A sufficient condition

to ensure the inverse uncontinuous

Theorem Let X, Y be a Banach space, $\dim X = \infty$, $A : X \to Y$ be compact, linear, and injective. Then A^{-1} is not continuous.

Proof Argue by contradiction. Suppose A^{-1} is continous, then $I = A^{-1}A : X \to X$ is compact, which implies X is of finite dimension, a contradiction.