parameters used in this paper are listed in Table 1. Through looking at the space-time evolutions of the planetary- and synoptic-scale fields of a NAO event for its two phases, the mutual interaction relationship between the planetary- and synoptic-scale structures associated with the NAO event can be understood. In the next section, we will present some results to clarify such a two-way interaction.

5. Mutual relationship between planetary- and synoptic-scale fields during the NAO life cycle.

As an example, $h_0 = 0.4$ and $x_T = 0$ ($h_0 = -0.4$ and $x_T = -1.5$) are chosen as the fixed 2 amplitude and position parameters of the CSW anomaly for the negative (positive) phase of 3 NAO. Moreover, without the loss of generality B(x,0) = 0.35 is chosen as the initial 4 amplitude of the NAO anomaly in order to understand if a uniform planetary-scale wave is 5 reinforced into a zonally isolated NAO mode. For the preexisting synoptic-scale eddies, we 6 may choose $\alpha = -1$ ($\alpha = 1$) so that $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ favors the growth of the NAO⁻ (NAO⁺) 7 anomaly. For $a_0 = 0.17$ and $\Delta u = 0$, we show the instantaneous planetary-scale, synoptic-scale 8 and total fields of an NAO event in Fig. 7 (Fig. 8) for the eddy position $x_0 = 2.87/2$ 9 $(x_0 = 2.87/4)$ for the negative (positive) phase. 10

It is seen in Fig. 7 that for the negative phase there is a weak ridge centered on x = 0 at day 11 0 in the planetary-scale field. This ridge is intensified due to the forcing of $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ and 12 undergoes a westward displacement. During the period from day 6 to 12, this blocking flow 13 exhibits a typical Omega-type block with the largest amplitude at day 9 (Fig.7a). Thus, the 14 composite NAO⁻ pattern corresponds actually to a blocking flow in the Atlantic basin. The 15 amplitude variation and retrograde movement of the blocking flow can also be clearly seen 16 from the planetary-scale anomaly field (Fig. 7b) that resembles the composite NAO⁻ anomaly 17 pattern as shown in Fig. 3c. On the other hand, we can see that the synoptic- scale eddies are 18 split into two branches as the NAO⁻ pattern intensifies, in which the northern branch is stronger 19 than the southern one (Fig. 7c). The amplitude of the synoptic-scale eddies tend to exhibit a 20 consistent variation with that of the NAO⁻ pattern. When the amplitude of the NAO⁻ anomaly 21

reaches the maximum, the synoptic-scale EKE is strongest. The EKE becomes weak once the
NAO⁻ amplitude weakens. In other words, there is a positive correlation between the EKE and
NAO⁻ strengths during the NAO⁻ life cycle. This result confirms our observational finding
(Fig. 5c). In addition, the total field of the NAO⁻ event exhibits a meandering westerly flow
structure comprised of several isolated anticyclonic and cyclonic vortices (Fig.5d), which was
first noted by Berggren et al. (1949) and Rex (1950).

7 For the positive phase, the occurrence of the NAO⁺ event corresponds to a process of the intensification of a zonal westerly wind in the planetary-scale field (Fig. 8a). In an anomaly 8 field, the NAO⁺ pattern exhibits a low-over-high dipole structure, whose largest amplitude is at 9 day 9 (Fig.8b). It is interesting to see that the EKE strength is weakest at day 9 (Fig. 8c). This 10 hints that the EKE weakens as the NAO+ anomaly intensifies, thus suggesting that there is a 11 negative correlation between the EKE and NAO⁺ strengths during the NAO⁺ life cycle, as 12 found in Fig. 5d. On the other hand, we can see from Fig. 8d that in the total field, upstream 13 synoptic-scale ridges and troughs are absorbed by the zonal flow so that the zonal westerly 14 wind in the Atlantic basin is intensified during the NAO⁺ growing stage. Following the decay 15 of the NAO⁺ event, a blocking event is easily seen to occur over the European continent 16 (downstream side of the NAO⁺ region). 17

18 b) Physical cause of the presence (absence) of CWB during NAO⁻ (NAO⁺) episodes

Generally speaking, the CWB may be defined to represent the northward (southward) displacement of warm (cold) airs in the blocking or NAO system (Thorncroft et al. 1993; Benedict et al. 2004; Woollings et al. 2008). It is clear that the CWB can be seen in the total field of the NAO⁻ event (Fig. 7d), but invisible for the NAO⁺ event (Fig. 8d). Thus, the occurrence of the NAO⁻(NAO⁺) event corresponds essentially to the presence (absence) of CWB. As we will find here, the NAO feedback term ψ'_2 is important for the presence (absence) of CWB during the NAO⁻(NAO⁺) episodes.

For the same parameters as in Figs. 7 and 8, we show the instantaneous ψ'_2 , ψ'_1 and Ψ_T 5 $(\Psi_T = \psi_P + \psi'_1)$ fields of NAO⁻ and NAO⁺ events in Figs. 9 and 10, respectively. It is found 6 that the deformed eddies, as denoted by the NAO feedback term ψ'_2 , are intensified (weakened) 7 as the NAO⁻(NAO⁺) anomaly strengthens (Fig. 9). For the negative phase the deformed 8 eddies ψ_2' tend to exhibit a clear tripole meridional structure and tilt along the southwest 9 (SW)-northeast (NE) direction during its growing process (Fig. 9a), while they tend to have a 10 dominant dipole structure and tilt along the southeast (SE)-northwest (NW) direction for the 11 positive phase (Fig. 9b). For both phases of NAO, the preexisting eddies ψ'_1 always have a 12 monopole meridional structure, although weakened with time for the positive phase as they 13 propagate eastward (Figs. 10a and 10c). For the negative phase, because the deformed eddies 14 ψ_2' have a tripole meridional structure and intensify with the growth of the NAO⁻ anomaly, the 15 superimposition of preexisting and deformed eddies is able to make the synoptic-scale eddies 16 $(\psi' = \psi'_1 + \psi'_2)$ organized by the intensified NAO – pattern exhibit a strengthening and 17 meridional straining (Fig. 7c). Such an eddy straining is a concomitant phenomenon of the 18 NAO-onset and results from the feedback of the NAO-anomaly. In the presence of eddy 19 straining, the CWB can be seen in the total field of the NAO⁻ event (Fig. 7d). In contrast, no 20 evident CWB can be seen if the NAO feedback term ψ'_2 is neglected in the total field (Fig. 21

1 10b). Thus, the feedback of the intensified planetary-scale NAO⁻ anomaly field on preexisting 2 eddies that can lead to the initial growth of the NAO⁻ anomaly is the main cause of CWB. This 3 result is different from previous findings of Benedict et al. (2004), Franzke et al. (2004) and 4 Woollings et al. (2008), who noted that the NAO⁻ events result from the CWB. Even so, the 5 NAO⁻ event may be identified as corresponding to the presence of CWB.

For the positive phase, both preexisting (Fig. 10c) and deformed (Fig. 9b) eddies exhibit a 6 7 weakening as the NAO + anomaly intensifies. Their superimposition tends to make the synoptic-scale eddies propagating in the NAO+region weaken and tilt along the SE-NW 8 direction (Fig. 8c). In this process, the CWB cannot be detected in the total field (Fig. 8d). 9 However, in the downstream side (x > 0) of the NAO⁺ region, the synoptic-scale ridges 10 amplify, merge and then break anticyclonically that is characterized by a southwest-northeast 11 (SW-NE) tilted trough-ridge pair being advected anticyclonically (Fig. 8d for days 3 and 6), as 12 noted by Benedict et al. (2004). This wave breaking is the so-called anticyclonic wave breaking 13 (AWB) observed by Benedict et al. (2004) and Franzke et al. (2004). The AWB is significantly 14 reduced due to the almost disappearance of the SW-NE tilted tough-ridge pair in the 15 downstream side of the NAO region if the NAO feedback term is neglected from the total 16 field solution (Fig. 10d). Thus, the NAO feedback term ψ'_2 is also important for the 17 occurrence of AWB during the NAO+life process. In some sense, NAO+events correspond 18 essentially to the absence of CWB as suggested by Woollings et al (2008), or the presence of 19 AWB suggested by Benedict et al. (2004) and Franzke et al. (2004). Thus, it is concluded that 20 the feedback of the intensified NAO⁻(NAO⁺) anomaly on synoptic-scale eddies plays a 21

crucial role in the presence of CWB (AWB) for the negative (positive) phase of NAO. On the
other hand, we can see from Figs. 8 and 9 that the feedback of the intensified NAO⁻ and
NAO⁺ anomalies on synoptic-scale eddies is opposite between their eddy intensity evolutions.
Such a NAO⁻ (NAO⁺) feedback can lead to an enhanced positive (negative) correlation
between the EKE and NAO strengths for the negative (positive) phase of NAO, as found in the
above section.

7 6. Is the NAO event free mode?

In section 4, our proposed driving mechanism of the NAO event is based on the assumption 8 of the free mode that satisfies $J(\psi - u_0 y, \nabla^2 \psi - F \psi + \beta y + h) \approx 0$. Here, we define 9 $\psi_P = -u_0 y + \psi$, $q_P = \nabla^2 \psi - F \psi + \beta y + h$ and $q_T = q_P + \nabla^2 \psi'$ to understand if there is a 10 linear relation between q_P and ψ_P during the NAO life cycle. We show the instantaneous 11 scatter diagrams of planetary-scale potential vorticity q_p plotted against planetary-scale 12 streamfunction ψ_p in the domain $-2.0 \le x \le 2.0$ and $0 \le y \le 5.0$ during the NAO life cycle 13 in Fig. 11 for the same parameters as in Figs. 7 and 8. It is seen that there is an approximate 14 linear functional relationship between ψ_{p} and q_{p} during the whole life processes for two 15 phases of a NAO event (Fig. 11), consistent with the diagnostic result of Luo et al. (2007b). 16 Two straight lines are visible in a small piecewise region for the negative phase and particularly 17 evident at its strongest stage (Fig. 11a). Such a functional relationship is invisible for the 18 positive phase (Fig. 11b). However, a single valued linear functional relationship is dominant 19 during the NAO life cycle for its two phases. Thus, in the planetary scale field the NAO event 20 may be thought of as a free mode although it is both eddy-driven and time-dependent. However, 21

the nearly linear functional relationship between the total streamfunction Ψ_r and its potential 1 vorticity q_T is broken once the synoptic-scale eddies are included in the total field (Fig. 12). 2 This shows that the presence of synoptic-scale eddies may destroy the linear relationship 3 between ψ_P and q_P . This result was first noted by Luo et al. (2007b). However, they didn't 4 examined the role of the NAO feedback in the departure of the linear functional relationship 5 between Ψ_T and q_T . For $\Psi_T = \psi_P + \psi'_1$ and $q_T = q_P + \nabla^2 \psi'_1$, we show the instantaneous 6 scatter diagrams of Ψ_T vs q_T of a NAO event in Fig. 13 for its two phases. In this figure, 7 the NAO feedback term ψ'_2 has been removed from the total streamfunction field. It is clear 8 that in the absence of ψ'_2 the functional $q_T = F(\Psi_T)$ of the NAO⁻ event can exhibit one 9 straight line at day 0, but two approximate straight lines at day 9 (Fig. 13a). In other words, the 10 functional $q_T = F(\Psi_T)$ satisfies approximately a two valued linear relationship. For the 11 positive phase, the functional $q_T = F(\Psi_T)$ exhibits a thick straight line at day 0 and a thin 12 straight line at day 9 (Fig. 13b). For this case, the single valued line relation of $q_T = F(\Psi_T)$ is 13 most evident as the synoptic eddies are weakest (Figs. 10a and 10c). That is to say, when the 14 deformation of synoptic eddies induced by the intensified NAO anomaly is not considered, 15 there is a double (single) valued linear relationship between Ψ_T and q_T for two phases of 16 NAO event even when the preexisting synoptic eddies are involved. Thus, the presence of 17 deformed synoptic eddies due to the NAO feedback will significantly distort the linear 18 relationship between Ψ_{T} and q_{T} . This is a main cause of why the instantaneous scatter 19 diagrams of the observed streamfunction field and its potential vorticity do not strictly satisfy a 20 linear functional relationship (Butchart et al. 1989; Haines et al. 1994). Its main cause is that 21

the synoptic-scale eddies in observed fields are not sufficiently filtered out. However, our theoretical calculation indicates that in planetary-scale fields the NAO event may be considered a free mode, though eddy driven. Thus, our assumption $J(\psi - u_0 y, \nabla^2 \psi - F\psi + \beta y + h) \approx 0$ may be approximately used to establish the driving mechanism of the NAO growth proposed in section 4.

7. Does the wave breaking affect the persistence of the NAO event?

7 As described in above sections, the feedback of the NAO anomaly on synoptic eddies is able to result in the breaking of synoptic waves such as the presence (absence) of CWB for the 8 negative (positive) phase. In previous studies (Luo et al. 2007a,c) we have pointed out that the 9 spatial structure of the eddy vorticity forcing induced by preexisting synoptic-scale eddies is 10 crucial for what phase of the NAO event is produced. Here, we will demonstrate that the eddy 11 vorticity flux divergence induced by the different synoptic wave breaking for different NAO 12 phases may affect the persistence time of NAO events through altering the dissipation. To 13 clearly see this point, we consider the case without topography. For the same parameters as in 14 Figs. 7 and 8 the horizontal distribution of $-\nabla \cdot (\mathbf{v}'q')$ is shown in Fig. 14 for two phases of 15 NAO under the condition without topography. Correspondingly, the wavenumber 2 component 16 of $-\nabla \cdot (\mathbf{v}'q')$ is shown in Fig. 15. It is obvious that for the negative phase the strength of 17 $-\nabla \cdot (\mathbf{v}'q')$ is increased as the NAO - anomaly intensifies. At day 0 (initial time) the 18 $-\nabla \cdot (\mathbf{v}'q')$ has a negative-over-positive dipole in the west side of x = 0. Also, there is an 19 opposite dipole in the upstream side of the NAO⁻ region. The $-\nabla \cdot (\mathbf{v}'q')$ dipole patterns are 20 intensified with the growth of the NAO⁻ anomaly and reach their maximum strengths at day 9 21

(Fig. 14a). Such an enhanced eddy vorticity flux divergence is generated by the enhanced CWB 1 during the NAO⁻ episode. While the dissipation $A_{\mu}\nabla^2 q$ becomes strong with the growth of 2 the NAO – anomaly, it can be in part counteracted by $-\nabla \cdot (\mathbf{v}'q')$ because $-\nabla \cdot (\mathbf{v}'q')$ is 3 intensified and has a spatial structure opposite to that of $A_H \nabla^2 q$ in the NAO⁻region. This is 4 particularly evident in the $-\nabla \cdot (\mathbf{v}'q')_p$ field ("P" denotes the wavenumber 2), as shown in Fig. 5 15a. As a result, the NAO⁻ anomaly can be maintained because of the weakening of the 6 7 dissipation due to the presence of CWB. This mechanism was noted by Shutts (1983) in a numerical model experiment. 8

For the positive phase, the $-\nabla \cdot (\mathbf{v}'q')$ strength decreases as the NAO ⁺ anomaly is 9 amplified. At day 0 $-\nabla \cdot (\mathbf{v}'q')$ exhibits a negative-over-positive dipole anomaly in the west 10 side of x = 0 in the NAO⁺ region and a relatively weak opposite dipole in the east side of 11 x = 0. When the amplitude of the NAO⁺ anomaly is increased, the positive-over-negative 12 dipole of $-\nabla \cdot (\mathbf{v}'q')$ seems to be rapidly intensified and shifts westward into the NAO⁺ region 13 (Fig.14a at day 3). However, it is seen in the wavenumber 2 field of $-\nabla \cdot (\mathbf{v}'q')$ that the 14 positive-over-negative dipole of $-\nabla \cdot (\mathbf{v}'q')_p$ weakens and shift eastward into the NAO⁺ 15 region (Fig. 15b). Such a $-\nabla \cdot (\mathbf{v}'q')_p$ dipole pattern can amplify the NAO⁺ anomaly. Because 16 $A_H \nabla^2 q$ becomes strong and $-\nabla \cdot (\mathbf{v}' q')_p$ is weakened during the NAO ⁺ growth, the 17 dissipation $A_{\mu}\nabla^2 q$ can be enhanced so that it can reach the strongest amplitude at day 9. Such 18 an enhanced dissipation process is generated by the AWB during the NAO⁺ episode, which is 19 able to cause the rapid decay of the NAO⁺ anomaly. 20

The above results indicate that the presence of CWB (AWB) during NAO⁻(NAO⁺) 1 episodes can weaken (strengthen) the dissipation so as to enhance (reduce) the persistence of 2 the NAO⁻(NAO⁺) event. Thus, the asymmetry of the NAO persistence between two phases is 3 related to different types of synoptic-scale wave breaking. 4

5

8. Conclusion and discussions

The dynamics of NAO events has been widely investigated in previous studies. However, 6 7 the causal relationship between time-dependent planetary- and synoptic-scales of NAO events is unclear. Observations first show that the NAO⁻ event is more persistent than the NAO⁺ 8 event, consistent with previous findings of Barnes and Hartmann (2010). During the NAO life 9 cycle there is a positive (negative) correlation between the eddy kinetic energy (EKE) and 10 NAO⁻ (NAO⁺) anomaly strengths, as well. 11

On the other hand, we have used a nonlinear multi-scale interaction (NMI) model to 12 propose an eddy-NAO matching (ENM) mechanism of NAO events to clarify the mutual 13 relationship between planetary and synoptic scales during the NAO life process and why there 14 is a strong asymmetry of the NAO persistence between two phases. In this NMI model, 15 synoptic-scale eddies are divided into preexisting eddies ψ'_1 and deformed eddies ψ'_2 from the 16 feedback of the NAO anomaly (the NAO feedback term, hereafter). Our theoretical result 17 indicates that although preexisting eddies drive the NAO growth, the NAO feedback term is 18 important for whether there is a strong positive (negative) correlation between the EKE and 19 NAO⁻ (NAO⁺) anomaly strengths. In particular, when the NAO feedback term is excluded, 20 no synoptic wave breaking is seen. Moreover, we may find that the NAO⁻ (NAO⁺) anomaly 21

feedback is intensified (weakened) as the NAO anomaly increases. Such an enhanced (reduced) NAO⁻ (NAO⁺) feedback will increase (decrease) the eddy vorticity flux divergence to weaken (strengthen) the dissipation so that the NAO⁻ (NAO⁺) event is more (less) persistent. As a result, the NAO⁻ (NAO⁺) event should be long-lived (short-lived). Thus, the enhanced (reduced) eddy vorticity flux divergence associated with cyclonic (anticyclonic) wave breaking is able to generate a strong asymmetry of the NAO persistence between two phases through the weakening (strengthening) of dissipation.

Although our NMI model is based upon a highly idealized barotropic model, it can capture main characteristics of NAO events. However, because the baroclinic processes are neglected, there is a quantitative difference between our theoretical results and observations. Even so, the results of the NMI model can be used to improve our understanding of the basic physics of the NAO dynamics. In the future work, the NMI model used here should be extended to include baroclinic processes.

14

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Figure 1. Event numbers of (a) NAO⁻ (black) and associated NAO⁻ to NAO⁺ (gray) events,
and (b) NAO⁺ (gray) and associated NAO⁺ to NAO⁻ (black) transition events in winter
during 1950-2013.

















Figure 3. Composite 300-hPa geopotential height fields for individual (a) NAO⁻ and (b)
NAO⁺ events during 1950-2013. The anomaly fields are shown in panels (c) and (d) for
negative (c) and positive (d) phases.

90N -90N Lag-9 Lag-75N 75N 60N 60N 45N 45N 0 30N 30N \circ 0 15N ↓ > 120W △ _{15N}↓∑ 60E 120W 1 30E 90W 60W 30W ò 30E 90W 6ÖW 3ÓW ò 6ÒE 90N ______ 90N -0> 75N 75N 0 60N 60N () 0 45N 0 45N 30N 30N 15N | 120W ☐ 15N 60E 120W 2 30E 60W 90w 60W зо́м 30E 60E 90W 3ÓW ò 90N 90N 1Lag0 75N 75N 60N 60N 45N 45N 30N 30N 15N | 120W ☐ 15N 60E 120W 3 30W ò 30E 90W 6ÓW 30E 90M 60W 3ÓW 6ÒE 90N Lag+3 90N 5 75N 75N 60N 60N 45N 45N 30N 30N 15N ▶ 120W 0 24 15N ↓ / 60E 120W ^₀, 4 30E 90M 60W 30W 30E 90M 60M зо́ж ό 60E Ò 90N 90N 75N 75N 60N 60N 0 6 45N 45N 0 30N 30N $\overline{}$ △ 15N ► 60E 120W 5 3ÓE 30E 60W 3ÓW ò 90W 6ÓW 30W 60E эċw ò (a) 6 7 8

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Figure 4. Instantaneous fields of composite 300-hPa EKE for individual (a) NAO⁻ and (b)
NAO⁺ events during 1950-2013. The shading denotes the region above the 95% confidence
level for a two-sided student's *t*-test.





Figure 5. Time-latitude evolution of the composite EKE averaged over 90°W-20°E for
individual (a) NAO⁻ and (b) NAO⁺ events. The black (gray) shading denotes the region
above the 95% confidence level for a two-sided student's t-test. The time sequences of daily
EKE and NAO strengths are shown in panels (c) and (d) for two phases of the NAO event.



Figure 6. Schematic picture of the eddy-NAO matching (ENM) mechanism due to the interaction between an initial NAO pattern and upstream synoptic-scale eddies: (a) Negative phase and (b) positive phase. The grey region denotes the eddy vorticity forcing pattern $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$. In panel 6a (6b), $-\nabla \cdot (\mathbf{v}'_1 q'_1)_p$ has a negative-over-positive (positive-overnegative) dipole upstream of the NAO⁻(NAO⁺) region required to amplify the NAO⁻(NAO⁺) anomaly.





Figure 7, Instantaneous fields of NAO⁻ event from the NMI model: (a) Planetary-scale field (contour interval (CI) =0.15); (b) planetary-scale anomaly field (CI=0.2); (c) synoptic-scale field (CI=0.3) and (d) total field (CI=0.3)



Figure 8, Same as Fig. 7 but for a NAO⁺ event.

(c)

(d)



5 Figure 9, Instantaneous fields of the NAO feedback term ψ'_2 of a NAO event for its (a)









Figure 10, (a, c) Synoptic-scale (ψ'_1) and (b, d) total $(\Psi_T = \psi_P + \psi'_1)$ fields of an NAO event without the NAO feedback term (i.e., $\psi'_2 = 0$) for the same parameters in Figs. 7 and 8. Panels (a, b): negative phase, and panels (c, d): positive phase.



Figure 11, Instantaneous scatter diagrams of ψ_p vs q_p during the life process of a NAO event for the same parameters in Figs. 7 and 8: (a) negative phase and (b) positive phase.



Figure 12, Instantaneous scatter diagrams of the total streamfunction field Ψ_T vs q_T during the life process of a NAO event for the same parameters in Figs. 7 and 8: (a) negative phase and (b) positive phase.





Figure 13, Instantaneous scatter diagrams of the total streamfunction field $\Psi_T (\Psi_T = \psi_P + \psi'_1)$ vs $q_T (q_T = q_P + \nabla^2 \psi'_1)$ without including the NAO anomaly feedback during the life process of a NAO event for the same parameters in Figs. 7 and 8: (a) negative phase and (b) positive phase.

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Figure 14, Instantaneous fields (CI=2) of eddy vorticity flux divergence $-\nabla \cdot (\mathbf{v}'q')$ during the life process of a NAO event for the same parameters in Figs. 7 and 8 but in the absence of topography: (a) negative phase and (b) positive phase.



Figure 15, Instantaneous fields (CI=0.15) of the wavenumber 2 component of eddy vorticity 6 flux divergence $-\nabla \cdot (\mathbf{v}'q')$ shown in Fig. 14.