Classical Invariant Theory(Peter J.Olver),Exercise 1.1

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Exercise (1.1). *Find the explicit formulas for α,β that will reduce a quadratic polynomial Q to its canonical form.Is the residual coefficient k uniquely determined?Determine the formula(e) for k in terms of the original coefficients of Q.*

Solve. $\overline{p}=\alpha p+\beta$,so $p=\frac{\overline{p}-\beta}{\alpha}$.Then

$$a(\frac{\overline{p}-\beta}{\alpha})^2+2b(\frac{\overline{p}-\beta}{\alpha})+c=0.$$

So

$$a\overline{p}^2+(2\alpha b-2a\beta)\overline{p}+a\beta^2-2\alpha\beta b+\alpha^2c=0.$$

Let $a\beta^2-2\alpha\beta b+\alpha^2c=0$,then we have

$$\frac{\beta}{\alpha}=\frac{b\pm\sqrt{b^2-ac}}{a}.$$

Then we let

$$2\alpha\frac{b}{a}-2\beta=-1,$$

So $\alpha=\frac{a}{2\sqrt{b^2-ac}}$,$\beta=\frac{b}{2\sqrt{b^2-ac}}+\frac{1}{2}$.Or $\alpha=\frac{-a}{2\sqrt{b^2-ac}}$, $\beta=\frac{-b}{2\sqrt{b^2-ac}}+\frac{1}{2}$.At this time,$k=a$. □

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