Searching Gravitational Waves with Satellite Timing

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ABSTRACT
The precision of space-based experiments is sufficient to inspect gravitational effects. Similar to the natural pulsar timing array, satellites with atomic clocks located at the Lagrangian points, L_4 or L_5, could be employed to detect the gravitational waves. We estimate the signal of timing residual and the sensitivity for such a system. If the gravitational waves come from merging of black holes resided in distant galaxies. Under current technology, the detection limit corresponds to the supermassive ($\geqslant 10^8$ $\mathrm{M}_\odot$) binary block holes. If the timing precision is improved to 10^{-14} second, supermassive binary black holes of 10^6 $\mathrm{M}_\odot$ could be detected. The requiement of engineering is feasible. The satellite timing method covers the frequency range of 10^{-6} to 10^{-2} Hz, between pulsar timing array and eLISA.

Key words: gravitational waves

1 INTRODUCTION

Gravitation wave (GW) is the fluctuation of space-time, with two polarization modes, h_+ and $h_\times$. It is extremely hard to be detected due to the lack of mechanism of interaction. For the same reason, GWs from very distant sources propagate with few dilution or dispersion. In the weak field approximation, the amplitude of GWs is determined by the second order derivatives of the quadrupole moment of the sources with respect of time. It could be generated by massive explosion of supernovae, coalescence of compact binary objects etc.

In the framework of Λ cold dark matter (ΛCDM) cosmology, the hierarchical structure formation scenario is supported by a number of theoretical and observational works. The galaxy merger rate peaks at about redshift of $z \sim 1-2$ (Guo & White 2008). During the galaxy merging, the supermassive black holes situated at the center of individual galaxies would form binary systems. They co-rotate with each other for a long period. As losing energy, they inspiral and finally coalesce (Centrella et al. 2010). So far, the details of the binary black hole merging is still unclear. The observation of GWs can provide plenty of information about the properties and orbital parameters of the binary black hole. It is estimated that tens of events will occur in the whole observable universe (Sesana et al. 2004; Sesana & Vecchio 2010; Seto 2016).

The post Newtonian approximation can be used to predict the gravitational emission in the weak field limit, depending on the the orbital angular momentum and the spin of each black hole. The orbiting properties are important in understanding the growth history of the black hole. For convenience, we define chirp mass

$$M_\mathrm{c} = (m_1 + m_2)\left[\frac{m_1 m_2}{(m_1+m_2)^2}\right]^{\frac{3}{5}}, \tag{1}$$

where m_1 and m_2 are the masses of individual black holes in the binary, in geometrized units, $G = c = 1$. The strain can be written as

$$h \propto M_\mathrm{c}^{5/3} D^{-1} \omega^{-1/3}, \tag{2}$$

where D is the luminosity distance of the binary black holes, and ω is the orbit frequency.

In order to measure the strain of extremely tiny amplitude, two strategies are considered. The first one makes use of the laser interferometer. For instance, the experiment of advanced Laser Interferometer Gravitational Observatory (aLIGO) employed two 40-kilometer base arms to measure the shift of interference fringes. In September 2015, aLIGO discovers the first

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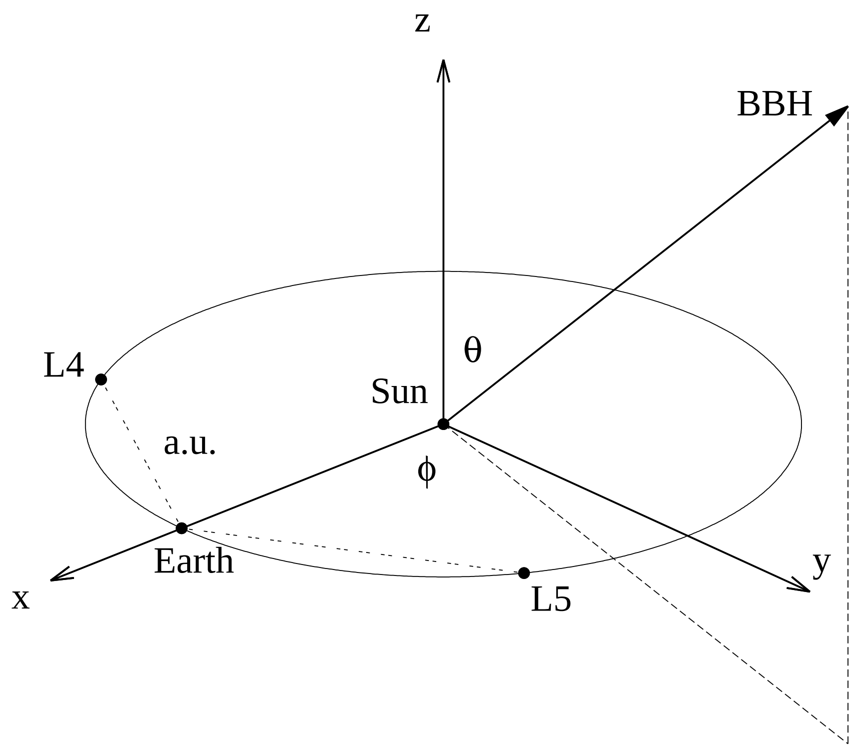

Figure 1. configuration scheme

merging event of a binary black hole system, a 36 $M_\odot$ and 29 $M_\odot$ pair 400 Mpc ($z \sim 0.1$) away (GW150914; (Abbott et al. 2016)). The signal lasts for about 0.5 seconds. A space project can extend the base line over the Earth scale. The project of Evolved Laser Interferometer Space Antenna (eLISA)(Consortium et al. 2013), has million-kilometer arms, which would be launched in the 2030s. There are several space plans, such as Big Bang Observer(BBO), DECIGO (Kawamura et al. 2011), TianQin (Luo et al. 2016)) And the ASTROD has a very long baseline over astronomical unit (Ni 2013).

The second strategy is to measure the timing residual of a pulsar timing array. Some pulsars are excellent natural timing systems. The long time stability of millisecond pulsars is $\sigma \sim 10^{-14}$ second. The time of arrival (ToA) of pulses will be affected by GW. The measurement of the timing residual could derive the information of the binary black hole (Lee et al. 2011). The International Pulsar Timing Array (IPTA; (Hobbs et al. 2010; Verbiest et al. 2016)) combines 49 pulsars mainly from European PTA(EPTA; (Desvignes et al. 2016)), North-American Nanohertz Observatory for GWs (NANOGrav; (The NANOGrav Collaboration et al. 2015)), Parkes PTA(PPTA; (Reardon et al. 2016)) etc..

Deep space satellites with atomic clocks could be another GW probe analog of pulsars timing. In this wrok, we estimate the signal(time residual), frequency range and sensitivity of satellite timing method for the satellites located at the Lagrangian points L_4 and L_5.

2 SPECIFICATION OF THE SATELLITES

There are 5 Lagrangian points of the Earth-solar system, in which L_4 and L_5 are stable points. As shown in Fig. 1, the origin is set to the location of the sun and x axis points to the Earth. The angle between binary black hole and z-axis is θ. L_4 locates at $\phi = -\pi/3$ and L_5 at $\phi = \pi/3$. μ denotes the direction cosine of the angle between the GW sources and the satellites. The direction cosine of L_4 is $\mu_4 = \sin\theta\cos(5\pi/3 - \phi)$, and L_5 is $\mu_5 = \sin\theta\cos(\pi/3 - \phi)$.

Assume that satellites with atomic clock are locked to the Lagrangian points L_4 and L_5. They keep constant separation with the Earth. When a GW pass through the solar system from the direction of θ in Fig. 1, it could change the separation between the satellites and the Earth. As for the signal to link the satellite with the Earth, the time of arrival (ToA) would be delayed or ahead of time. The observable quantity induced by GW is the timing residual. In this ideal case, it is the difference between clock time on-board and on the Earth.

Instead of the one direction reception of pulsar timing, the satellite communication is able to build two-way links. The signal emits from the Earth at wall clock time of t^e_1 and it arrives the satellite at on-board clock time of t^s. Then this signal has been replied to the Earth and finally arrives to the Earth at time of t^e_2. The timing residual of one single measurement is given by $R_{\rm t} = (t^e_2 + t^e_1)/2 - t^s + \Delta t_{\rm geom} + \Delta t_{\rm grav} + \Delta t_{\rm m}$, with three main corrections for geometry effects, the gravitational delay by the solar system, and the motion of satellites.

The singal links could be measured in two bands:

1. Radio band: Vessot(Vessot 1991) proposed S-band chain satellites to measure the timing residual. Three chains are employed using one single microwave antenna. The first chain is responsible for Doppler motion. The second chain is to receive the signal from the Earth, then it is transmitted down to the Earth immediately, with the third chain. The frequency ratio of the 3 chains must be fine tuned for canceled atmosphere effect and Doppler velocity can be derived from the first chain and the second chain. Specifically, the 3 chains uses 2205.1 MHz, 2117.7 MHz, and 2299.7 MHz, respectively. The ratio of 2nd to 3rd frequencies is 221/240. For the near Earth satellite, the accuracy of the residual is about nanosecond for a single communication.

2. Optical band: A laser link can take place in the optical band. In general, the round trip of laser usually has higher precision. In 2008, the satellite Jason-2 carries a Time Transfer by Laser Link(T2L2) for synchronize the clocks at different stations. Its time stability is better than 1 ps over 1000 s and an accuracy is sub-nanosecond. The Laser time transfer in the Galileo programme has the timing precision of the order of 10^{-12} s (Prochazka et al. 2011; Berceau & Hollberg 2014; Berceau et al. 2015). Therefor a practical laser link has potential in a precision of $10^{-10} - 10^{-12}$ second. But the laser is sensitive to the weather conditions.

The propagation also experiences time delay, $\Delta t_{\rm grav}$, induced by the gravitational field of the Sun, which is approximately $[1-2GM_\odot/(1{\rm AU})c^2]^{1/2} \sim 10^{-8}$. The gravitational redshift induced by the Earth is about $[1-2GM_\oplus/R_\oplus c^2]^{1/2} \sim 0.7\times10^{-8}$.

Even the relative position between the Earth and the satellite is fixed, but the signal propagation is not symmetric due to the revolution of the system. The round trip time cost about 1000 seconds, then the Earth will move about 30,000 kilometers along the orbit. That means asymmetry contributes to $\Delta t_{\rm geom} \sim 0.1$ second.

Besides the accuracy of the single measurement, the other key is the timing stability σ_y. Though the satellites resides on the Lagrangian points, they still move about the points. The relativistic effect induced by this motion will affect the time stability. Thus a long term monitor is necessary to correct for the motion effect. The line of sight projection of this velocity could be measured by the Doppler effect. It is helpful to reconstruct the orbit and motion of the satellites.

3 TIMING RESIDUAL

In this section, we estimate the amplitude of the timing residual of artificial pulse sources on L_4 and(or) L_5. It is a similar analyses as pulsar timing (Detweiler 1979; Wahlquist 1987; Jenet et al. 2004). The residual of ToA reads

$$R(t) = \frac{1+\mu}{2}[(r^e_+ - r^s_+)\cos 2\psi + (r^e_\times - r^s_\times)\sin 2\psi], \tag{3}$$

where μ is the direction cosine of the opening angle, ψ is the phase angle of the polarization, and the Earth term r^e and satellite term r^s are given by the integral

$$r^{e,s}_{+,\times}(t) = \int_0^t h^{e,s}_{+,\times}(\tau)\mathrm{d}\tau. \tag{4}$$

In general, the h is time-dependent, but the GW sources we considered evolve slowly. Thus we calculate the residual for a monochromatic harmonic wave.

The term r^e and r^s have the same form, but different emission time. The delay for propagation is

$$t^s = t^e - \frac{|d|}{c}(1-\mu), \tag{5}$$

where $|d|$ is the separation between the observer and the Earth (about 1 AU).

We consider the most optimistic GW source. Its inclination and ellipticity of binary black hole orbits vanishes. The amplitude of GWs also depends on the opening angles μ. If $\mu = 1$, Eq. 5 vanishes. It means that the photons has no relative motion to the GW, having no contribution to the timing residual. On the other hand, If $\mu = -1$, the timing residual is also zero, according to Equ. 3. Thus the opening angle of $\pi/2$ corresponds to the strongest signals when other parameters are fixed. If base line $|d|$ is equal to integer times of the wavelength, the residual is also cancelled, because the photons just experience a complete period.

For GWs with specific frequencies, we change the opening angle and record the maximum of the amplitude during a whole period. The maximum is referred to as the characteristic residual. Figure 2 demonstrates the characteristic residual

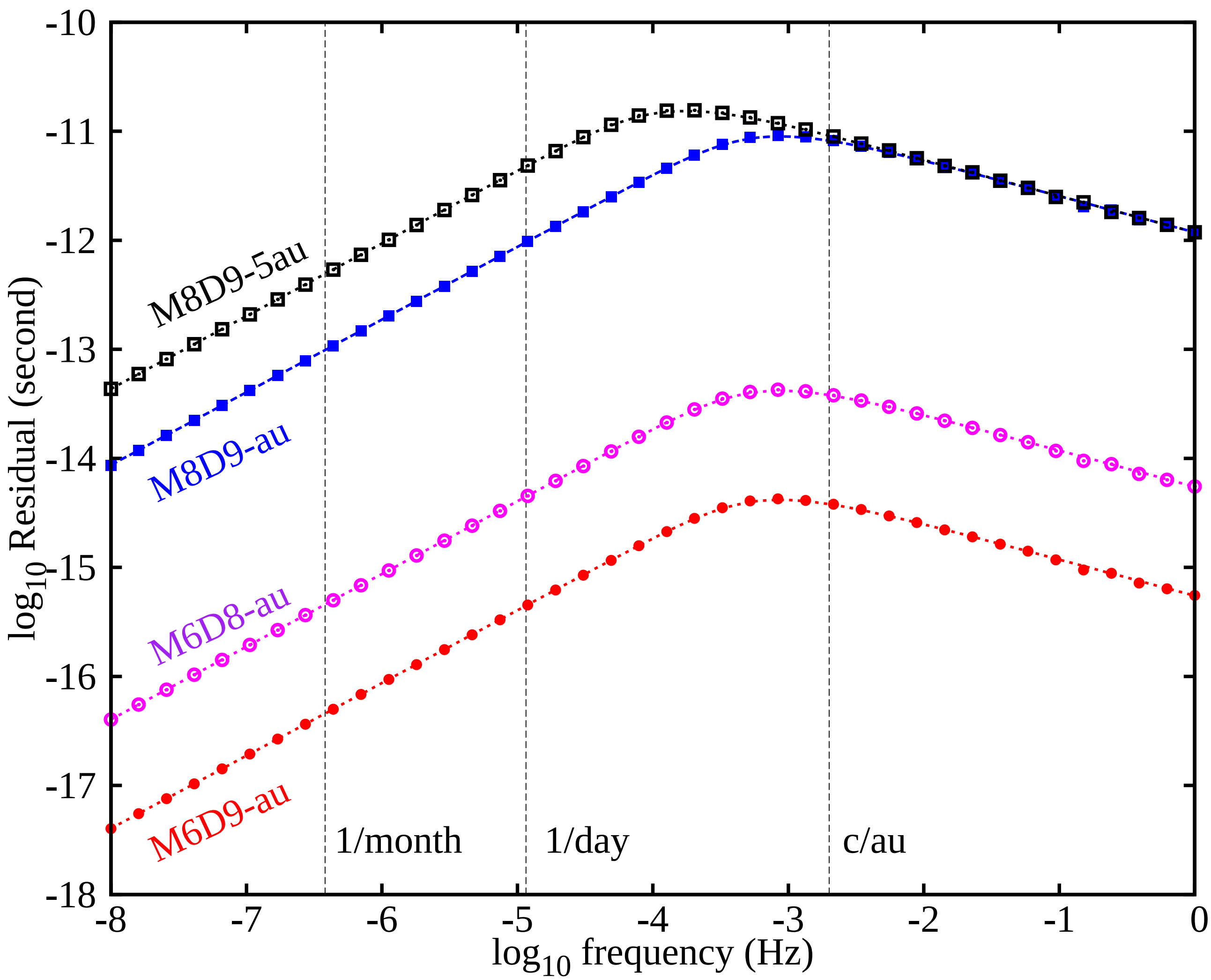

Figure 2. The residual of timing induced by GWs: the (red) filled points denote $M_c = 10^6$ $\mathrm{M}_\odot$ and $D = 1$ Gpc, the open circles denote $M_c = 10^6$ $\mathrm{M}_\odot$ abut $D = 100$ Mpc. The (blue) filled square denote $M_c = 10^8\mathrm{M}_\odot$ and $D = 1$ Gpc. The open squares also measure the $M_c = 10^8$ $\mathrm{M}_\odot$ and $D = 1$ Gpc with a baseline of 5 au. The corresponding curves are fitting formula6. The vertical lines the characteristic frequencies.

with different conditions of GW sources. The reference model is a binary black hole system with chirp mass of 10^6 $M_\odot$ at 1 Gpc far away (red filled points). It is apparent that the peak of the signal corresponds to the half wavelength.

Assume the precision level of ~ 10 ps, the detection limit of the BBH is $\sim 10^8$ $\mathrm{M}_\odot$ for a distance of 1 Gpc and signal-to-noise ratio of 1. If the precision is improved to fs, the most SBBH of interest($\geqslant 10^6$ $\mathrm{M}_\odot$) in the universe can be detectable.

We find the following fitting formula can well describe the characteristic residual. It can be written as

$$R(t) = 0.35 \left(\frac{M_c}{10^6\mathrm{M}_\odot}\right)^{\frac{5}{3}} \left(\frac{\mathrm{Gpc}}{D}\right) f^{-\frac{1}{3}} \tan^{-1}\left(\frac{5f}{\lambda_\star}\right) \tag{6}$$

in the unit of femto second, where $\lambda_\star$ denotes the wavelength of GWs(The dashed curves in Fig. 2).

4 SENSITVITY

In order to evaluate the artificial satellite timing method, we transfer the time residual to the effective strain. In the absence of information of equipments, we just estimate the effective strain by a simple relation of $cR_t/\lambda_\star$.

Figure 3 demonstrates that the predicted strains(red dashed curve) binary black hole at 1 Gpc, with $M_c = 10^4$, 10^6, 10^8 and 10^{10} $M_\odot$. If the precision of measurement is better than nanosecond, picosecond, and femtosecond, the signal of $M_c = 10^{10}$, 10^8, and 10^6 should be detectable. The dark blue area denotes the detection limit of current level, say 50 ps. The light blue area is of the limit from expectation in the future. It requires the single measurement of timing signal to be better

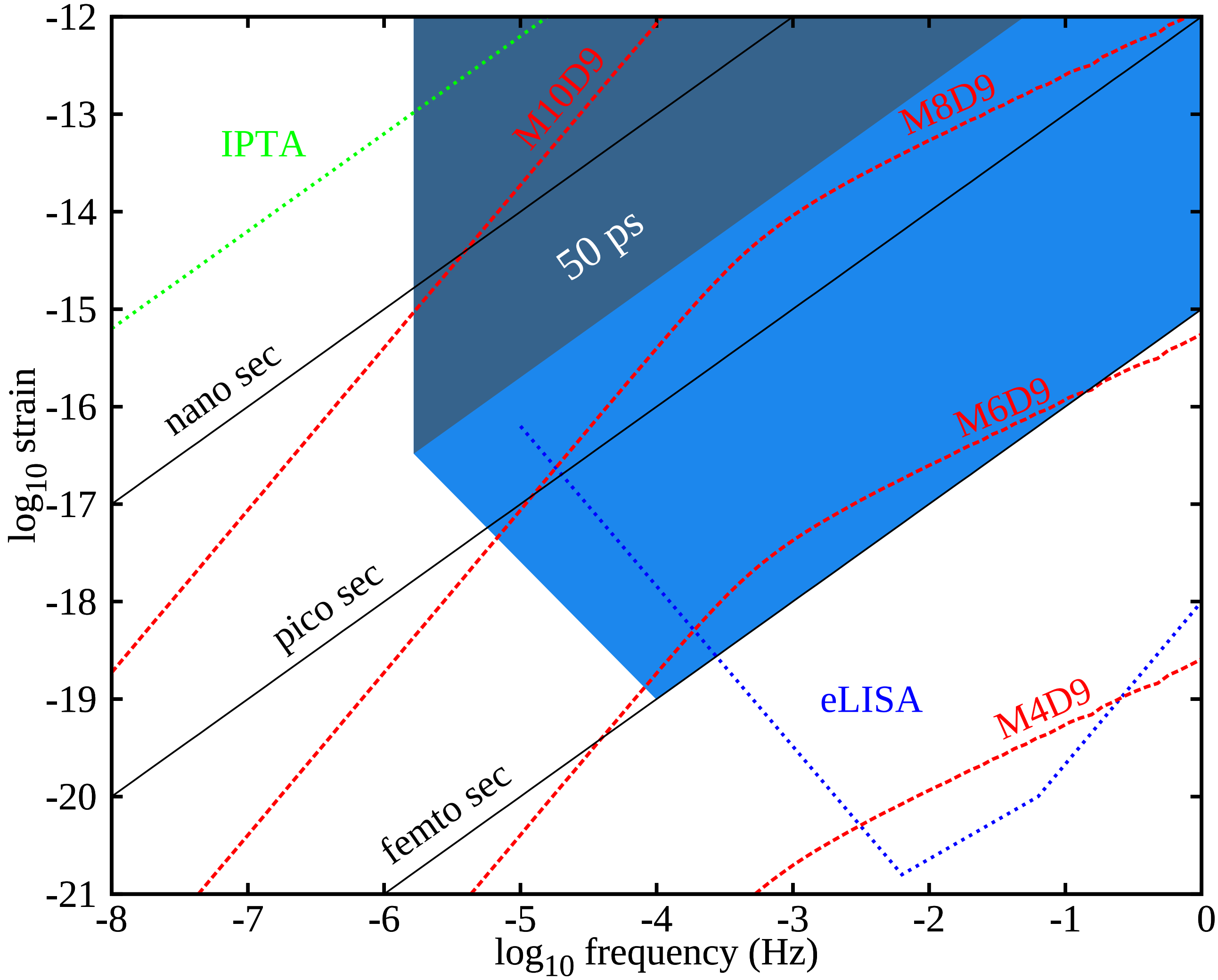

Figure 3. The strain of GWs for satellite timing method with a base line of AU, the dark blue area is the detection limit of current engineering level. If the precision of single measurement achieve to fs, the most of supermassive binary black hole $\geqslant 10^6 M_\odot$ could be observable. For comparison, the sensitivity curves of eLISA and IPTA are plotted in the same figure.

than fs, near the proper stability of on-board clock, and time stability be kept longer than the frequency of interest by an order. The space clock in the project of Atomic Clock Ensemble in Space (ACES) has precision about 10^{-16} level[1]. Its time deviation enduring 1000 second is about 10^{-12} second (Cacciapuoti & Salomon 2011).

Note that the slope of monochromatic GW in our calculation is proportional to 2/3 (Thorne 1987). The slope of the characteristic strain spectrum for stochastic background is $-2/3$ (Sesana et al. 2008). We simply plot the curves of eLISA and PTA for references (Moore et al. 2015)[2] in the Fig. 3.

Most coalescence events happens for the chirp mass less than 10^5 $M_\odot$ (Sesana et al. 2004). But the small binary black hole system, say $10^4 M_\odot$, is hard to be detected unless it is very close to Earth. The signal at the frequency around $10^{-3} - 10^{-5}$ Hz comes from the long period systems of merging binary black hole. Thus there are still sufficient candidates to expect.

5 DISCUSSION

We present a satellite timing method, which could be powerful to detect the gravitational wave, and understand the physics of supermassive binary black holes and their merging rate. It has the same principle as pulsar timing, but a shorter base line for higher frequencies. The sensitive range of frequency is lower than the eLISA-like projects. The technology requirement is

[1] http://issresearchproject.nasa.gov/JPL/ACES/
[2] http://rhcole.com/apps/GWplotter

relatively modest compared with projects based on space laser Interferometer, though the sensitivity of laser Interface could be better at some frequency. The current engineering condition is ready for the most massive BBH merger systems. Compare with the eLISA-like space project, satellite timing method is simpler and faster to implement.

There are some special cases to enhance the signal. If $\theta = 0$, two individual satellites at L_4 and L_5 could be strongly anti-correlated with each other. It will increase the reliability of the signal. Considering the satellite moving along the direction of the Earth, the separation between the Earth and the satellite decreases, so the delay of ToA is negative. In such a case, the delay of ToA comes entirely from the GWs effects. The correlation of ToA and Doppler shift is helpful to infer the GW signal. The revolution of the system naturally covers the more area of sky.

The orbit frequency of BBH evolves as $\mathrm{d}f/\mathrm{d}t \propto f^{11/3}$(Seto 2016; Sesana 2016). The elapsed time at the frequency band of satellite timing method is much longer than higher frequency like eLISA. It is an advantage to measure lower frequency by the baseline of AU. If the baseline extends to 5 AU, which will be a better complement for the gap between eLISA and PTA.

A potential issue is the effects of the atmosphere turbulence. Instead of the ground observatories, a low-orbit space or moon-based telescope is an alternative.

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