

Classical Invariant Theory(Peter J.Olver),Exercise 1.2

Luqing Ye *

2014 年 10 月 20 日

Exercise (1.2). *An alternative canonical form for such quadratics is $\tilde{Q}(\tilde{p}) = \tilde{k}(\tilde{p}^2 + 1)$. Do the same exercise for this canonical form, and describe what is happening to the roots of Q .*

Solve. $\tilde{p} = \alpha p + \beta$. So

$$\tilde{Q}(\tilde{p}) = a\tilde{p}^2 + (-2a\beta + 2b\alpha)\tilde{p} + a\beta^2 - 2\alpha\beta b + c\alpha^2.$$

Let $-2a\beta + 2b\alpha = 0$, and $a\beta^2 - 2\alpha\beta b + c\alpha^2 = a$, so we have

$$\alpha = \frac{a}{\sqrt{ca - b^2}}, \beta = \frac{b}{\sqrt{ca - b^2}},$$

or

$$\alpha = \frac{-a}{\sqrt{ca - b^2}}, \beta = \frac{-b}{\sqrt{ca - b^2}}.$$

□

*叶卢庆 (1992-), 男, 杭州师范大学理学院数学与应用数学专业大四. 学号:1002011005.E-mail:yeluqingmathematics@gmail.com