Chapter 1 Radar Systems - An Overview

This chapter presents an overview of radar systems operation and design. The approach is to introduce few definitions first, followed by detailed derivation of the radar range equation. Different radar parameters are analyzed in the context of the radar equation. The search or surveillance radar equation will also be derived. Where appropriate, a few examples are introduced. Special topics that affect radar signal processing are also presented and analyzed in the context of the radar equation. This includes the effects of system noise, wave propagation, jamming, and target Radar Cross Section (RCS).

1.1. Range Measurements

Consider a radar systems that transmits a periodic sequence, with period T, of square pulses, each of width τ, shown in Fig. 1.1. The period is referred to as the Pulse Repetition Interval (PRI) and the inverse of the PRI is called the Pulse Repetition Frequency (PRF), denoted by f_r. If the peak transmitted power for each pulse is referred to as P_t, then the average transmitted power over one full period is

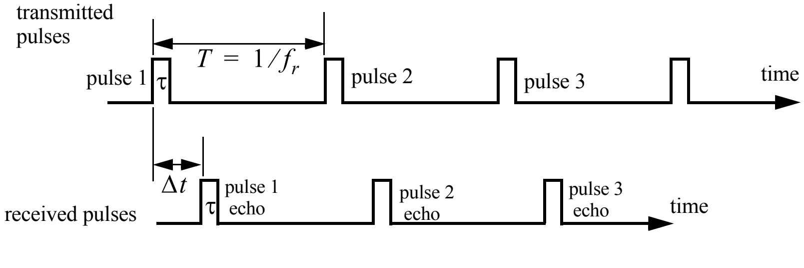


Figure 1.1. Train of transmitted and received pulses.

$$P_{av} = P_t \times \frac{\tau}{T} \tag{1.1}$$

The ratio of the pulse width to the PRI is called transmit duty cycle, denoted by dt. The pulse energy is $E_x = P_t\tau = P_{av}T = P_{av}/f_r$.

The top portion of Fig. 1.1 represents the transmitted sequence of pulses, while the lower portion represents the received radar echoes reflected from a target at some range R. By measuring the two-way time delay, Δt, the radar receiver can determine the range as follows:

$$R = \frac{c\Delta t}{2} \tag{1.2}$$

where: $c = 3 \times 10^8 m/s$ is the speed of light, and the factor 2 is used to account for the round trip (two-way) delay.

The range corresponding to the two-way time delay $\Delta t = T$, where T is the pulse repetition interval is referred to as the radar unambiguous range, R_u. Consider the case shown in Fig. 1.2. Echo 1 represents the radar return from a target at range $R_1 = c\Delta t/2$ due to pulse 1. Echo 2 could be interpreted as the return from the same target due to pulse 2, or it may be the return from a far-away target at range R_2 due to pulse 1 again. That is,

$$R_{2a} = \frac{c\Delta t}{2} \qquad or \qquad R_{2b} = \frac{c(T+\Delta t)}{2} \tag{1.3}$$

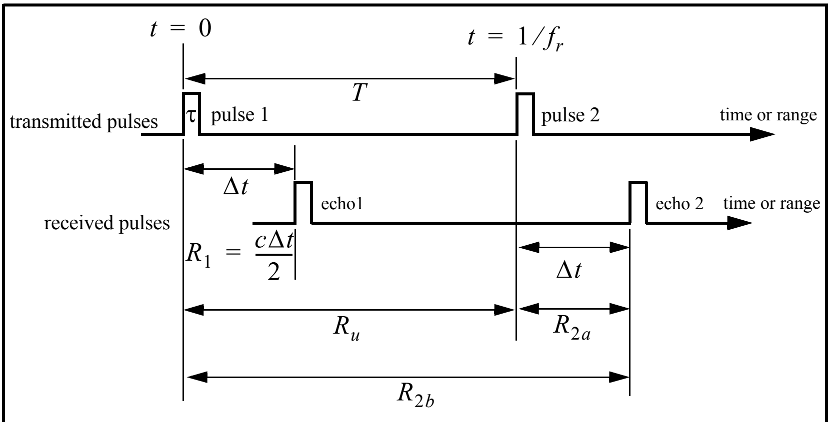


Figure 1.2. Illustrating range ambiguity.

Clearly, range ambiguity is associated with echo 2. Once a pulse is transmitted, the radar must wait a sufficient length of time so that returns from targets at maximum range are back before the next pulse is emitted. It follows that the maximum unambiguous range must correspond to half of the PRI:

$$R_u = c\frac{T}{2} = \frac{c}{2f_r} \tag{1.4}$$

Example:

A certain airborne pulsed radar has peak power $P_t = 10KW$ and uses two PRFs, $f_{r1} = 10KHz$ and $f_{r2} = 30KHz$. What are the required pulse widths for each PRF so that the average transmitted power is constant and is equal to $1500 Watts$? Compute the pulse energy in each case.

Solution:

Since P_{av} is constant, both PRFs have the same duty cycle,

$$d_t = \frac{1500}{10 \times 10^3} = 0.15$$

The pulse repetition intervals are

$$T_1 = \frac{1}{10 \times 10^3} = 0.1ms$$

$$T_2 = \frac{1}{30 \times 10^3} = 0.0333ms$$

It follows that

$$\tau_1 = 0.15 \times T_1 = 15\mu s$$

$$\tau_2 = 0.15 \times T_2 = 5\mu s$$

$$E_{x1} = P_t\tau_1 = 10 \times 10^3 \times 15 \times 10^{-6} = 0.15 \ \ Joules$$

$$E_{x2} = P_2\tau_2 = 10 \times 10^3 \times 5 \times 10^{-6} = 0.05 \ \ Joules$$

1.2. Range Resolution

Range resolution, denoted as ΔR, is a radar metric that describes its ability to detect targets in close proximity to each other as distinct objects. Radar systems are normally designed to operate between a minimum range R_{min} and maximum range R_{max}. The distance between R_{min} and R_{max} along the radar line of sight is divided into M range bins (gates), each of width ΔR,

$$M = \frac{R_{max} - R_{min}}{\Delta R} \tag{1.5}$$

Targets separated by at least ΔR will be completely resolved in range.

In order to derive an exact expression for ΔR, consider two targets located at ranges R_1 and R_2, corresponding to time delays t_1 and t_2, respectively. This is illustrated in Fig. 1.3. Denote the difference between those two ranges as ΔR:

$$\Delta R = R_2 - R_1 = c\frac{(t_2 - t_1)}{2} = c\frac{\delta t}{2} \tag{1.6}$$

The question that needs to be answered is: What is the minimum time, δt, such that target 1 at R_1 and target 2 at R_2 will appear completely resolved in range (different range bins)? In other words, what is the minimum ΔR?

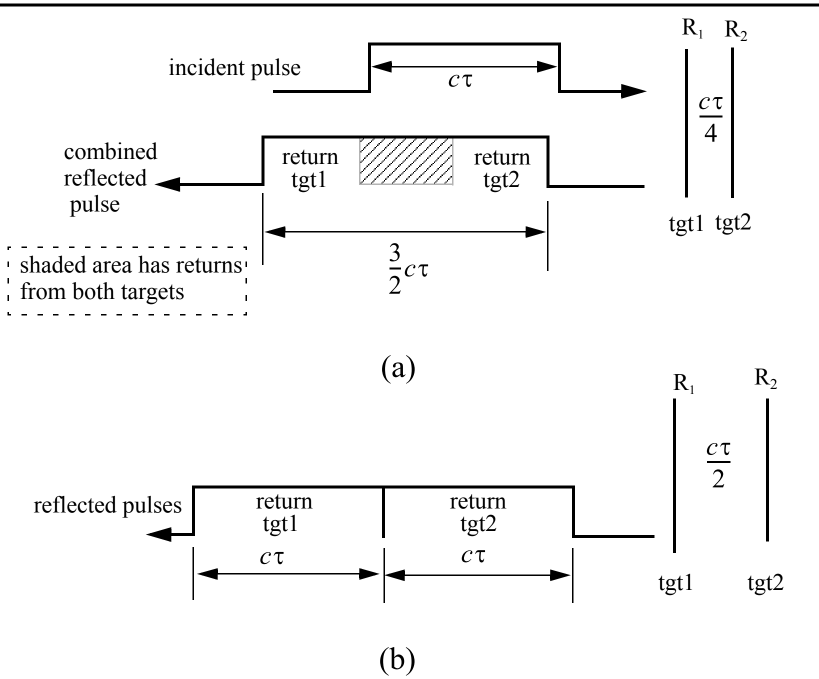


Figure 1.3. (a) Two unresolved targets. (b) Two resolved targets.

First, assume that the two targets are separated by $c\tau/4$, τ is the pulse width. In this case, when the pulse trailing edge strikes target 2, the leading edge would have traveled backward a distance $c\tau$, and the returned pulse would be composed of returns from both targets (i.e., unresolved return), as shown in Fig. 1.3a. If the two targets are at least $c\tau/2$ apart, then as the pulse trailing edge strikes the first target, the leading edge will start to return from target 2, and two distinct returned pulses will be produced, as illustrated by Fig. 1.3b. This means ΔR should be greater or equal to $c\tau/2$. Since the radar bandwidth B is equal to $1/\tau$, then

$$\Delta R = \frac{c\tau}{2} = \frac{c}{2B} \tag{1.7}$$

In general, radar users and designers alike seek to minimize ΔR in order to enhance the radar performance. As suggested by Eq. (1.7), in order to achieve fine range resolution one must minimize the pulse width. This will reduce the average transmitted power and increase the operating bandwidth. Achieving fine range resolution while maintaining adequate average transmitted power can be accomplished by using pulse compression techniques.

Example:

A radar system has an unambiguous range of 100 Km and a bandwidth 0.5 MHz. Compute the required PRF, PRI, ΔR, and τ.

Solution:

$$PRF = \frac{c}{2R_u} = \frac{3 \times 10^8}{2 \times 10^5} = 1500 \quad Hz$$

$$PRI = \frac{1}{PRF} = \frac{1}{1500} = 0.6667 \quad ms$$

It follows,

$$\Delta R = \frac{c}{2B} = \frac{3 \times 10^8}{2 \times 0.5 \times 10^6} = 300 \quad m$$

$$\tau = \frac{2\Delta R}{c} = \frac{2 \times 300}{3 \times 10^8} = 2 \quad \mu s$$

1.3. Doppler Frequency

Radars use Doppler frequency to extract target radial velocity (range rate), as well as to distinguish between moving and stationary targets or objects, such as clutter. The Doppler phenomenon describes the shift in the center frequency of an incident waveform due to the target motion with respect to the source of radiation. Depending on the direction of the target's motion, this frequency shift may be positive or negative. A waveform incident on a target has equiphase wavefronts separated by λ, the wavelength. A closing target will cause the reflected equiphase wavefronts to get closer to each other (smaller wavelength). Alternatively, an opening or receding target (moving away from the radar) will cause the reflected equiphase wavefronts to expand (larger wavelength), as illustrated in Fig. 1.4.

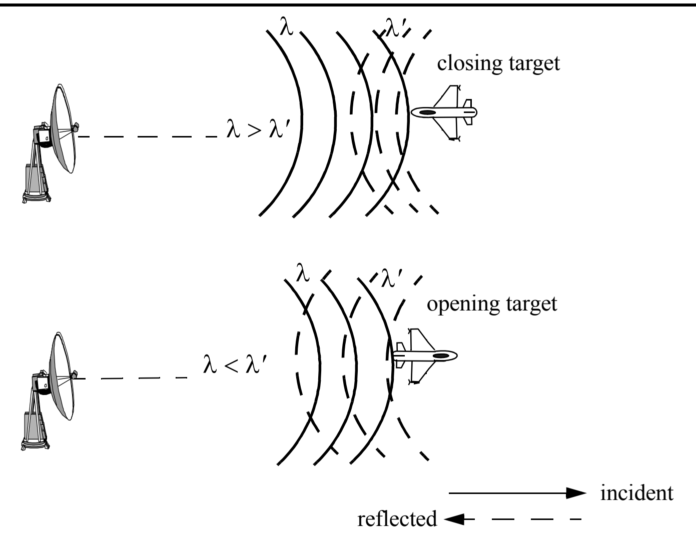


Figure 1.4. Effect of target motion on the reflected equiphase waveforms.

The result formula for the Doppler frequency can be derived with the help of Fig. 1.5. Assume a target closing on the radar with radial velocity (target velocity component along the radar line of sight) v . Let R_0 refer to the range at time t_0 (time reference); then the range to the target at any time t is

$$R(t) = R_0 - vt \tag{1.8}$$

Assume a radar transmitted signal given by

$$x(t) = A\cos(2\pi f_0 t) \tag{1.9}$$

where f_0 is the radar operating center frequency. It follows that the signal received by the radar is

$$x_r(t) = x(t - \phi(t)) \tag{1.10}$$

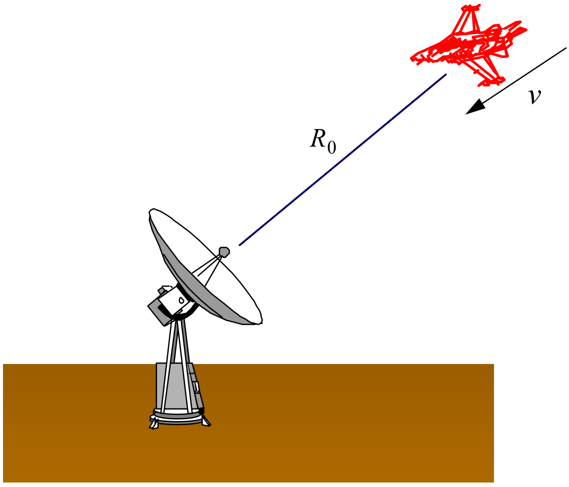


Figure 1.5. Closing target with velocity *v*.

where

$$\phi(t) = \frac{2}{c}(R_0 - vt) \tag{1.11}$$

Substituting Eq. (1.9) and Eq. (1.11) into Eq. (1.10) and collecting terms yields

$$x_r(t) = A_r \cos\left[2\pi\left(f_0 t - f_0 \frac{2R_0}{c} + \frac{2f_0 vt}{c}\right)\right] \tag{1.12}$$

where A_r is a constant. The phase term

$$\psi_0 = 2\pi f_0 \frac{2R_0}{c} \tag{1.13}$$

is used to measure initial target detection range, and the term $2f_0 v/c$ represents a frequency shift due to target velocity (i.e., Doppler frequency shift). The Doppler frequency is given by

$$f_d = \frac{2f_0 v}{c} = \frac{2v}{\lambda} \tag{1.14}$$

where λ is the wavelength given by

$$\lambda = \frac{c}{f_0} \tag{1.15}$$

Note that if the target were going away from the radar (opening or receding target), then

$$f_d = -\frac{2f_0 v}{c} = -\frac{2v}{\lambda} \tag{1.16}$$

as illustrated in Fig. 1.6.

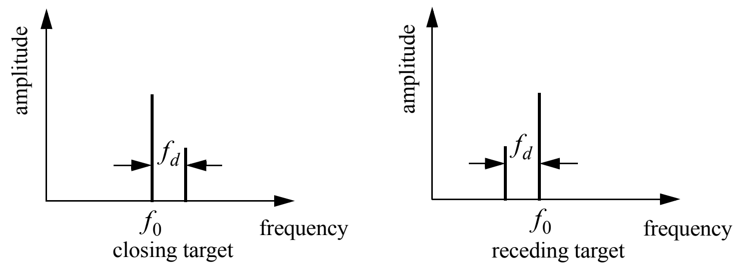


Figure 1.6. Spectra of received signal showing Doppler shift.

In general the target Doppler frequency depends on the target velocity component in the direction of the radar (radial velocity). Figure 1.7 shows three targets all having velocity v. Target 1 has zero Doppler shift; target 2 has maximum Doppler frequency as defined in Eq. (1.15). The amount of Doppler frequency of target 3 is $f_d = 2v\cos\theta/\lambda$, where $v\cos\theta$ is the radial velocity; and θ is the total angle between the radar line of sight and the target.

A more general expression for f_d that accounts for the total angle between the radar and the target is

$$f_d = \frac{2v}{\lambda}\cos\theta \tag{1.17}$$

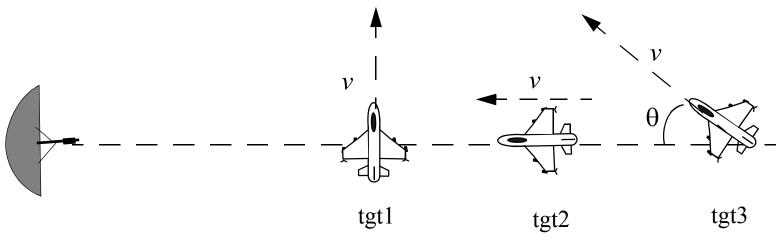


Figure 1.7. Target 1 generates zero Doppler. Target 2 generates maximum Doppler. Target 3 is in between.

and for an opening target is

$$f_d = \frac{-2v}{\lambda}\cos\theta \qquad \textbf{(1.18)}$$

where $\cos\theta = \cos\theta_e \ \cos\theta_a$. The angles θ_e and θ_a are, respectively, the elevation and azimuth angles; see Fig. 1.8.

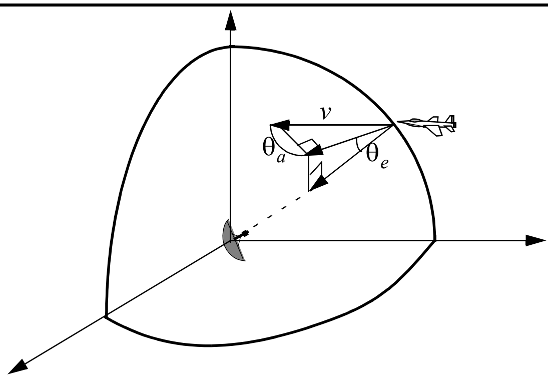
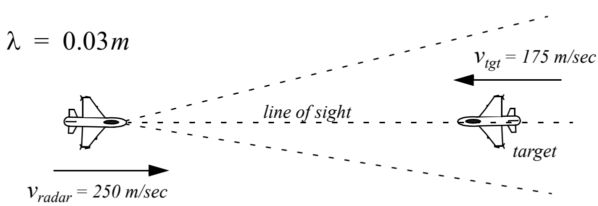


Figure 1.8. Radial velocity is proportional to the azimuth and elevation angles.

Example:

Compute the Doppler frequency measured by the radar shown in the figure below.



Solution:

The relative radial velocity between the radar and the target is $v_{radar} + v_{tgt}$. *Using Eq. (1.15) yields*

$$f_d = 2\frac{(250 + 175)}{0.03} = 28.3KHz$$

Similarly, if the target were opening, the Doppler frequency is

$$f_d = 2\frac{250 - 175}{0.03} = 5KHz$$

1.4. Coherence

A radar is said to be coherent if the phase of any two transmitted pulses is consistent; i.e., there is a continuity in the signal phase from one pulse to the next. One can view coherence as the radar's ability to maintain an integer multiple of wavelengths between the equiphase wavefront from the end of one pulse to the equiphase wavefront at the beginning of the next pulse. Coherency can be achieved by using a STAble Local Oscillator (STALO). A radar is said to be coherent-on-receive or quasi-coherent if it stores in its memory a record of the phases of all transmitted pulses. In this case, the receiver phase reference is normally the phase of the most recently transmitted pulse.

Coherence also refers to the radar's ability to accurately measure (extract) the received signal phase. Since Doppler represents a frequency shift in the received signal, only coherent or coherent-on-receive radars can extract Doppler information. This is because the instantaneous frequency of a signal is proportional to the time derivative of the signal phase.

1.5. The Radar Equation

Consider a radar with an isotropic antenna (one that radiates energy equally in all directions). Since these kinds of antennas have a spherical radiation pattern, we can define the peak power density (power per unit area) at any point in space as

$$P_D = \frac{Peak\ \ transmitted\ \ power}{area\ \ of\ \ a\ \ sphere} \qquad \frac{watts}{m^2} \tag{1.19}$$

The power density at range R away from the radar (assuming a lossless propagation medium) is

$$P_D = P_t / (4\pi R^2) \tag{1.20}$$

where P_t is the peak transmitted power and $4\pi R^2$ is the surface area of a sphere of radius R. Radar systems utilize directional antennas in order to increase the power density in a certain direction. Directional antennas are usually characterized by the antenna gain G and the antenna effective aperture A_e. They are related by

$$G = (4\pi A_e) / \lambda^2 \tag{1.21}$$

where λ is the wavelength. The relationship between the antenna's effective aperture A_e and the physical aperture A is

$$A_e = \rho A \qquad 0 \le \rho \le 1 \tag{1.22}$$

ρ is referred to as the aperture efficiency, and good antennas require $\rho \to 1$. In this book we will assume, unless otherwise noted, that A and A_e are the same. We will also assume that antennas have the same gain in the transmitting and receiving modes. In practice, $\rho \approx 0.7$ is widely accepted.

The gain is also related to the antenna's azimuth and elevation beam widths by

$$G = K\frac{4\pi}{\theta_e \theta_a} \tag{1.23}$$

where $K \le 1$ and depends on the physical aperture shape; the angles θ_e and θ_a are the antenna's elevation and azimuth beam widths, respectively, in radians. When the antenna has a continuous aperture, an excellent approximation of Eq. (1.23) can be written as

$$G \approx \frac{26000}{\theta_e \theta_a} \tag{1.24}$$

where in this case the azimuth and elevation beam widths are given in degrees.

The power density at a distance R away from a radar using a directive antenna of gain G is then given by

$$P_D = \frac{P_t G}{4\pi R^2} \tag{1.25}$$

When the radar radiated energy impinges on a target, the induced surface currents on that target radiate electromagnetic energy in all directions. The amount of the radiated energy is proportional to the target size, orientation, physical shape, and material, which are all lumped together in one target-specific parameter called the Radar Cross Section (RCS) denoted by σ.

The radar cross section is defined as the ratio of the power reflected back to the radar to the power density incident on the target,

$$\sigma = \frac{P_r}{P_D} \ m^2 \tag{1.26}$$

where P_r is the power reflected from the target. The total power delivered to the radar receiver at the back-end of the antenna is

$$P_r = \frac{P_t G \sigma}{(4\pi R^2)^2} A_e \tag{1.27}$$

Substituting the value of A_e from Eq. (1.21) into Eq. (1.27) yields

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \tag{1.28}$$

Let S_{min} denote the minimum detectable signal power. It follows that the maximum radar range R_{max} is

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right)^{1/4} \tag{1.29}$$

Equation (1.29) suggests that in order to double the radar maximum range one must increase the peak transmitted power P_t sixteen times; or equivalently, one must increase the effective aperture four times.

In practical situations the returned signals received by the radar will be corrupted with noise, which introduces unwanted voltages at all radar frequencies. Noise is random in nature and can be described by its Power Spectral Density (PSD) function. The noise power N is a function of the radar operating bandwidth, B. More precisely

$$N = Noise \;\; PSD \times B \tag{1.30}$$

The receiver input noise power is

$$N_i = kT_0B \tag{1.31}$$

where $k = 1.38 \times 10^{-23}$ $Joule/degree \;\; Kelvin$ is Boltzmann's constant, and $T_0 = 290$ is the receiver input noise temperature in degrees Kelvin. It is always desirable that the minimum detectable signal (S_{min}) be greater than the noise power. The sensitivity of a radar receiver is normally described by a figure of merit called the noise figure F (see Section 1.9 for details). The noise figure is defined as

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i/N_i}{S_o/N_o} \tag{1.32}$$

$(SNR)_i$ and $(SNR)_o$ are, respectively, the Signal to Noise Ratios (SNR) at the input and output of the receiver. The input signal power is S_i; and the input noise power immediately at the antenna terminal is N_i. The values S_o and N_o are, respectively, the output signal and noise power.

The receiver effective noise temperature excluding the antenna is (see Section 1.9)

$$T_e = T_0(F-1) \tag{1.33}$$

where F is the receiver noise figure. It follows that the total effective system noise temperature T_s is given by

$$T_s = T_e + T_a = T_0(F-1) + T_a = T_0F - T_0 + T_a \tag{1.34}$$

where T_a is the antenna temperature.

In many radar applications it is desirable to set the antenna temperature T_a to T_0 and thus, Eq. (1.34) is reduced to

$$T_s = T_0F \tag{1.35}$$

Using Eq. (1.35) and Eq. (1.31) in Eq. (1.32) yields

$$S_i = kT_0BF(SNR)_o \tag{1.36}$$

The minimum detectable signal power can be written as

$$S_{min} = kT_0BF(SNR)_{o_{min}} \tag{1.37}$$

The radar detection threshold is set equal to the minimum output SNR, $(SNR)_{o_{min}}$. Substituting Eq. (1.37) in Eq. (1.29) gives

$$R_{max} = \left(\frac{P_tG^2\lambda^2\sigma}{(4\pi)^3kT_0BF(SNR)_{o_{min}}}\right)^{1/4} \tag{1.38}$$

or equivalently,

$$(SNR)_{o_{min}} = \frac{P_tG^2\lambda^2\sigma}{(4\pi)^3kT_0BFR^4_{max}} \tag{1.39}$$

In general, radar losses denoted as L reduce the overall SNR, and hence

$$(SNR)_o = \frac{P_tG^2\lambda^2\sigma}{(4\pi)^3kT_0BFLR^4} \tag{1.40}$$

Equivalently, Eq. (1.40) can be rewritten using Eq. (1.35) as

$$(SNR)_o = \frac{P_tG^2\lambda^2\sigma}{(4\pi)^3kT_sBLR^4} \tag{1.41}$$

In this book, the antenna temperature is assumed to be negligible; therefore, Eq. (1.40) will be dominantly used as the Radar Equation.

Example:

Given a certain C-band radar with the following parameters: Peak power $P_t = 1.5MW$, *operating frequency* $f_0 = 5.6GHz$, *antenna gain* $G = 45dB$, *effective temperature* $T_0 = 290K$, *noise figure* $F = 3dB$, *pulse width* $\tau = 0.2\mu\sec$. *The radar threshold is* $(SNR)_{min} = 20dB$. *Assume target cross section* $\sigma = 0.1m^2$. *Compute the maximum range.*

Solution:

The radar bandwidth is

$$B = \frac{1}{\tau} = \frac{1}{0.2 \times 10^{-6}} = 5MHz$$

The wavelength is

$$\lambda = \frac{c}{f_0} = \frac{3 \times 10^8}{5.6 \times 10^9} = 0.054m$$

From Eq. (1.40) we have

$$(R^4)_{dB} = (P_t + G^2 + \lambda^2 + \sigma - (4\pi)^3 - kT_0B - F - (SNR)_{o_{min}})_{dB}$$

where, before summing, the dB calculations are carried out for each of the individual parameters on the right-hand side. We can now construct the following table with all parameters computed in dB:

P_t	λ^2	G^2	kT_0B	$(4\pi)^3$	F	$(SNR)_{o_{min}}$	σ
61.761	–25.421	$90dB$	–136.987	32.976	$3dB$	$20dB$	–10

It follows that

$$R^4 = 61.761 + 90 - 25.352 - 10 - 32.976 + 136.987 - 3 - 20 = 197.420dB$$

$$R^4 = 10^{(197.420/10)} = 55.208 \times 10^{18} m^4$$

$$R = \sqrt[4]{55.208 \times 10^{18}} = 86.199Km$$

Thus, the maximum detection range is $86.2Km$.

Figure 1.9 shows plots of the SNR versus detection range for the following parameters: Peak power $P_t = 1.5MW$, operating frequency $f_0 = 5.6GHz$, antenna gain $G = 45dB$, radar losses $L = 6dB$, and noise figure $F = 3dB$. The radar bandwidth is $B = 5MHz$. The radar minimum and maximum detection ranges are $R_{min} = 25Km$ and $R_{max} = 165Km$. This figure can be reproduced using the following MATLAB code which utilizes MATLAB function *"radar_eq.m."*

```
close all;
clear all
pt = 1.5e+6; % peak power in Watts
freq = 5.6e+9; % radar operating frequency in Hz
g = 45.0; % antenna gain in dB
sigma = 0.1; % radar cross section in m squared
b = 5.0e+6; % radar operating bandwidth in Hz
nf = 3.0; %  noise figure in dB
loss = 6.0; % radar losses in dB
range = linspace(25e3,165e3,1000);
snr = radar_eq(pt, freq, g, sigma, b, nf, loss, range);
rangekm  = range ./ 1000;
plot(rangekm,snr,'linewidth',1.5)
grid;
xlabel ('Detection range in Km');
ylabel ('SNR in dB');
```

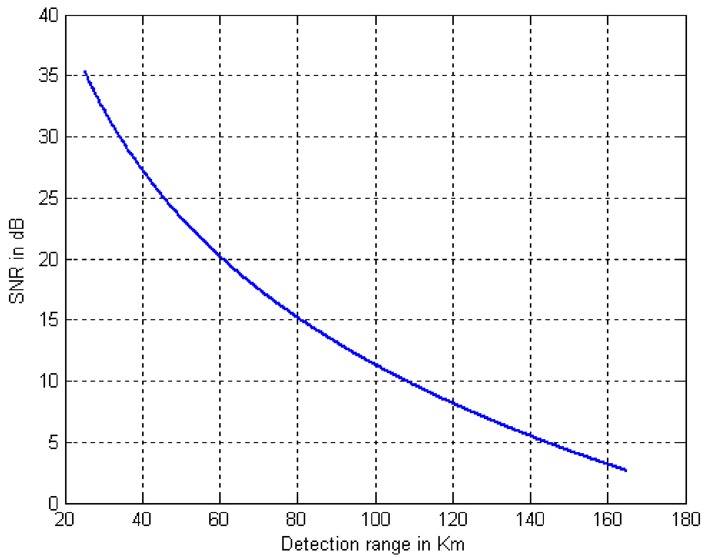


Figure 1.9. SNR versus detection range.

1.6. Surveillance Radar Equation

The first task a certain radar system has to accomplish is to continuously scan a specified volume in space searching for targets of interest. Once detection is established, target information such as range, angular position, and possibly target velocity are extracted by the radar signal and data processors. Depending on the radar design and antenna, different search patterns can be adopted.

Search volumes are normally specified by a search solid angle Ω in steradians, as illustrated in Fig. 1.10. Define the radar search volume extent for both azimuth and elevation as Θ_A and Θ_E. Consequently, the search volume is computed as

$$\Omega = (\Theta_A \Theta_E)/(57.296)^2 \ steradians \tag{1.42}$$

where both Θ_A and Θ_E are given in degrees. The radar antenna $3dB$ beamwidth can be expressed in terms of its azimuth and elevation beam widths θ_a and θ_e, respectively. It follows that the antenna solid angle coverage is $\theta_a \theta_e$ and, thus, the number of antenna beam positions n_B required to cover a solid angle Ω is

$$n_B = \frac{\Omega}{\theta_a \theta_e} \tag{1.43}$$

In order to develop the search radar equation, start with Eq. (140). Using the relations $\tau = 1/B$ and $P_t = P_{av} T/\tau$, where T is the PRI and τ is the pulse width, yields

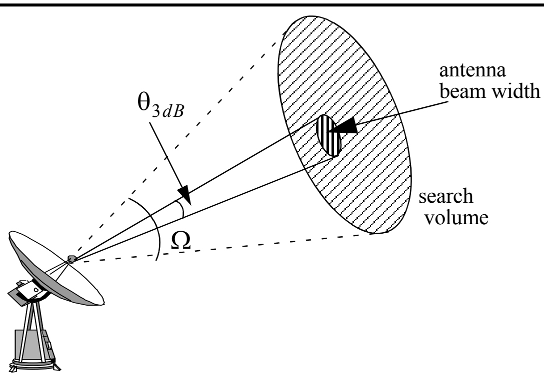


Figure 1.10. A cut in space showing the antenna beam width and the search volume.

$$SNR = \frac{T}{\tau} \frac{P_{av} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 k T_0 F L R^4} \tag{1.44}$$

Define the time it takes the radar to scan a volume defined by the solid angle Ω as the scan time T_{sc}. The time on target can then be expressed in terms of T_{sc} as

$$T_i = \frac{T_{sc}}{n_B} = \frac{T_{sc}}{\Omega} \theta_a \theta_e \tag{1.45}$$

Assume that during a single scan only one pulse per beam per PRI illuminates the target. It follows that $T_i = T$ and, thus, Eq. (1.44) can be written as

$$SNR = \frac{P_{av} G^2 \lambda^2 \sigma}{(4\pi)^3 k T_0 F L R^4} \frac{T_{sc}}{\Omega} \theta_a \theta_e \tag{1.46}$$

Substituting Eq. (1.21) and Eq. (1.45) into Eq. (1.46) and collecting terms yield the search radar equation (based on a single pulse per beam per PRI) as

$$SNR = \frac{P_{av} A_e \sigma}{4\pi k T_0 F L R^4} \frac{T_{sc}}{\Omega} \tag{1.47}$$

The quantity $P_{av} A$ in Eq. (1.47) is known as the power aperture product. In practice, the power aperture product (PAP) is widely used to categorize the radar's ability to fulfill its search mission. Normally, a power aperture product is computed to meet a predetermined SNR and radar cross section for a given search volume defined by Ω.

Figure 1.11 shows a plot of the PAP versus detection range. using the following parameters:

σ	T_{sc}	$\theta_e = \theta_a$	R	$F + L$	SNR
$0.1 \ m^2$	$2.5 \sec$	$2°$	$250 Km$	$13 dB$	$15 dB$

This figure can be reproduced using the following MATLAB code which utilizes the MATLAB function *"power_aperture.m."*

```
close all;
clear all;
tsc = 2.5; % scan time is 2.5 seconds
sigma = 0.1; % radar cross section in m squared
te = 900.0; % effective noise temperature in Kelvin
snr = 15; % desired SNR in dB
nf = 6.0; % noise figure in dB
```

```
loss = 7.0; % radar losses in dB
az_angle = 2; % search volume azimuth extent in degrees
el_angle = 2; % search volume elevation extent in degrees
range = linspace(20e3,250e3,1000);
pap = power_aperture(snr,tsc,sigma/10,range,nf,loss,az_angle,el_angle);
rangekm  = range ./ 1000;
plot(rangekm,pap,'linewidth',1.5)
grid
xlabel ('Detection range in Km');
ylabel ('Power aperture product in dB');
```

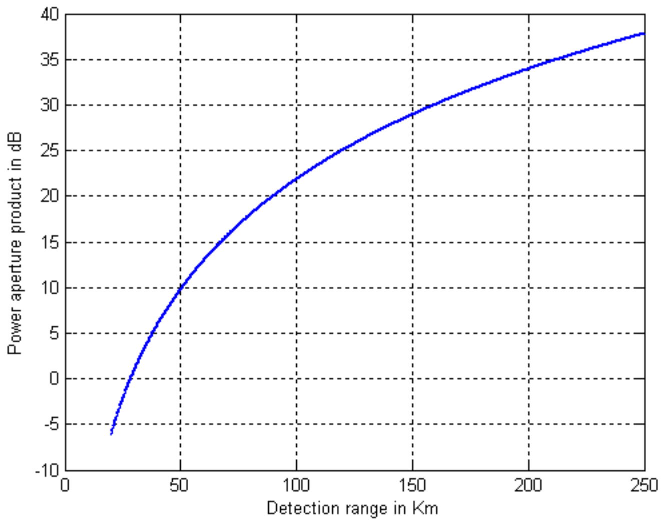


Figure 1.11. Power aperture product versus detection range.

Example:

Compute the power aperture product corresponding to the radar that has the following parameters: Scan time $T_{sc} = 2s$*, noise figure* $F = 8dB$*, losses* $L = 6dB$*, search volume* $\Omega = 7.4\ steradians$*, range of interest* $R = 75Km$*, and required SNR* $20dB$*. Assume that* $\sigma = 3.162m^2$.

Solution:

Note that $\Omega = 7.4\ steradians$ *corresponds to a search sector that is three fourths of a hemisphere. Thus, we conclude that* $\Theta_a = 180°$ *and* $\Theta_e = 135°$*. Using the MATLAB function "power_aperture.m" with the following syntax:*

PAP = power_aperture(20, 2, 3.162, 75e3, 8, 6, 180, 135)

one computes the power aperture product as 36.2 dB.

Example:

Compute the power aperture product for an X-band radar with the following parameters: Signal-to-noise ratio $SNR = 15dB$; *losses* $L = 8dB$; *search volume* $\Omega = 2°$; *scan time* $T_{sc} = 2.5s$; *noise figure* $F = 5dB$. *Assume a* $-10dBsm$ *target cross section, and range* $R = 250Km$. *Also, compute the peak transmitted power corresponding to 30% duty factor if the antenna gain is 45 dB. Assume a circular aperture.*

Solution:

The angular coverage is $2°$ *in both azimuth and elevation. It follows that the solid angle coverage is*

$$\Omega = \frac{2 \times 2}{(57.23)^2} = -29.132dB$$

Note that the factor $360/2\pi = 57.23$ *converts degrees into steradians. When the aperture is circular Eq. (1.47) is reduced to (details are left as an exercise)*

$$(SNR)_{dB} = (P_{av} + A + \sigma + T_{sc} - 16 - R^4 - kT_0 - L - F - \Omega)_{dB}$$

σ	T_{sc}	16	R^4	kT_0
–10	3.979	12.041	215.918	–203.977

It follows that

$$15 = P_{av} + A - 10 + 3.979 - 12.041 - 215.918 + 203.977 - 5 - 8 + 29.133$$

Then the power aperture product is

$$P_{av} + A = 38.716dB$$

Now, assume the radar wavelength to be $\lambda = 0.03m$, *then*

$$A = \frac{G\lambda^2}{4\pi} = 3.550dB$$

$$P_{av} = -A + 38.716 = 35.166dB$$

$$P_{av} = 10^{3.5166} = 3285.489W$$

$$P_t = \frac{P_{av}}{d_t} = \frac{3285.489}{0.3} = 10.9512KW$$

1.7. Radar Cross Section

Electromagnetic waves are normally diffracted or scattered in all directions when incident on a target. These scattered waves are broken down into two parts. The first part is made of waves that have the same polarization as the receiving antenna. The other portion of the scattered waves will have a different polarization to which the receiving antenna does not respond. The two polarizations are orthogonal and are referred to as the Principal Polarization (PP) and Orthogonal Polarization (OP), respectively. The intensity of the *backscattered* energy that has the same polarization as the radar's receiving antenna is used to define the target RCS. When a target is illuminated by RF energy, it acts like a virtual antenna and will have near and far scattered fields. Waves reflected and measured in the near field are, in general, spherical. Alternatively, in the far field the wavefronts are decomposed into a linear combination of plane waves. Assume the power density of a wave incident on a target located at range R away from the radar is P_{Di}, as illustrated in Fig. 1.12. The amount of reflected power from the target is

$$P_r = \sigma P_{Di} \tag{1.48}$$

where σ denotes the target cross section. Define P_{Dr} as the power density of the scattered waves at the receiving antenna. It follows that

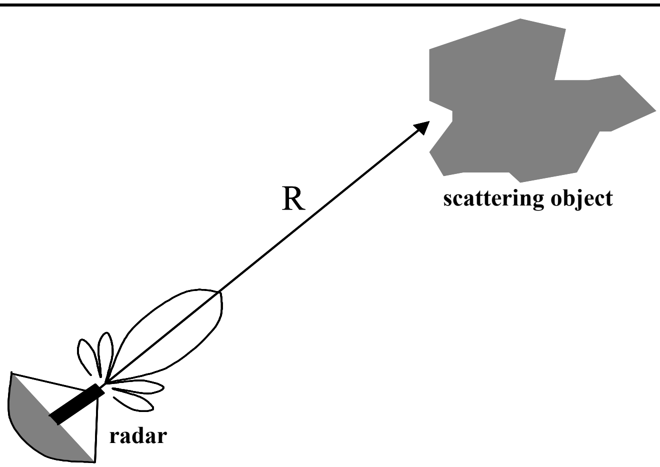


Figure 1.12. Scattering object located at range R.

$$P_{Dr} = P_r/(4\pi R^2) \tag{1.49}$$

Equating Eqs. (1.48) and (1.49) yields

$$\sigma = 4\pi R^2 \left(\frac{P_{Dr}}{P_{Di}}\right) \tag{1.50}$$

and in order to ensure that the radar receiving antenna is in the far field (i.e., scattered waves received by the antenna are planar), Eq. (1.50) is modified to

$$\sigma = 4\pi R^2 \lim_{R \to \infty} \left(\frac{P_{Dr}}{P_{Di}}\right) \tag{1.51}$$

The RCS defined by Eq. (1.51) is often referred to as either the monostatic RCS, the backscattered RCS, or simply target RCS.

The backscattered RCS is measured from all waves scattered in the direction of the radar and has the same polarization as the receiving antenna. It represents a portion of the total scattered target RCS σ_t, where $\sigma_t > \sigma$. Assuming a spherical coordinate system defined by (ρ, θ, φ), then at range ρ the target scattered cross section is a function of (θ, φ). Let the angles (θ_i, φ_i) define the direction of propagation of the incident waves. Also, let the angles (θ_s, φ_s) define the direction of propagation of the scattered waves. The special case, when $\theta_s = \theta_i$ and $\varphi_s = \varphi_i$, defines the monostatic RCS. The RCS measured by the radar at angles $\theta_s \neq \theta_i$ and $\varphi_s \neq \varphi_i$ is called the bistatic RCS.

The total target scattered RCS is given by

$$\sigma_t = \frac{1}{4\pi} \int_{\varphi_s = 0}^{2\pi} \int_{\theta_s = 0}^{\pi} \sigma(\theta_s, \varphi_s) \sin\theta_s \; d\theta \; d\varphi_s \tag{1.52}$$

The amount of backscattered waves from a target is proportional to the ratio of the target extent (size) to the wavelength, λ, of the incident waves. In fact, a radar will not be able to detect targets much smaller than its operating wavelength. The frequency region, where the target extent and the wavelength are comparable, is referred to as the Rayleigh region. Alternatively, the frequency region where the target extent is much larger than the radar operating wavelength is referred to as the optical region.

1.7.1. RCS Dependency on Aspect Angle and Frequency

Radar cross section fluctuates as a function of radar aspect angle and frequency. For the purpose of illustration, isotropic point scatterers are considered. Consider the geometry shown in Fig. 1.13. In this case, two unity ($1m^2$)

isotropic scatterers are aligned and placed along the radar line of sight (zero aspect angle) at a far field range R. The spacing between the two scatterers is 1 meter. The radar aspect angle is then changed from zero to 180 degrees, and the composite RCS of the two scatterers measured by the radar is computed.

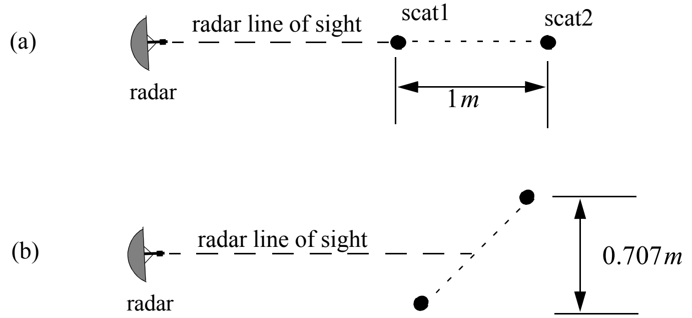


Figure 1.13. RCS dependency on aspect angle. (a) Zero aspect angle, zero electrical spacing. (b) $45°$ aspect angle, 1.414λ electrical spacing.

This composite RCS consists of the superposition of the two individual radar cross sections. At zero aspect angle, the composite RCS is $2m^2$. Taking scatterer-1 as a phase reference, when the aspect angle is varied, the composite RCS is modified by the phase that corresponds to the electrical spacing between the two scatterers. For example, at aspect angle $10°$, the electrical spacing between the two scatterers is

$$elec\text{–}spacing = \frac{2 \times (1.0 \times \cos(10°))}{\lambda} \tag{1.53}$$

λ is the radar operating wavelength.

Figure 1.14 shows the composite RCS corresponding to this experiment. This plot can be reproduced using the MATLAB code listed below. As clearly indicated by Fig. 1.14, RCS is dependent on the radar aspect angle; thus, knowledge of this constructive and destructive interference between the individual scatterers can be very critical when a radar tries to extract the RCS of complex or maneuvering targets. This is true for two reasons. First, the aspect angle may be continuously changing. Second, complex target RCS can be viewed to be made up from contributions of many individual scattering points distributed on the target surface. These scattering points are often called scattering centers. Many approximate RCS prediction methods generate a set of scattering centers that define the backscattering characteristics of such complex targets. The figures can be reproduced using the following MATLAB program.

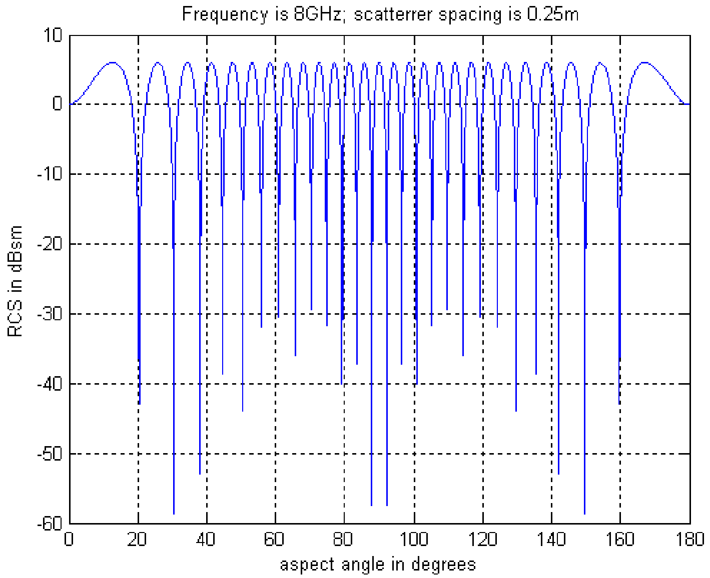


Figure 1.14. Illustration of RCS dependency on aspect angle.

```
clear all; close all;
% This program produces Fig. 1.14. This code demonstrates the effect of aspect angle
% on RCS. The radar is observing two unity point scatterers separated by scat_spacing.
% Initially the two scatterers are aligned with radar line of sight. The aspect angle is
% changed from 0 degrees to 180 degrees and the equivalent RCS is computed.
% The RCS as measured by the radar versus aspect angle is then plotted.
scat_spacing = 0.25; % 0.25 meter scatterers spacing
freq = 8e9; % operating frequency
eps = 0.00001;
wavelength = 3.0e+8 / freq;
% Compute aspect angle vector
aspect_degrees = linspace(0, 180, 500);
aspect_radians = (pi/180) .* aspect_degrees;
% Compute electrical scatterer spacing vector in wavelength units
elec_spacing = (2.0 * scat_spacing / wavelength) .* cos(aspect_radians);
% Compute RCS (rcs = RCS_scat1 + RCS_scat2)
% Scat1 is taken as phase reference point
rcs = abs(1.0 + cos((2.0 * pi) .* elec_spacing) + i * sin((2.0 * pi) .* elec_spacing));
rcs = rcs + eps;
rcs = 20.0*log10(rcs); % RCS in dBsm
% Plot RCS versus aspect angle
figure (1);
plot(aspect_degrees,rcs);
```

```
grid; xlabel('aspect angle in degrees'); ylabel('RCS in dBsm');
title(' Frequency is 8GHz; scatterer spacing is 0.25m');
```

Next, to demonstrate RCS dependency on frequency, consider the experiment shown in Fig. 1.15. In this case, two far field unity isotropic scatterers are aligned with radar line of sight, and the composite RCS is measured by the radar as the frequency is varied from 8 GHz to 12.5 GHz (X-band). Figs. 1.16 and 1.17 show the composite RCS versus frequency for scatterer spacing of 0.25 and 0.75 meters. The figures can be reproduced using the following MATLAB function.

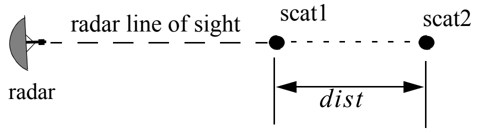


Figure 1.15. Experiment setup which demonstrates RCS dependency on frequency; dist = 0.25, or 0.75 m.

```
clear all; close all;
% This program demonstrates the dependency of RCS on wavelength
% The radar line of sight is aligned with the two scatterers
% A plot of RCS variation versus frequency if produced
eps = 0.0001;
scat_spacing = 0.25;
freql = 8e9;
frequ = 12.5e9;
freq = linspace(freql,frequ,500);
wavelength = 3.0e+8 ./ freq;
% Compute electrical scatterer spacing vector in wavelength units
elec_spacing = 2.0 * scat_spacing ./ wavelength;
% Compute RCS (RCS = RCS_scat1 + RCS_scat2)
rcs = abs (  1 + cos((2.0 * pi) .* elec_spacing) ...
        + i * sin((2.0 * pi) .* elec_spacing));
rcs = rcs + eps;
rcs = 20.0*log10(rcs); % RCS ins dBsm
% Plot RCS versus frequency
figure (1);
plot(freq./1e9,rcs);
grid;
xlabel('Frequency in GHz');
ylabel('RCS in dBsm');
% title(' X=Band; scatterer spacing is 0.25 m'); % Fig. 1.16
% title(' X=Band; scatterer spacing is 0.75 m'); % Fig. 1.17
```

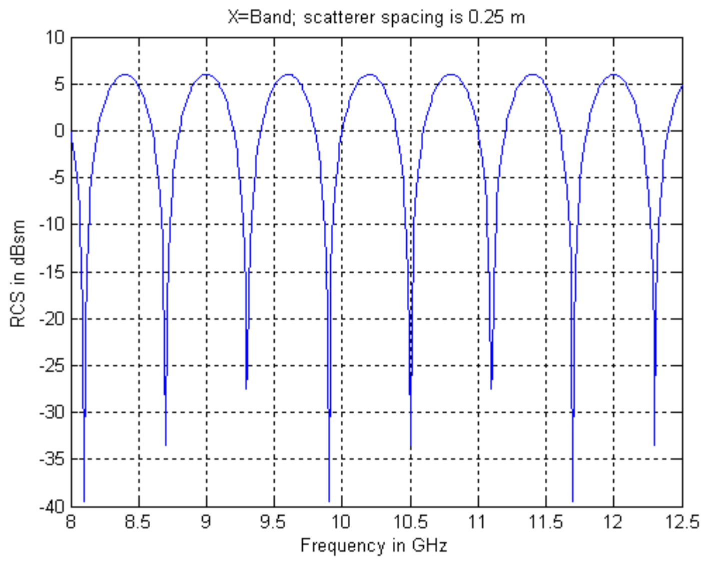


Figure 1.16. Illustration of RCS dependency on frequency.

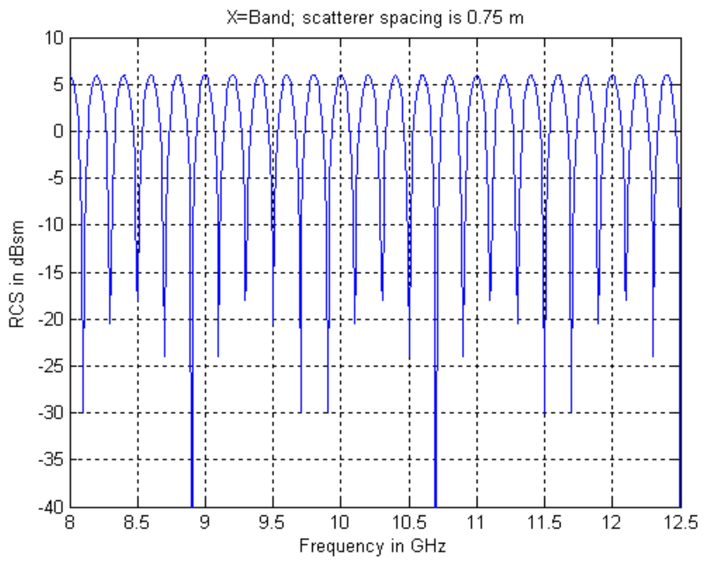


Figure 1.17. Illustration of RCS dependency on frequency.

1.7.2. RCS Dependency on Polarization

Normalized Electric Field

In most radar simulations, it is desirable to obtain the complex-valued electric field scattered by the target at the radar. In such cases, it is useful to use a quantity called the normalized electric field. It is assumed that the incident electric field has a magnitude of unity and is phase centered at a point at the target (usually the center of gravity). More precisely,

$$E_i = e^{jk(\vec{r}_i \cdot \vec{r})} \tag{1.54}$$

where $\vec{r}_i$ is the direction of incidence and $\vec{r}$ a location at the target, each with respect to the phase center. The normalized scattered field is then given by

$$E_s = \sigma E_i \tag{1.55}$$

The quantity E_s is independent of radar and target location. It may be combined with an incident magnitude and phase.

Polarization

The x and y electric field components for a wave traveling along the positive z direction are given by

$$E_x = E_1 \sin(\omega t - kz) \tag{1.56}$$

$$E_y = E_2 \sin(\omega t - kz + \delta) \tag{1.57}$$

where $k = 2\pi/\lambda$, ω is the wave frequency, the angle δ is the time phase angle at which E_y leads E_x, and finally, E_1 and E_2 are, respectively, the wave amplitudes along the x and y directions. When two or more electromagnetic waves combine, their electric fields are integrated vectorially at each point in space for any specified time. In general, the combined vector traces an ellipse when observed in the x-y plane. This is illustrated in Fig. 1.18.

The ratio of the major to the minor axes of the polarization ellipse is called the Axial Ratio (AR). When AR is unity, the polarization ellipse becomes a circle, and the resultant wave is then called circularly polarized. Alternatively, when $E_1 = 0$ and $AR = \infty$, the wave becomes linearly polarized.

Equations (1.56) and (1.57) can be combined to give the instantaneous total electric field,

$$\vec{E} = \hat{a}_x E_1 \sin(\omega t - kz) + \hat{a}_y E_2 \sin(\omega t - kz + \delta) \tag{1.58}$$

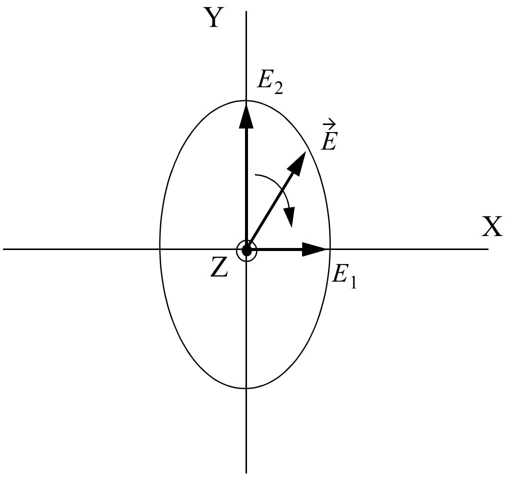


Figure 1.18. Electric field components along the x and y directions. The positive z direction is out of the page.

where $\hat{a}_x$ and $\hat{a}_y$ are unit vectors along the x and y directions, respectively. At $z = 0$, $E_x = E_1 \sin(\omega t)$ and $E_y = E_2 \sin(\omega t + \delta)$, then by replacing $\sin(\omega t)$ by the ratio E_x / E_1 and by using trigonometry properties Eq. (1.58) can be rewritten as

$$\frac{E_x^2}{E_1^2} - \frac{2E_xE_y\cos\delta}{E_1E_2} + \frac{E_y^2}{E_2^2} = (\sin\delta)^2 \tag{1.59}$$

which has no dependency on ωt.

In the most general case, the polarization ellipse may have any orientation, as illustrated in Fig. 1.19. The angle ξ is called the tilt angle of the ellipse. In this case, AR is given by

$$AR = \frac{OA}{OB} \qquad (1 \le AR \le \infty) \tag{1.60}$$

When $E_1 = 0$, the wave is said to be linearly polarized in the y direction, while if $E_2 = 0$, the wave is said to be linearly polarized in the x direction. Polarization can also be linear at an angle of $45°$ when $E_1 = E_2$ and $\xi = 45°$. When $E_1 = E_2$ and $\delta = 90°$, the wave is said to be Left Circularly Polarized (LCP), while if $\delta = -90°$ the wave is said to Right Circularly Polarized (RCP). It is a common notation to call the linear polarizations along the x and y directions by the names horizontal and vertical polarizations, respectively.

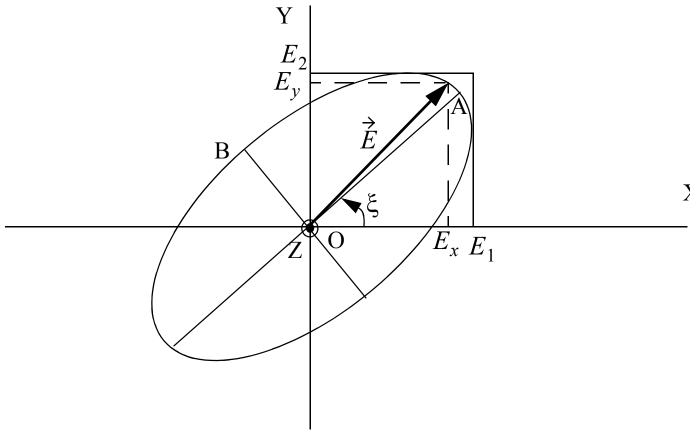


Figure 1.19. Polarization ellipse in the general case.

In general, an arbitrarily polarized electric field may be written as the sum of two circularly polarized fields. More precisely,

$$\vec{E} = \overrightarrow{E_R} + \overrightarrow{E_L} \tag{1.61}$$

where $\overrightarrow{E_R}$ and $\overrightarrow{E_L}$ are the RCP and LCP fields, respectively. Similarly, the RCP and LCP waves can be written as

$$\overrightarrow{E_R} = \overrightarrow{E_V} + j\overrightarrow{E_H} \tag{1.62}$$

$$\overrightarrow{E_L} = \overrightarrow{E_V} - j\overrightarrow{E_H} \tag{1.63}$$

where $\overrightarrow{E_V}$ and $\overrightarrow{E_H}$ are the fields with vertical and horizontal polarizations, respectively. Combining Eqs. (1.62) and (1.63) yields

$$E_R = \frac{E_H - jE_V}{\sqrt{2}} \tag{1.64}$$

$$E_L = \frac{E_H + jE_V}{\sqrt{2}} \tag{1.65}$$

Using matrix notation, Eqs. (1.64) and (1.65) can be rewritten as

$$\begin{bmatrix} E_R \\ E_L \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix} = [T] \begin{bmatrix} E_H \\ E_V \end{bmatrix} \tag{1.66}$$

$$\begin{bmatrix} E_H \\ E_V \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \begin{bmatrix} E_R \\ E_L \end{bmatrix} = [T]^{-1} \begin{bmatrix} E_H \\ E_V \end{bmatrix} \tag{1.67}$$

For many targets the scattered waves will have different polarization than the incident waves. This phenomenon is known as depolarization or cross-polarization. However, perfect reflectors reflect waves in such a fashion that an incident wave with horizontal polarization remains horizontal, and an incident wave with vertical polarization remains vertical but is phase shifted $180°$. Additionally, an incident wave that is RCP becomes LCP when reflected, and a wave that is LCP becomes RCP after reflection from a perfect reflector. Therefore, when a radar uses LCP waves for transmission, the receiving antenna needs to be RCP polarized in order to capture the PP RCS, and LCP to measure the OP RCS.

Target Scattering Matrix

Target backscattered RCS is commonly described by a matrix known as the scattering matrix and is denoted by $[S]$. When an arbitrarily linearly polarized wave is incident on a target, the backscattered field is then given by

$$\begin{bmatrix} E_1^s \\ E_2^s \end{bmatrix} = [S] \begin{bmatrix} E_1^i \\ E_2^i \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} E_1^i \\ E_2^i \end{bmatrix} \tag{1.68}$$

The superscripts i and s denote incident and scattered fields. The quantities s_{ij} are in general complex and the subscripts 1 and 2 represent any combination of orthogonal polarizations. More precisely, $1 = H, R$, and $2 = V, L$. From Eq. (1.50), the backscattered RCS is related to the scattering matrix components by the following relation:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = 4\pi R^2 \begin{bmatrix} |s_{11}|^2 & |s_{12}|^2 \\ |s_{21}|^2 & |s_{22}|^2 \end{bmatrix} \tag{1.69}$$

It follows that once a scattering matrix is specified, the target backscattered RCS can be computed for any combination of transmitting and receiving polarizations. The reader is advised to see Ruck et al. (1970) for ways to calculate the scattering matrix $[S]$. Rewriting Eq. (1.69) in terms of the different possible orthogonal polarizations yields

$$\begin{bmatrix} E_H^s \\ E_V^s \end{bmatrix} = \begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} \begin{bmatrix} E_H^i \\ E_V^i \end{bmatrix} \tag{1.70}$$

$$\begin{bmatrix} E_R^s \\ E_L^s \end{bmatrix} = \begin{bmatrix} s_{RR} & s_{RL} \\ s_{LR} & s_{LL} \end{bmatrix} \begin{bmatrix} E_R^i \\ E_L^i \end{bmatrix} \tag{1.71}$$

By using the transformation matrix $[T]$ in Eq. (1.66), the circular scattering elements can be computed from the linear scattering elements

$$\begin{bmatrix} s_{RR} & s_{RL} \\ s_{LR} & s_{LL} \end{bmatrix} = [T] \begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} [T]^{-1} \tag{1.72}$$

and the individual components are

$$s_{RR} = \frac{-s_{VV} + s_{HH} - j(s_{HV} + s_{VH})}{2} \tag{1.73}$$

$$s_{RL} = \frac{s_{VV} + s_{HH} + j(s_{HV} - s_{VH})}{2} \tag{1.74}$$

$$s_{LR} = \frac{s_{VV} + s_{HH} - j(s_{HV} - s_{VH})}{2} \tag{1.75}$$

$$s_{LL} = \frac{-s_{VV} + s_{HH} + j(s_{HV} + s_{VH})}{2} \tag{1.76}$$

Similarly, the linear scattering elements are given by

$$\begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} = [T]^{-1} \begin{bmatrix} s_{RR} & s_{RL} \\ s_{LR} & s_{LL} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} [T] \tag{1.77}$$

and the individual components are

$$s_{HH} = \frac{-s_{RR} + s_{RL} + s_{LR} - s_{LL}}{2} \tag{1.78}$$

$$s_{VH} = \frac{j(s_{RR} - s_{LR} + s_{RL} - s_{LL})}{2} \tag{1.79}$$

$$s_{HV} = \frac{-j(s_{RR} + s_{LR} - s_{RL} - s_{LL})}{2} \tag{1.80}$$

$$s_{VV} = \frac{s_{RR} + s_{LL} + js_{RL} + s_{LR}}{2} \tag{1.81}$$

1.8. Radar Equation with Jamming

Any deliberate electronic effort intended to disturb normal radar operation is usually referred to as an Electronic Countermeasure (ECM). This may also include chaff, radar decoys, radar RCS alterations (e.g., radio frequency absorbing materials), and of course, radar jamming.

Jammers can be categorized into two general types: (1) barrage jammers and (2) deceptive jammers (repeaters). When strong jamming is present, detection capability is determined by receiver signal-to-noise plus interference ratio rather than SNR. In fact, in most cases, detection is established based on the signal-to-interference ratio alone.

Barrage jammers attempt to increase the noise level across the entire radar operating bandwidth. Consequently, this lowers the receiver SNR and, in turn, makes it difficult to detect the desired targets. This is the reason barrage jammers are often called maskers (since they mask the target returns). Barrage jammers can be deployed in the main beam or in the sidelobes of the radar antenna. If a barrage jammer is located in the radar main beam, it can take advantage of the antenna maximum gain to amplify the broadcasted noise signal. Alternatively, sidelobe barrage jammers must either use more power or operate at a much shorter range than main-beam jammers. Main-beam barrage jammers can either be deployed on-board the attacking vehicle or act as an escort to the target. Sidelobe jammers are often deployed to interfere with a specific radar, and since they do not stay close to the target, they have a wide variety of stand-off deployment options.

Repeater jammers carry receiving devices on board in order to analyze the radar's transmission and then send back false target-like signals in order to confuse the radar. There are two common types of repeater jammers: spot noise repeaters and deceptive repeaters. The spot noise repeater measures the transmitted radar signal bandwidth and then jams only a specific range of frequencies. The deceptive repeater sends back altered signals that make the target appear in some false position (ghosts). These ghosts may appear at different ranges or angles than the actual target. Furthermore, there may be several ghosts created by a single jammer. By not having to jam the entire radar bandwidth, repeater jammers are able to make more efficient use of their jamming power. Radar frequency agility may be the only way possible to defeat spot noise repeaters.

In general a jammer can be identified by its effective operating bandwidth B_J and by its Effective Radiated Power (ERP), which is proportional to the jammer transmitter power P_J. More precisely,

$$ERP = P_J G_J / L_J \tag{1.82}$$

where G_J is the jammer antenna gain and L_J is the total jammer loss. The effect of a jammer on a radar is measured by the Signal-to-Jammer ratio (S/J).

Consider a radar system whose detection range R in the absence of jamming is governed by

$$SNR = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_s B_r L R^4} \tag{1.83}$$

The term Range Reduction Factor (RRF) refers to the reduction in the radar detection range due to jamming. More precisely, in the presence of jamming the effective radar detection range is

$$R_{dj} = R \times RRF \tag{1.84}$$

In order to compute RRF, consider a radar characterized by Eq. (1.83) and a barrage jammer whose output power spectral density is J_o (i.e., Gaussian-like). Then the amount of jammer power in the radar receiver is

$$J = k T_J B_r \tag{1.85}$$

where T_J is the jammer effective temperature. It follows that the total jammer plus noise power in the radar receiver is given by

$$N_i + J = k T_s B_r + k T_J B_r \tag{1.86}$$

In this case, the radar detection range is now limited by the receiver signal-to-noise plus interference ratio rather than SNR. More precisely,

$$\left(\frac{S}{J+N}\right) = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k (T_s + T_J) B_r L R^4} \tag{1.87}$$

The amount of reduction in the signal-to-noise plus interference ratio because of the jammer effect can be computed from the difference between Eqs. (1.83) and (1.87). It is expressed (in dB) by

$$\Upsilon = 10.0 \times \log\left(1 + \frac{T_J}{T_s}\right) \tag{1.88}$$

Consequently, the RRF is

$$RRF = 10^{\frac{-\Upsilon}{40}} \tag{1.89}$$

Figures 1.20 a and b show typical value for the RRF versus the radar wavelength and detection range using the following parameters

Symbol	Value
te	*500 kelvin*
pj	*500 KW*
gj	*3 dB*
g	*45 dB*
freq	*10 GHz*
bj	*10 MHZ*
rangej	*750 Km*
lossj	*1 dB*

This figure can be reproduced using the following MATLAB code

```
clear all;
close all;
te = 730.0; % radar effective temp in Kelvin
pj  = 15; % jammer peak power in W
gj = 3.0; % jammer antenna gain in dB
g = 40.0; % radar antenna gain
freq = 10.0e+9; % radar operating frequency in Hz
bj  = 1.0e+6; % radar operating bandwidth in Hz
rangej = 400.0; % radar to jammer range in Km
lossj = 1.0; % jammer losses in dB
c = 3.0e+8;
k = 1.38e-23;
lambda = c / freq;
gj_10 = 10^( gj/10);
g_10 = 10^( g/10);
lossj_10 = 10^(lossj/10);
index = 0;
for wavelength = .01:.001:1
  index = index +1;
  jamer_temp = (pj * gj_10 * g_10 *wavelength^2) / ...
    (4.0^2 * pi^2 * k * bj * lossj_10 * (rangej * 1000.0)^2);
  delta = 10.0 * log10(1.0 + (jamer_temp / te));
  rrf(index) = 10^(-delta /40.0);
end
w = 0.01:.001:1;
figure (1)
semilogx(w,rrf,'k')
grid
xlabel ('Wavelength in meters')
ylabel ('Range reduction factor')
index = 0;
```

```
for ran =rangej*.3:10:rangej*2
   index = index + 1;
   jamer_temp = (pj * gj_10 * g_10 *lambda^2) / ...
      (4.0^2 * pi^2 * k * bj * lossj_10 * (ran * 1000.0)^2);
   delta = 10.0 * log10(1.0 + (jamer_temp / te));
   rrf1(index) = 10^(-delta /40.0);
end
figure(2)
ranvar = rangej*.3:10:rangej*2 ;
plot(ranvar,rrf1,'k')
grid
xlabel ('Radar to jammer range in Km')
ylabel ('Range reduction factor')
index = 0;
for pjvar = pj*.01:100:pj*2
   index = index + 1;
   jamer_temp = (pjvar * gj_10 * g_10 *lambda^2) / ...
      (4.0^2 * pi^2 * k * bj * lossj_10 * (rangej * 1000.0)^2);
   delta = 10.0 * log10(1.0 + (jamer_temp / te));
   rrf2(index) = 10^(-delta /40.0);
end
```

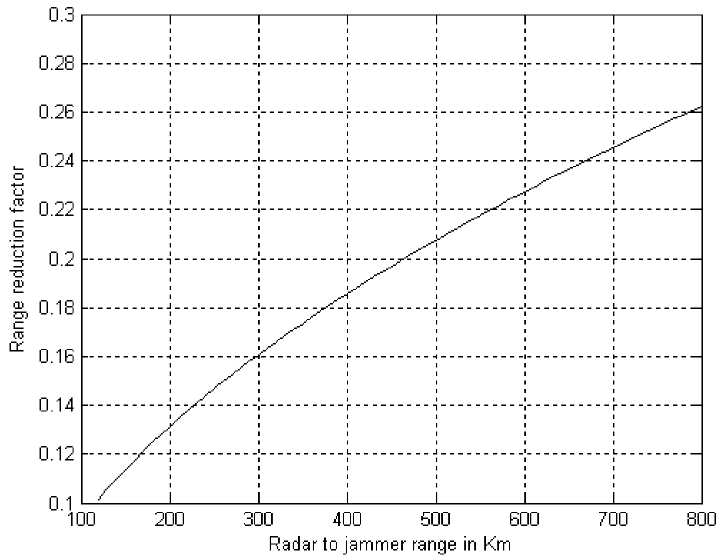


Figure 1.20a. Range reduction factor versus radar to jammer range. This plot was generated using the function *"range_red_factor.m."*

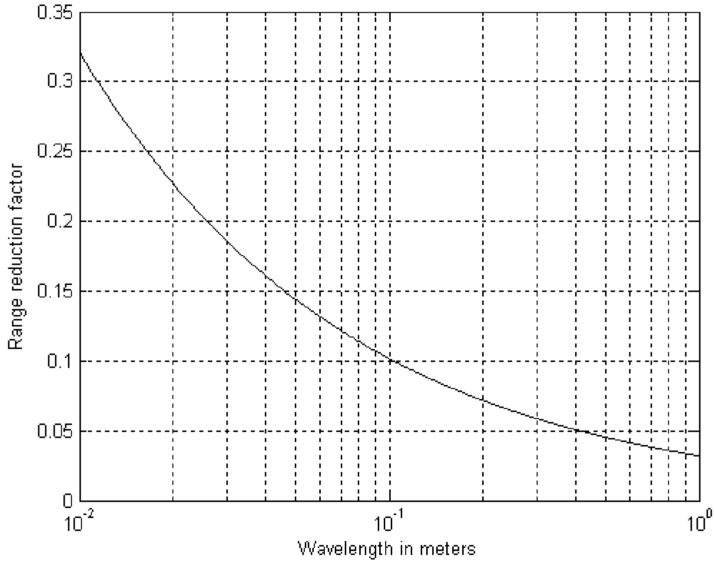


Figure 1.20b. Range reduction factor versus radar operating wavelength. This plot was generated using the function *"range_red_factor.m."*

1.9. Noise Figure

Any signal other than the target returns in the radar receiver is considered to be noise. This includes interfering signals from outside the radar and thermal noise generated within the receiver itself. Thermal noise (thermal agitation of electrons) and shot noise (variation in carrier density of a semiconductor) are the two main internal noise sources within a radar receiver.

The power spectral density of thermal noise is given by

$$S_n(\omega) = \frac{|\omega|h}{\pi\left[\exp\left(\frac{|\omega|h}{2\pi kT}\right) - 1\right]} \tag{1.90}$$

where $|\omega|$ is the absolute value of the frequency in radians per second, T is the temperature of the conducting medium in degrees Kelvin, k is Boltzman's constant, and h is Plank's constant ($h = 6.625 \times 10^{-34}\ Joules$). When the condition $|\omega| \ll 2\pi kT/h$ is true, it can be shown that Eq. (1.90) is approximated by

$$S_n(\omega) \approx 2kT \tag{1.91}$$

This approximation is widely accepted, since, in practice, radar systems operate at frequencies less than $100GHz$; and, for example, if $T = 290K$, then $2\pi kT/h \approx 6000GHz$.

The mean-square noise voltage (noise power) generated across a 1 *ohm* resistance is then

$$\langle n^2 \rangle = \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} 2kT \quad d\omega = 4kTB \tag{1.92}$$

where B is the system bandwidth. Any electrical system containing thermal noise and having input resistance R_{in} can be replaced by an equivalent noiseless system with a series combination of a noise equivalent voltage source and a noiseless input resistor R_{in} added at its input. This is illustrated in Fig. 1.21.

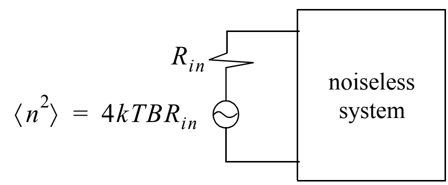


Figure 1.21. Noiseless system with an input noise voltage source.

The amount of noise power that can physically be extracted from $\langle n^2 \rangle$ is one fourth the value computed in Eq. (1.92). Consider a noisy system with power gain A_P, as shown in Fig. 1.22. The noise figure is defined by

$$F_{dB} = 10 \ \log \frac{total \ \ noise \ \ power \ \ out}{noise \ \ power \ \ out \ \ due \ \ to \ \ R_{in} \ \ alone} \tag{1.93}$$

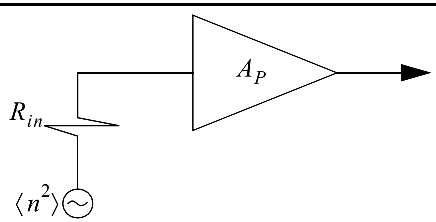


Figure 1.22. Noisy amplifier replaced by its noiseless equivalent and an input voltage source in series with a resistor.

More precisely,

$$F_{dB} = 10 \ \log \frac{N_o}{N_i \ A_p} \tag{1.94}$$

where N_o and N_i are, respectively, the noise power at the output and input of the system.

If we define the input and output signal power by S_i and S_o, respectively, then the power gain is

$$A_P = \frac{S_o}{S_i} \tag{1.95}$$

It follows that

$$F_{dB} = 10\log\left(\frac{S_i/Ni}{S_o/N_o}\right) = \left(\frac{S_i}{N_i}\right)_{dB} - \left(\frac{S_o}{N_o}\right)_{dB} \tag{1.96}$$

where

$$\left(\frac{S_i}{N_i}\right)_{dB} > \left(\frac{S_o}{N_o}\right)_{dB} \tag{1.97}$$

Thus, the noise figure is the loss in the signal-to-noise ratio due to the added thermal noise of the amplifier ($(SNR)_o = (SNR)_i - F \ \ in \ \ dB$).

One can also express the noise figure in terms of the system's effective temperature T_e. Consider the amplifier shown in Fig. 1.22, and let its effective temperature be T_e. Assume the input noise temperature is T_0. Thus, the input noise power is

$$N_i = kT_0B \tag{1.98}$$

and the output noise power is

$$N_o = kT_0B \ \ A_p + kT_eB \ \ A_p \tag{1.99}$$

where the first term on the right-hand side of Eq. (1.99) corresponds to the input noise, and the latter term is due to thermal noise generated inside the system. It follows that the noise figure can be expressed as

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i}{kT_0B} \ kBA_p \ \frac{T_0 + T_e}{S_o} = 1 + \frac{T_e}{T_0} \tag{1.100}$$

Equivalently, we can write

$$T_e = (F-1)T_0 \tag{1.101}$$

Example:

An amplifier has a 4dB noise figure; the bandwidth is $B = 500\ KHz$. *Calculate the input signal power that yields a unity SNR at the output. Assume* $T_0 = 290K$ *and an input resistance of one ohm.*

Solution:

The input noise power is

$$kT_0B = 1.38 \times 10^{-23} \times 290 \times 500 \times 10^3 = 2.0 \times 10^{-15} W$$

Assuming a voltage signal, then the input noise mean squared voltage is

$$\langle n_i^2 \rangle = kT_oB = 2.0 \times 10^{-15}\ v^2$$

$$F = 10^{0.4} = 2.51$$

From the noise figure definition we get

$$\frac{S_i}{N_i} = F\left(\frac{S_o}{N_o}\right) = F$$

and

$$\langle s_i^2 \rangle = F\langle n_i^2 \rangle = 2.51 \times 2.0 \times 10^{-15} = 5.02 \times 10^{-15}\ v^2$$

Finally,

$$\sqrt{\langle s_i^2 \rangle} = 70.852 nv$$

Consider a cascaded system as in Fig. 1.23. Network 1 is defined by noise figure F_1, power gain G_1, bandwidth B, and temperature T_{e1}. Similarly, network 2 is defined by F_2, G_2, B, and T_{e2}. Assume the input noise has temperature T_0.

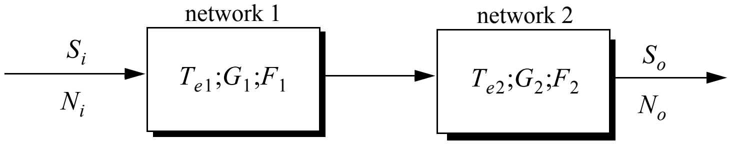


Figure 1.23. Cascaded linear system.

The output signal power is

$$S_o = S_i G_1 G_2 \tag{1.102}$$

The input and output noise powers are, respectively, given by

$$N_i = kT_0 B \tag{1.103}$$

$$N_o = kT_0 B G_1 G_2 + kT_{e1} B G_1 G_2 + kT_{e2} B G_2 \tag{1.104}$$

where the three terms on the right-hand side of Eq. (1.104), respectively, correspond to the input noise power, thermal noise generated inside network 1, and thermal noise generated inside network 2.

Now if we use the relation $T_e = (F - 1)T_0$ along with Eq. (1.02), we can express the overall output noise power as

$$N_o = F_1 N_i G_1 G_2 + (F_2 - 1) N_i G_2 \tag{1.105}$$

It follows that the overall noise figure for the cascaded system is

$$F = \frac{(S_i/N_i)}{(S_o/N_o)} = F_1 + \frac{F_2 - 1}{G_1} \tag{1.106}$$

In general, for an n-stage system we get

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 G_3 \cdot \ \cdot \ \cdot G_{n-1}} \tag{1.107}$$

Also, the n-stage system effective temperatures can be computed as

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \dots + \frac{T_{en}}{G_1 G_2 G_3 \cdot \ \cdot \ \cdot G_{n-1}} \tag{1.108}$$

As suggested by Eq. (1.107) and Eq. (1.108), the overall noise figure is mainly dominated by the first stage. Thus, radar receivers employ low noise power amplifiers in the first stage in order to minimize the overall receiver noise figure. However, for radar systems that are built for low RCS operations every stage should be included in the analysis.

Example:

A radar receiver consists of an antenna with cable loss $L = 1dB = F_1$, an RF amplifier with $F_2 = 6dB$, and gain $G_2 = 20dB$, followed by a mixer whose noise figure is $F_3 = 10dB$ and conversion loss $L = 8dB$, and finally, an integrated circuit IF amplifier with $F_4 = 6dB$ and gain $G_4 = 60dB$. Find the overall noise figure.

Solution:

From Eq. (1.107) we have

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3}$$

G_1	G_2	G_3	G_4	F_1	F_2	F_3	F_4
$-1dB$	$20dB$	$-8dB$	$60dB$	$1dB$	$6dB$	$10dB$	$6dB$
0.7943	100	0.1585	10^6	1.2589	3.9811	10	3.9811

It follows that

$$F = 1.2589 + \frac{3.9811 - 1}{0.7943} + \frac{10 - 1}{100 \times 0.7943} + \frac{3.9811 - 1}{0.158 \times 100 \times 0.7943} = 5.3629$$

$$F = 10\log(5.3628) = 7.294dB$$

1.10. Effects of the Earth's Surface on the Radar Equation

So far, in developing the radar equation it was implicitly assumed that the radar electromagnetic waves travel as if they were in free space. Furthermore, all analysis presented did not account for the effects of the earth's atmosphere nor the effects of the earth's surface. Despite the fact that *"free space analysis"* may be adequate to provide a general understanding of radar systems, it is only an approximation. In order to accurately predict radar performance, we must modify free space analysis to include the effects of the earth and its atmosphere. Radar clutter is not considered to be part of this analysis. This is true because clutter is almost always assumed to be a distributed target that can be dealt with by the radar signal processor separately. Clutter is the subject of discussion in a later chapter of this book.

In this chapter, the effects of the earth's atmosphere are considered first. Then, the effect of the earth' surface on the radar equation is analyzed. The earth's surface impact on the radar equation manifests itself by introducing an additional power term in the radar equation. This term is called the *pattern propagation factor* and is denoted by symbol F. The propagation factor, can actually introduce constructive as well as distructive interference in the SNR depending on the radar frequency and the geometry under consideration. In general, the pattern propagation factor is defined by

$$F = |E/E_0| \tag{1.109a}$$

where E is the electric field in the medium and E_0 is the free space electric field. In this case the radar equation is now given by

$$(SNR)_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_0 B F L R^4} F^4 \qquad \textbf{(1.109b)}$$

1.10.1. Earth's Atmosphere

The earth's atmosphere is composed of several layers, as illustrated in Fig. 1.24. The first layer, which extends in altitude to about 20 Km, is known as the troposphere. Electromagnetic waves refract (bend downward) as they travel in the troposphere. The troposphere refractive effect is related to its dielectric constant, which is a function of pressure, temperature, water vapor, and gaseous content. Additionally, due to gases and water vapor in the atmosphere, radar energy suffers a loss. This loss is known as the atmospheric attenuation. Atmospheric attenuation increases significantly in the presence of rain, fog, dust, and clouds.

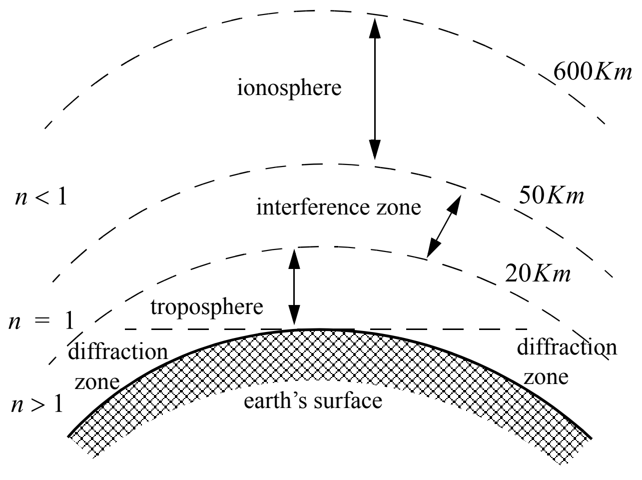


Figure 1.24. Earth's atmosphere geometry.

The region above the troposphere (altitude from 20 to 50 Km) behaves like free space, and thus little refraction occurs in this region. This region is known as the interference zone. The ionosphere extends from about 50 Km to about 600 Km. It has very low gas density compared to the troposphere. It contains a significant amount of ionized free electrons. The ionization is primarily caused by the sun's ultraviolet and X-rays. This presence of free electrons in the ionosphere affects electromagnetic wave propagation in different ways. These effects include refraction, absorption, noise emission, and polarization rotation. The degree of degradation depends heavily on the frequency of the incident waves. For example, frequencies lower than about 4 to 6 MHz are completely reflected from the lower region of the ionosphere. Frequencies higher than 30 MHz may penetrate the ionosphere with some level of attenuation. In general, as the frequency is increased the ionosphere's effects become less prominent. The region below the horizon, close to the earth's surface, is called the diffraction region. *Diffraction* is a term used to describe the bending of radar waves around physical objects. Two types of diffraction are common. They are knife edge and cylinder edge diffraction.

In order to effectively study the effects of the atmosphere on the propagation of radar waves, it is necessary to have accurate knowledge of the height-variation of the index of refracting in the troposphere and the ionosphere. The index of refraction is a function of the geographic location on the earth, weather, time of day or night, and the season of the year. Therefore, analyzing the atmospheric propagation effects under all parametric conditions is an overwhelming task. Typically, this problem is simplified by analyzing atmospheric models that are representative of an average of atmospheric conditions.

1.10.2. Refraction

In free space, electromagnetic waves travel in straight lines. However, in the presence of the earth's atmosphere, they bend (refract), as illustrated in Fig. 1.25. *Refraction* is a term used to describe the deviation of radar wave propagation from a straight line. The deviation from straight line propagation is caused by the variation of the index of refraction. The index of refraction is defined as

$$n = c/v \tag{1.110}$$

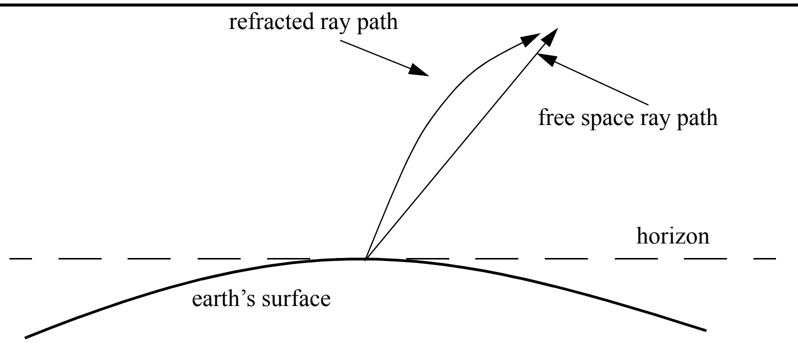


Figure 1.25. Bending of radio waves due to the variation in the atmosphere index of refraction.

where c is the velocity of electromagnetic waves in free space and v is the wave group velocity in the medium. Close to the earth's surface the index of refraction is almost unity; however, with increasing altitude the index of refraction decreases gradually.

The discussion presented in this chapter assumes a well-mixed atmosphere, where the index of refraction decreases in a smooth monotonic fashion with height. The rate of change of the earth's index of refraction n with altitude h is normally referred to as the refractivity gradient, dn/dh. As a result of the negative rate of change in dn/dh, electromagnetic waves travel at slightly higher velocities in the upper troposphere than in the lower part. As a result of this, waves traveling horizontally in the troposphere gradually bend downward. In general, since the rate of change in the refractivity index is very slight, waves do not curve downward appreciably unless they travel very long distances through the troposphere.

Refraction affects radar waves in two different ways depending on height. For targets that have altitudes typically above 100 meters, the effect of refraction is illustrated in Fig. 1.26. In this case, refraction imposes limitations on the radar's capability to measure target position. Refraction introduces an error in measuring the elevation angle. In a well mixed atmosphere, the refractivity gradient close to the earth's surface is almost constant. However, temperature changes and humidity lapses close to the earth's surface may cause serious changes in the refractivity profile. When the refractivity index becomes large enough, electromagnetic waves bend around the curve of the earth beyond the expected curvature due to earth surface. This phenomenon is called ducting and is illustrated in Fig. 1.27. Ducting can be extensive over the sea surface during a hot summer.

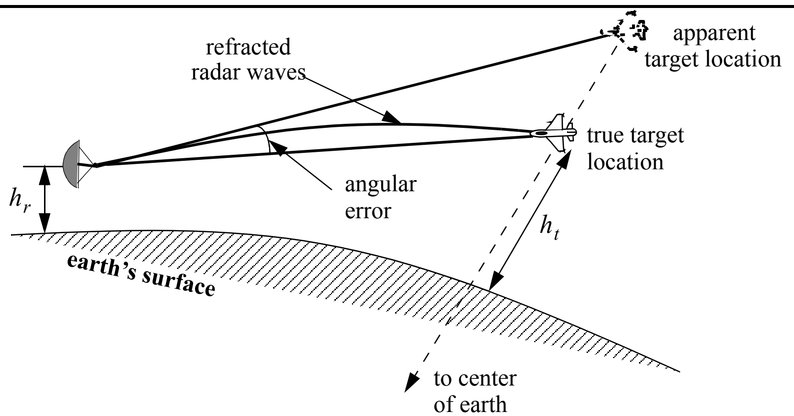


Figure 1.26. Refraction high altitude effect on electromagnetic waves.

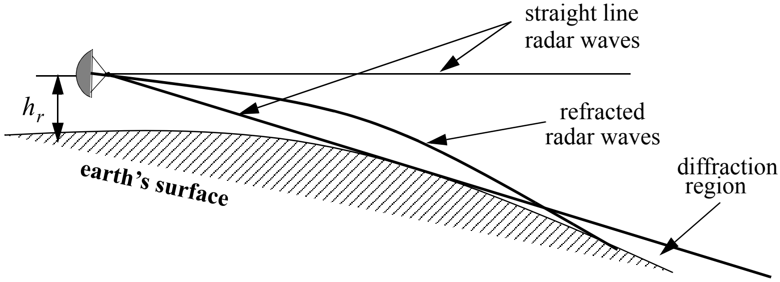


Figure 1.27. Refraction low altitude effect on electromagnetic waves.

Stratified Atmospheric Refraction Model

An approximation method for calculating the range measurement errors and the time-delay errors experienced by radar waves due to refraction is presented. This method is referred to as the "stratified atmospheric model" and is capable of producing very accurate theoretical estimates of the propagation errors. The basic assumption for this approach is that the atmosphere is stratified into M spherical layers, each of thickness $\{h_m;\ m = 0, 1, \ldots, M\}$, and a constant refractive index $\{n_m;\ m = 0, 1, \ldots, M\}$, as illustrated in Fig. 1.28. In this figure, β_0 is the apparent elevation angle and β_{0M} is the true elevation angle. The free space path is denoted by R_{0M}, while the refracted path is composed of $\{R_0, R_1, R_2, \ldots, R_M\}$. From the figure,

$$r_{(m+1)} = r_0 + \sum_{i=0}^{m} h_i \qquad ;\ m = 0, 1, \ldots, M \tag{1.111}$$

where r_0 is the actual radius of the earth and is equal to 6375 Km. Using the law of sines, the angle of incidence α_0 is given by

$$\frac{\sin\alpha_0}{r_0} = \frac{\sin(180 + \beta_0)}{r_1} \tag{1.112}$$

Using Snell's law for spherically symmetrical surfaces, the m^{th} angle, β_m, that the ray makes with the horizon in layer m is given by

$$n_{(m-1)} r_{(m-1)} \cos\beta_{(m-1)} = n_m r_m \cos\beta_m \tag{1.113}$$

Consequently,

$$\beta_m = \mathrm{acos}\left[\frac{n_{(m-1)} r_{(m-1)}}{n_m r_m} \cos\beta_{(m-1)}\right] \tag{1.114}$$

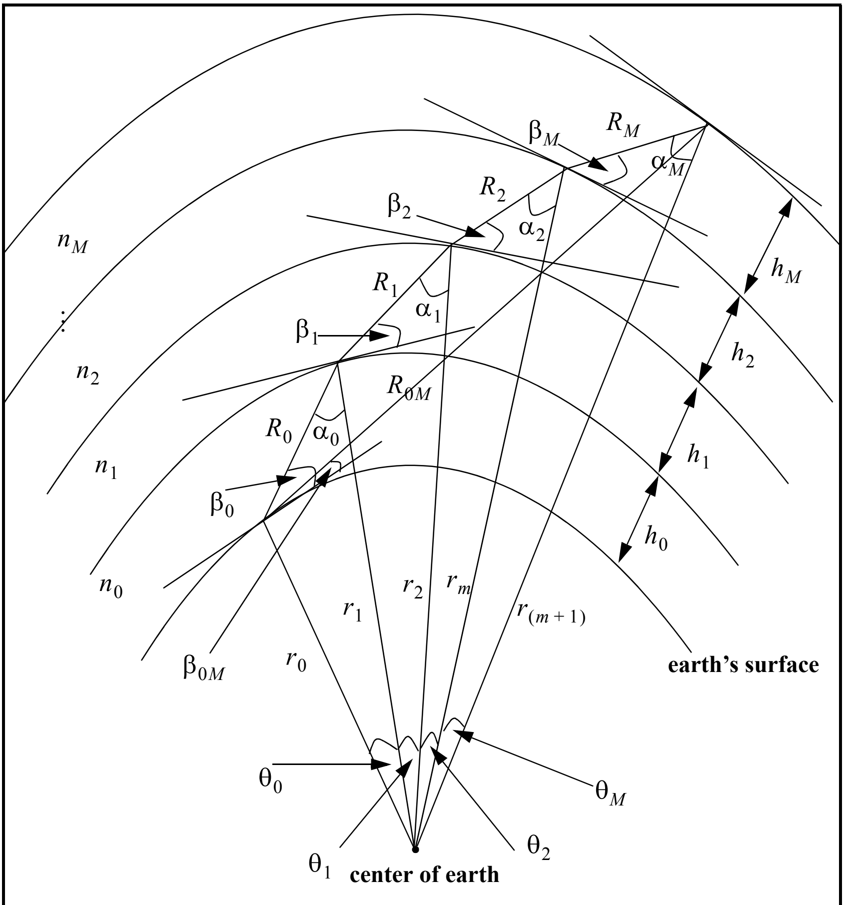


Figure 1.28. Atmosphere stratification.

From Eq. (1.112) one can write the general expression for the angle of incidence. More precisely,

$$\alpha_m = \operatorname{asin}\left[\frac{r_m}{r_{(m+1)}} \cos\beta_m\right] \tag{1.115}$$

Applying the law of sines of the direct path R_{0M} yields

$$\beta_{0M} = \operatorname{acos}\left\{\frac{r_{(M+1)}}{R_{0M}} \sin\left[\sum_{i=0}^{M}\theta_i\right]\right\} \tag{1.116}$$

where

$$R_{0M}^2 = r_0^2 + r_{(m+1)}^2 - 2r_0 r_{(m+1)} \cos\left[\sum_{i=0}^{M} \theta_i\right] \quad (1.117)$$

$$\theta_i = \frac{\pi}{2} - \beta_i - \alpha_i \quad (1.118)$$

The refraction angle error is measured as the difference between the apparent and true elevation angles. Thus, it is given by

$$\Delta\beta_M = \beta_0 - \beta_{0M} \quad (1.119)$$

Note that for $M = 0$,

$$R_{00} = R_0 \; and \; \Delta\beta_M = 0 \quad (1.120)$$

Furthermore, when $\beta_0 = 90°$,

$$R_{0M} = \sum_{i=0}^{M} h_i \quad (1.121)$$

Now, in order to determine the time-delay error due to refraction, refer again to Fig. 1.28. The time it takes an electromagnetic wave to travel through a given layer, $\{R_i; \; i = 0, 1, \ldots, M\}$, is defined as $\{t_i; \; i = 0, 1, \ldots, M\}$ where

$$t_i = R_i / \varphi_i \quad (1.122)$$

and where φ_i is the phase velocity in the ith layer and is defined by

$$\varphi_i = c / n_i \quad (1.123)$$

It follows that the total time of travel of the refracted wave in a stratified atmosphere is given by

$$t_T = \frac{1}{c} \sum_{i=0}^{M} n_i R_i \quad (1.124)$$

The free space travel time of an unrefracted wave is denoted by t_{0M},

$$t_{0M} = R_{0M} / c \quad (1.125)$$

Therefore, the range error ΔR that results from refraction is

$$\Delta R = \sum_{i=0}^{M} n_i R_i - R_{0M} \quad (1.126)$$

By using the law of cosines one computes R_i as

$$R_i^2 = r_i^2 + r_{(i+1)}^2 - 2r_i r_{(i+1)} \cos\theta_i \tag{1.127}$$

The results stated in Eqs. (1.125) and (1.26) are valid only in the troposphere. In the ionosphere, which is a dispersive medium, the index of refraction is also a function of frequency. In this case, the group velocity must be used when estimating the range errors of radar measurements. Thus, the total time of travel in the medium is now given by

$$t_T = \frac{1}{c}\sum_{i=0}^{M} \frac{R_i}{n_i} \tag{1.128}$$

Finally, the range error in the ionosphere is

$$\Delta R = \sum_{i=0}^{M} \frac{R_i}{n_i} - R_{0M} \tag{1.129}$$

1.10.3. Four-Third Earth Model

An effective and fairly accurate technique for dealing with refraction is to replace the actual earth with an imaginary earth whose radius is $r_e = kr_0$, where $r_0 = 6375Km$ is the actual earth radius, and k is

$$k = \frac{1}{1 + r_0(dn/dh)} \tag{1.130}$$

When the refractivity gradient is assumed to be constant with altitude and is equal to 39×10^{-9} per meter, then $k = 4/3$. Using an effective earth radius $r_e = (4/3)r_0$ produces what is known as the *"four-third earth model."* In general, choosing

$$r_e = r_0(1 + 6.37 \times 10^{-3}(dn/dh)) \tag{1.131}$$

produces a propagation model where waves travel in straight lines. Selecting the correct value for k depends heavily on the region's meteorological conditions. At low altitudes (typically less than 10 Km) when using the 4/3 earth model, one can assume that radar waves (beams) travel in straight lines and do not refract. This is illustrated in Fig. 1.29.

1.10.4. Ground Reflection

When radar waves are reflected from the earth's surface, they suffer a loss in amplitude and a change in phase. Three factors that contribute to these changes they are the smooth surface reflection coefficient, the divergence factor due to earth's curvature, and the surface roughness.

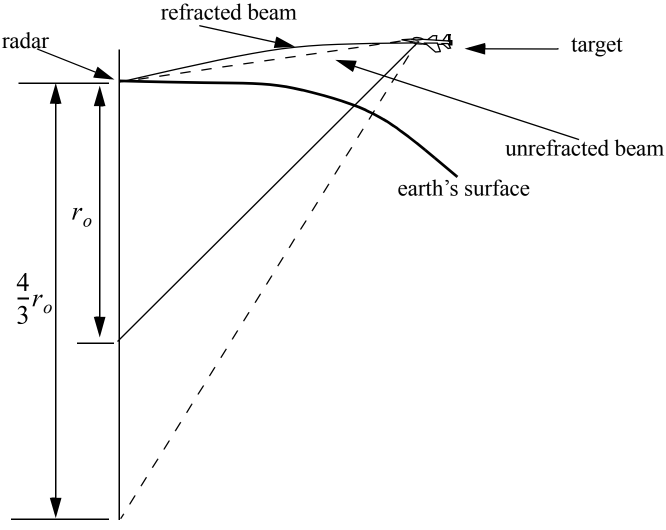


Figure 1.29. Geometry for 4/3 earth.

Smooth Surface Reflection Coefficient

The smooth surface reflection coefficient depends on the frequency, the surface dielectric coefficient, and the radar grazing angle. The vertical polarization and the horizontal polarization reflection coefficients are

$$\Gamma_v = \frac{\varepsilon \sin\psi_g - \sqrt{\varepsilon - (\cos\psi_g)^2}}{\varepsilon \sin\psi_g + \sqrt{\varepsilon - (\cos\psi_g)^2}} \tag{1.132}$$

$$\Gamma_h = \frac{\sin\psi_g - \sqrt{\varepsilon - (\cos\psi_g)^2}}{\sin\psi_g + \sqrt{\varepsilon - (\cos\psi_g)^2}} \tag{1.133}$$

where ψ_g is the grazing angle (incident angle) and ε is the complex dielectric constant of the surface, and are given by

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon' - j60\lambda\sigma \tag{1.134}$$

where λ is the wavelength and σ the medium conductivity in mhos/meter. Typical values of ε' and ε'' can be found tabulated in the literature.

Note that when $\psi_g = 90°$, we get

$$\Gamma_h = \frac{1 - \sqrt{\varepsilon}}{1 + \sqrt{\varepsilon}} = -\frac{\varepsilon - \sqrt{\varepsilon}}{\varepsilon + \sqrt{\varepsilon}} = -\Gamma_v \tag{1.135}$$

while when the grazing angle is very small ($\psi_g \approx 0$), we have

$$\Gamma_h = -1 = \Gamma_v \tag{1.136}$$

Tables 1.1 and 1.2 show some typical values for the electromagnetic properties of soil and sea water. Figure 1.30 shows the corresponding magnitude plots for Γ_h and Γ_v, while Fig. 1.31 shows the phase plots for seawater at $28°C$ where $\varepsilon' = 65$ and $\varepsilon'' = 30.7$ at X-band. The plots shown in these figures show the general typical behavior of the reflection coefficient.

Table 1.1. Electromagnetic properties of soil.

Frequency GHz	Moisture content by volume							
	0.3%		10%		20%		30%	
	ε'	ε''	ε'	ε''	ε'	ε''	ε'	ε''
0.3	2.9	0.071	6.0	0.45	10.5	0.75	16.7	1.2
3.0	2.9	0.027	6.0	0.40	10.5	1.1	16.7	2.0
8.0	2.8	0.032	5.8	0.87	10.3	2.5	15.3	4.1
14.0	2.8	0.350	5.6	1.14	9.4	3.7	12.6	6.3
24	2.6	0.030	4.9	1.15	7.7	4.8	9.6	8.5

Table 1.2. Electromagnetic properties of sea water.

Frequency GHz	Temperature					
	$T = 0^\circ C$		$T = 10^\circ C$		$T = 20^\circ C$	
	ε'	ε''	ε'	ε''	ε'	ε''
0.1	77.8	522	75.6	684	72.5	864
1.0	77.0	59.4	75.2	73.8	72.3	90.0
2.0	74.0	41.4	74.0	45.0	71.6	50.4
3.0	71.0	38.4	72.1	38.4	70.5	40.2
4.0	66.5	39.6	69.5	36.9	69.1	36.0
6.0	56.5	42.0	63.2	39.0	65.4	36.0
8.0	47.0	42.8	56.2	40.5	60.8	36.0

Observation of Fig. 1.30 indicates the following conclusions: (1) The magnitude of the reflection coefficient with horizontal polarization is equal to unity at very small grazing angles and it decreases monotonically as the angle is increased. (2) The magnitude of the vertical polarization has a well-defined minimum. The angle that corresponds to this condition is called Brewster's polarization angle. For this reason, airborne radars in the look-down mode utilize mainly vertical polarization to significantly reduce the terrain bounce reflections. (3) For horizontal polarization the phase is almost π; however, for vertical polarization the phase changes to zero around the Brewster's angle. (4) For very small angles (less than $2°$) both $|\Gamma_h|$ and $|\Gamma_v|$ are nearly one; $\angle\Gamma_h$ and $\angle\Gamma_v$ are nearly π. Thus, little difference in the propagation of horizontally or vertically polarized waves exists at low grazing angles. Figure 1.30 can be reproduced using the following MATLAB code.

```
close all; clear all
psi = 0.01:0.05:90;
[rh,rv] = ref_coef (psi, 65,30.7);
gamamodv = abs(rv); gamamodh = abs(rh); subplot(2,1,1)
plot(psi,gamamodv,'k',psi,gamamodh,'k -.','linewidth',1.5); grid
legend ('Vertical Polarization', 'Horizontal Polarization')
title('Reflection coefficient - magnitude')
pv = -angle(rv); ph = angle(rh); subplot(2,1,2)
plot(psi,pv,'k',psi,ph,'k -.','linewidth',1.5); grid
legend ('Vertical Polarizatio', 'Horizontal Polarization')
title('Reflection coefficient - phase'); xlabel('Grazing angle in degrees');
```

Figures 1.31 and 1.32 show the magnitudes of the horizontal and vertical reflection coefficients as a function of grazing angle for four soils at 8 GHz.

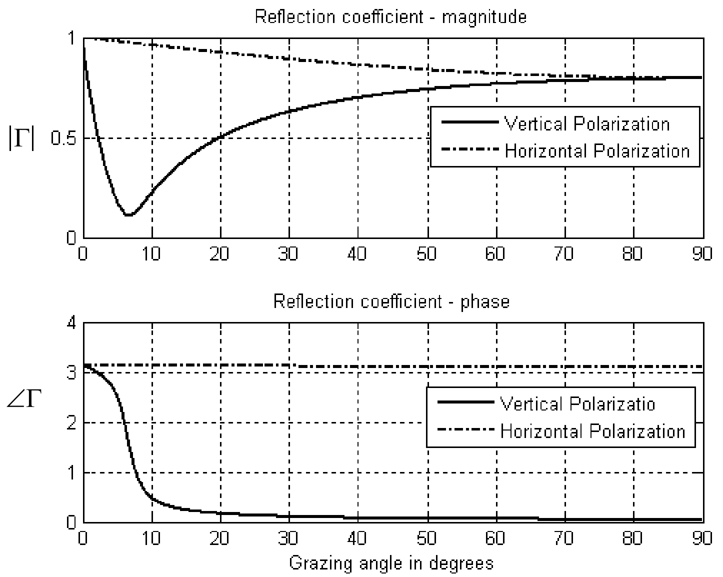


Figure 1.30. Reflection coefficient magnitude.

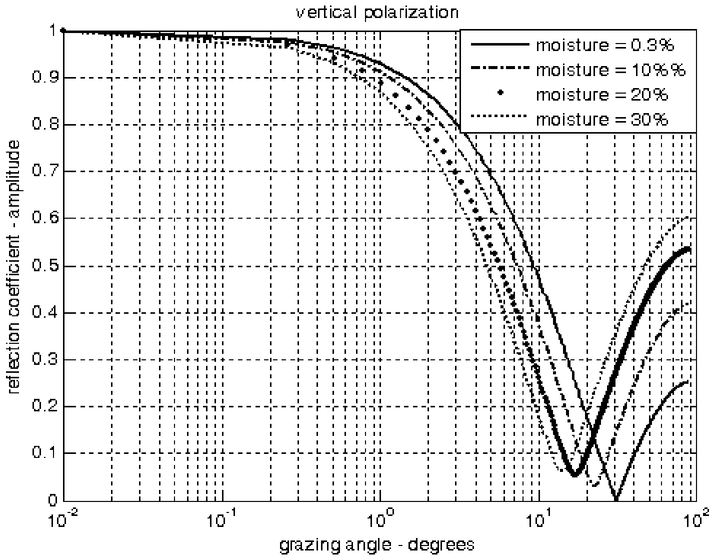


Figure 1.31. Vertical reflection coefficient for soil at 8 GHz.

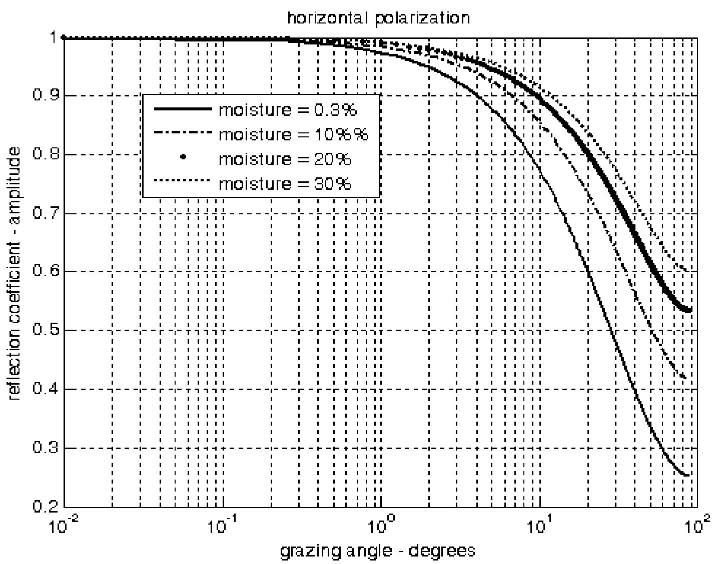


Figure 1.32. Horizontal reflection coefficient for soil at 8 GHz.

Divergence

The overall reflection coefficient is also affected by the round earth divergence factor, D. When an electromagnetic wave is incident on a round earth surface, the reflected wave diverges because of the earth's curvature. This is illustrated in Fig. 1.33. Due to divergence the reflected energy is defocused, and the radar power density is reduced. The divergence factor can be derived using geometrical considerations.

The divergence factor can be expressed as

$$D = \sqrt{\frac{r_e \; r \; \sin\psi_g}{[(2r_1r_2/\cos\psi_g) + r_e r\sin\psi_g](1 + h_r/r_e)(1 + h_t/r_e)}} \tag{1.137}$$

where all the parameters in Eq. (1.137) are defined in Fig. 1.34. Since the grazing ψ_g is always small when the divergence D is very large, the following approximation is adequate in most radar cases of interest:

$$D \approx \frac{1}{\sqrt{1 + \dfrac{4r_1r_2}{r_e r\sin 2\psi_g}}} \tag{1.138}$$

Rough Surface Reflection

In addition to divergence, surface roughness also affects the reflection coefficient. Surface roughness is given by

$$S_r = e^{-2\left(\frac{2\pi h_{rms}\sin\psi_g}{\lambda}\right)^2} \tag{1.139}$$

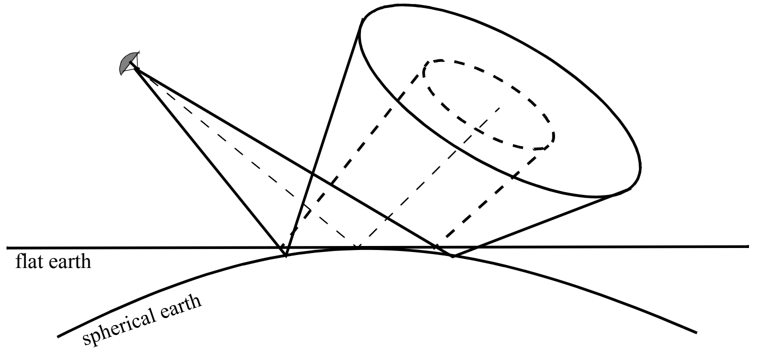


Figure 1.33. Illustration of divergence. Solid line: Ray perimeter for spherical earth. Dashed line: Ray perimeter for flat earth.

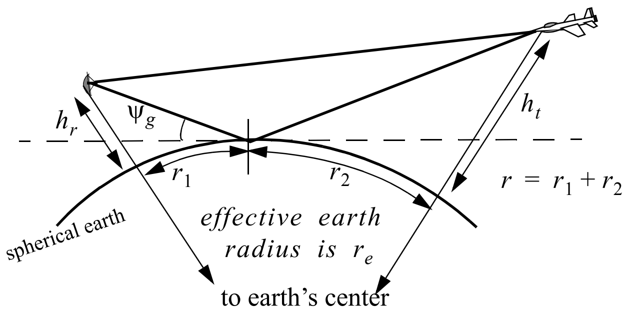


Figure 1.34. Definition of variables in Eq. (1.137).

where h_{rms} is the root mean square (rms) surface height irregularity. Another form for the rough surface reflection coefficient that is more consistent with experimental results is given by

$$S_r = e^{-z} I_0(z) \tag{1.140}$$

$$z = 2\left(\frac{2\pi h_{rms} \sin\psi_g}{\lambda}\right)^2 \tag{1.141}$$

where I_0 is the modified Bessel function of order zero.

Total Reflection Coefficient

In general, rays reflected from rough surfaces undergo changes in phase and amplitude, which results in the diffused (noncoherent) portion of the reflected signal. Combining the effects of smooth surface reflection coefficient, divergence, and the rough surface reflection coefficient, one express the total reflection coefficient Γ_t as

$$\Gamma_t = \Gamma_{(h,\, v)} D S_r \tag{1.142}$$

$\Gamma_{(h,\, v)}$ is the horizontal or vertical smooth surface reflection coefficient, D is divergence, and S_r is the rough surface reflection coefficient.

1.10.5. The Pattern Propagation Factor - Flat Earth

Consider the geometry shown in Fig. 1.35. The radar is located at height h_r. The target is at range R, and is located at a height h_t. The grazing angle is ψ_g. The radar energy emanating from its antenna will reach the target via two paths: the "direct path" AB and the "indirect path" ACB.

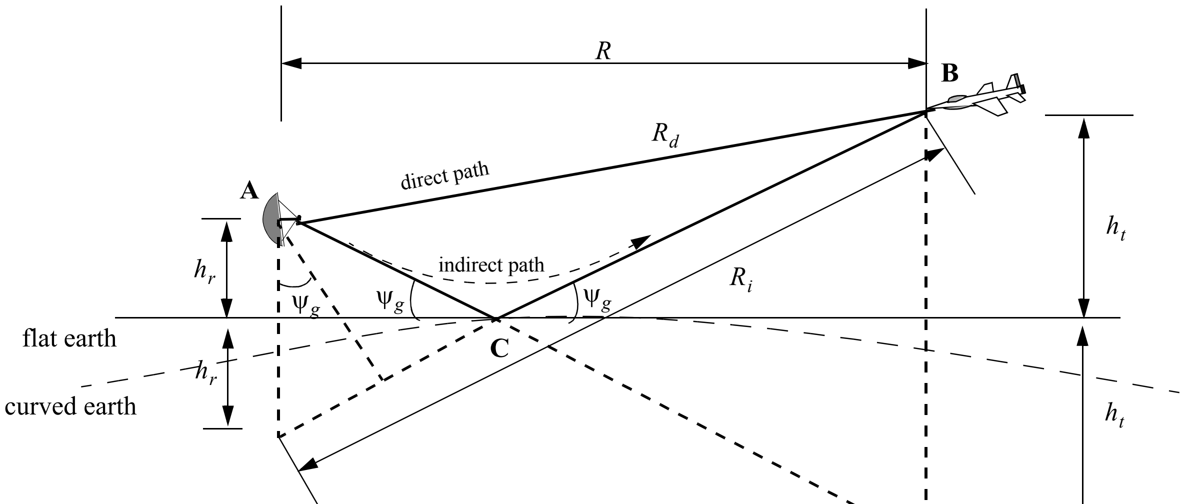


Figure 1.35. Geometry for multipath propagation.

The lengths of the paths AB and ACB are normally very close to one another and thus, the difference between the two paths is very small. Denote the direct path as R_d, the indirect path as R_i, and the difference as $\Delta R = R_i - R_d$. It follows that the phase difference between the two paths is given by

$$\Delta\Phi = \frac{2\pi\Delta R}{\lambda} \tag{1.143}$$

where λ is the radar wavelength.

The indirect signal amplitude arriving at the target is less than the signal amplitude arriving via the direct path. This is because the antenna gain in the direction of the indirect path is less than that along the direct path, and because the signal reflected from the earth's surface at point C is modified in amplitude and phase in accordance to the earth's reflection coefficient, Γ. The earth reflection coefficient is given by

$$\Gamma = \rho e^{j\varphi} \tag{1.144}$$

where ρ is less than unity and φ describes the phase shift induced on the indirect path signal due to surface roughness.

The direct signal (in volts) arriving at the target via the direct path can be written as

$$E_d = e^{j\omega_0 t} e^{j\frac{2\pi}{\lambda}R_d} \tag{1.145}$$

where the time harmonic term $\exp(j\omega_0 t)$ represents the signal's time dependency, and the exponential term $\exp(j(2\pi/\lambda)R_d)$ represents the signal spatial phase. The indirect signal at the target is

$$E_i = \rho e^{j\varphi} e^{j\omega_0 t} e^{j\frac{2\pi}{\lambda}R_i} \tag{1.146}$$

where $\rho\exp(j\varphi)$ is the surface reflection coefficient. Therefore, the overall signal arriving at the target is

$$E = E_d + E_i = e^{j\omega_0 t} e^{j\frac{2\pi}{\lambda}R_d}\left(1 + \rho e^{j\left(\varphi + \frac{2\pi}{\lambda}(R_i - R_d)\right)}\right) \tag{1.147}$$

Due to reflections from the earth's surface, the overall signal strength is then modified at the target by the ratio of the signal strength in the presence of earth to the signal strength at the target in free space. From Eq. (1.147) the modulus of this ratio is the propagation factor is

$$F = \left|\frac{E_d}{E_d + E_i}\right| = \left|1 + \rho e^{j\varphi} e^{j\Delta\Phi}\right| \quad (1.148)$$

which can be rewritten as

$$F = \left|1 + \rho e^{j\alpha}\right| \quad (1.149)$$

where $\alpha = \Delta\Phi + \varphi$. Using Euler's identity ($e^{j\alpha} = \cos\alpha + j\sin\alpha$), Eq. (1.149) can be written as

$$F = \sqrt{1 + \rho^2 + 2\rho\cos\alpha} \quad (1.150)$$

It follows that the signal power at the target is modified by the factor F^2. By using reciprocity, the signal power at the radar is computed by multiplying the radar equation by the factor F^4. In the following two sections we will develop exact expressions for the propagation factor for flat and curved earth.

In order to calculate the propagation factor defined in Eq. (1.150), consider the geometry of Fig. 1.35; the direct and indirect paths are computed as

$$R_d = \sqrt{R^2 + (h_t - h_r)^2} \quad (1.151)$$

$$R_i = \sqrt{R^2 + (h_t + h_r)^2} \quad (1.152)$$

which can be approximated using the truncated binomial series expansion as

$$R_d \approx R + \frac{(h_t - h_r)^2}{2R} \quad (1.153)$$

$$R_i \approx R + \frac{(h_t + h_r)^2}{2R} \quad (1.154)$$

This approximation is valid for low grazing angles, where $R \gg h_t, h_r$. It follows that

$$\Delta R = R_i - R_d \approx \frac{2h_t h_r}{R} \quad (1.155)$$

Substituting Eq. (1.155) into Eq. (1.143) yields the phase difference due to multipath propagation between the two signals (direct and indirect) arriving at the target. More precisely,

$$\Delta\Phi = \frac{2\pi}{\lambda}\Delta R \approx \frac{4\pi h_t h_r}{\lambda R} \quad (1.156)$$

As a special case, assume smooth surface with reflection coefficient $\Gamma = -1$. This assumption means that waves reflected from the surface suffer no amplitude loss, and that the induced surface phase shift is equal to $180°$. It follows that

$$F^2 = 2 - 2\cos\Delta\Phi = 4(\sin(\Delta\Phi/2))^2 \tag{1.157}$$

Substituting Eq. (1.156) into Eq. (1.157) yields

$$F^2 = 4\left(\sin\frac{2\pi h_t h_r}{\lambda R}\right)^2 \tag{1.158}$$

By using reciprocity, the expression for the propagation factor at the radar is then given by

$$F^4 = 16\left(\sin\frac{2\pi h_t h_r}{\lambda R}\right)^4 \tag{1.159}$$

Finally, the signal power at the radar is computed by multiplying the radar equation by the factor F^4:

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \, 16\left(\sin\frac{2\pi h_t h_r}{\lambda R}\right)^4 \tag{1.160}$$

Since the sine function varies between 0 and 1, the signal power will then vary between 0 and 16. Therefore, the fourth power relation between signal power and the target range results in varying the target range from 0 to twice the actual range in free space. In addition to that, the field strength at the radar will now have holes that correspond to the nulls of the propagation factor.

The nulls of the propagation factor occur when the sine is equal to zero. More precisely,

$$\frac{2h_r h_t}{\lambda R} = n \tag{1.161}$$

where $n = \{0, 1, 2, \ldots\}$. The maxima occur at

$$\frac{4h_r h_t}{\lambda R} = n + 1 \tag{1.162}$$

The target heights that produce nulls in the propagation factor are $\{h_t = n(\lambda R/2h_r); n = 0, 1, 2, \ldots\}$, and the peaks are produced from target heights $\{h_t = n(\lambda R/4h_r); n = 1, 2, \ldots\}$. Therefore, due to the presence of surface reflections, the antenna elevation coverage is transformed into a lobed pattern structure.

For small angles, Eq. (1.160) can be approximated by

$$P_r \approx \frac{4\pi P_t G^2 \sigma}{\lambda^2 R^8}\ (h_t h_r)^4 \tag{1.163}$$

Thus, the received signal power varies as the eighth power of the range instead of the fourth power. Also, the factor $G\lambda$ is now replaced by G/λ.

1.10.6. The Pattern Propagation Factor - Spherical Earth

In order to model the effects of multipath propagation on radar performance more accurately, we need to remove the flat earth condition and account for the earth's curvature. When considering round earth, electromagnetic waves travel in curved paths because of the atmospheric refraction. And as mentioned earlier, the most commonly used approach to mitigating the effects of atmospheric refraction is to replace the actual earth by an imaginary earth such that electromagnetic waves travel in straight lines. The fictitious effective earth radius is

$$r_e = kr_0 \tag{1.164}$$

where k is a constant and r_0 is the actual earth radius. Using the geometry in Fig. 1.36, the direct and indirect path difference is

$$\Delta R = R_1 + R_2 - R_d \tag{1.165}$$

The propagation factor is computed by using ΔR from Eq. (1.150). To compute (R_1, R_2, and R_d), the following cubic equation must first be solved for r_1:

$$2r_1^3 - 3rr_1^2 + (r^2 - 2r_e(h_r + h_t))r_1 + 2r_e h_r r = 0 \tag{1.166}$$

The solution is

$$r_1 = \frac{r}{2} - p\sin\frac{\xi}{3} \tag{1.167}$$

where

$$p = \frac{2}{\sqrt{3}}\sqrt{r_e(h_t + h_r) + \frac{r^2}{4}} \tag{1.168}$$

$$\xi = \operatorname{asin}\left(\frac{2r_e r(h_t - h_r)}{p^3}\right) \tag{1.169}$$

Next, we solve for R_1, R_2, and R_d. From Fig. 1.36 (assume flat 4/3 earth and use small angle approximation),

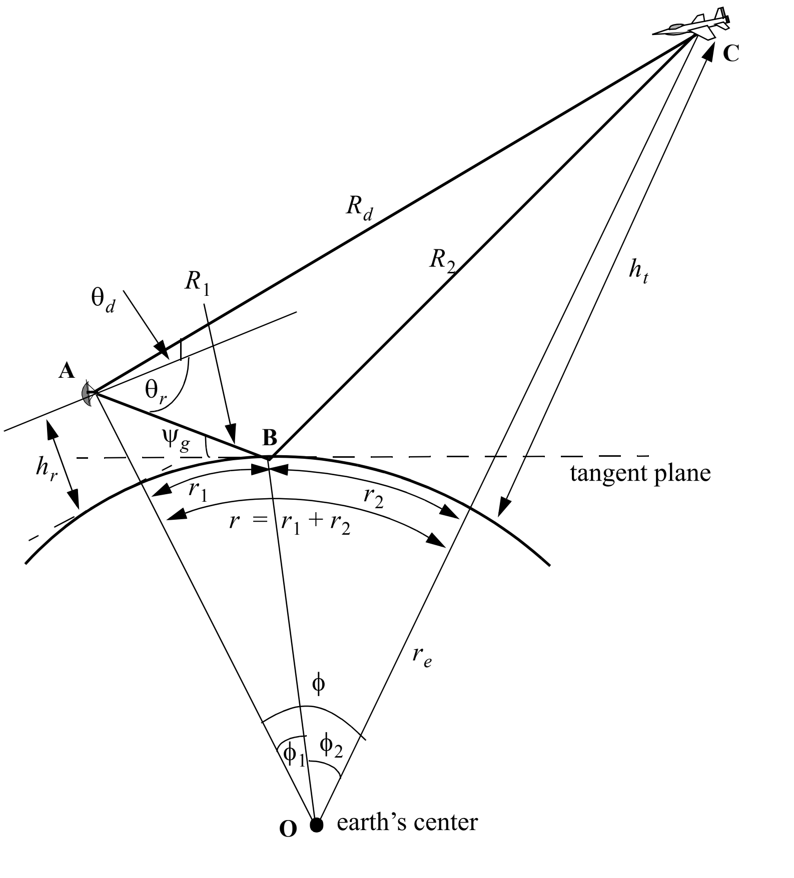


Figure 1.36. Geometry associated with multipath propagation over round earth.

$$\phi_1 = r_1 / r_e ; \ \ \phi_2 = r_2 / r_e \tag{1.170}$$

$$\phi = r / r_e \tag{1.171}$$

Using the law of cosines to the triangles ABO and BOC yields

$$R_1 = \sqrt{r_e^2 + (r_e + h_r)^2 - 2 r_e (r_e + h_r) \cos\phi_1} \tag{1.172}$$

$$R_2 = \sqrt{r_e^2 + (r_e + h_t)^2 - 2 r_e (r_e + h_t) \cos\phi_2} \tag{1.173}$$

Eqs. (1.172) and (1.173) can be written in the following simpler forms:

$$R_1 = \sqrt{h_r^2 + 4r_e(r_e + h_r)(\sin(\phi_1/2))^2} \tag{1.174}$$

$$R_2 = \sqrt{h_t^2 + 4r_e(r_e + h_t)(\sin(\phi_2/2))^2} \tag{1.175}$$

Using the law of cosines on the triangle AOC yields

$$R_d = \sqrt{(h_r - h_t)^2 + 4(r_e + h_t)(r_e + h_r)\left(\sin\left(\frac{\phi_1 + \phi_2}{2}\right)\right)^2} \tag{1.176}$$

Additionally

$$r = r_e \mathrm{acos}\left(\sqrt{\frac{(r_e + h_r)^2 + (r_e + h_t)^2 - R_d^2}{2(r_e + h_r)(r_e + h_t)}}\right) \tag{1.177}$$

Substituting Eqs. (1.174) through (1.176) directly into Eq. (1.165) may not be conducive to numerical accuracy. A more suitable form for the computation of ΔR is then derived. The detailed derivation is in Blake (1980). The results are listed below. For better numerical accuracy use the following expression to compute ΔR :

$$\Delta R = \frac{4R_1R_2(\sin\psi_g)^2}{R_1 + R_2 + R_d} \tag{1.178}$$

where

$$\psi_g = \mathrm{asin}\left(\frac{2r_eh_r + h_r^2 - R_1^2}{2r_eR_1}\right) \approx \mathrm{asin}\left(\frac{h_r}{R_1} - \frac{R_1}{2r_e}\right) \tag{1.179}$$

MATLAB Program *"multipath.m"*

The MATLAB program *"multipath.m"* calculates the two-way propagation factor using the 4/3 earth model for spherical earth. It assumes a known free space radar-to-target range. It can be easily modified to assume a known true spherical earth ground range between the radar and the target. Additionally, this program generates three types of plots. They are (1) the propagation factor as a function of range, (2) the free space relative signal level versus range, and (3) the relative signal level with multipath effects included. This program uses the equations presented in the previous few sections and includes the effects of the total surface reflection coefficient Γ_t. Finally, it can also be easily modified to plot the propagation factor versus target height at a fixed target range.

Using this program, Fig. 1.37 presents a plot for the propagation factor loss versus range using $f = 3GHz$, $h_r = 30.48m$, and $h_t = 60.96m$. In this example, vertical polarization is assumed. Divergence effects are not included; neither is the reflection coefficient. More precisely in this example $D = \Gamma_t = 1$ is assumed.

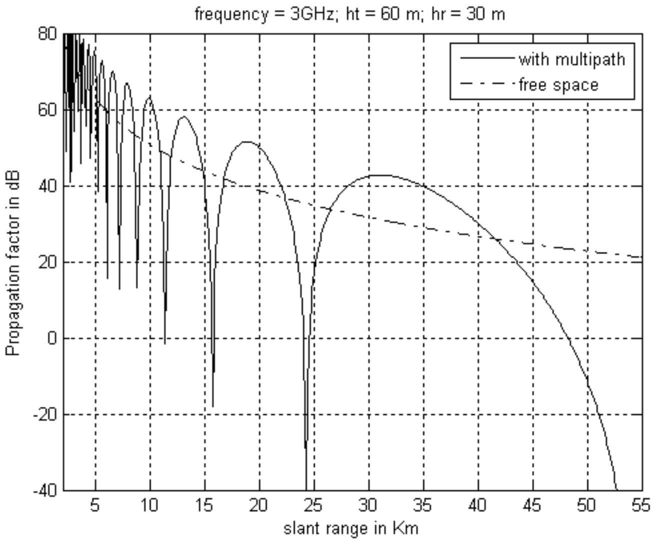


Figure 1.37. Effect of multipath on the radar sensitivity.

1.10.7. Diffraction

The analysis that led to creating the multipath model described in the previous section applies only to ground reflections from the intermediate region, as illustrated in Fig. 1.38. The effects of ground reflection below the radar horizon is governed by another physical phenomenon referred to as diffraction. The diffraction model requires calculations of the Airy function and its roots. For this purpose, the numerical approximation presented in Shatz and Polychronopoulos[1] is adopted. This numerical algorithm, described by Shatz and Polychronopoulos, is very accurate and its implementation using MATLAB is straight forward.

1. Shatz, M. P., and Polychronopoulos, G. H., *An Algorithm for Evaluation of Radar Propagation in the Spherical Earth Diffraction Region*. IEEE Transactions on Antenna and Propagation, Vol. 38, August 1990, pp. 1249-1252.

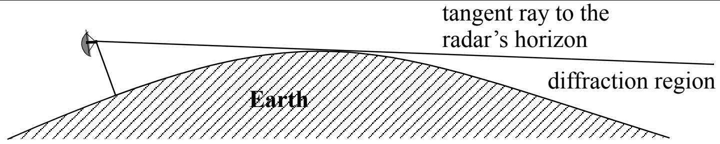


Figure 1.38. Diffraction region.

Define the following parameters,

$$x = \frac{R}{r_0} \quad , \quad y = \frac{h_r}{h_0} \quad , \quad t = \frac{h_t}{h_0} \tag{1.180}$$

where h_r is the radar altitude, h_t is target altitude, R is range to the target, h_0 and r_0 are normalizing factors given by

$$h_0 = \frac{1}{2}\left(\frac{r_e \lambda^2}{\pi^2}\right)^{1/3} \tag{1.181}$$

$$r_0 = \left(\frac{r_e^2 \lambda}{\pi}\right)^{1/3} \tag{1.182}$$

λ is the wavelength and r_e is the effective earth radius. Let $A_i(u)$ denote the Airy function defined by

$$A_i(u) = \frac{1}{\pi}\int_0^{\infty} \cos\left(\frac{q^3}{3} + uq\right) \, dq \tag{1.183}$$

The general expression for the propagation factor in the diffraction region is equal to

$$F = 2\sqrt{\pi x} \sum_{n=1}^{\infty} f_n(y) f_n(t) \exp[(e^{j\pi/6}) a_n x] \tag{1.184}$$

where (x, y, t) are defined in Eq. (1.180) and

$$f_n(u) = \frac{A_i(a_n + ue^{j\pi/3})}{e^{j\pi/3} A_i{}'(a_n)} \tag{1.185}$$

where a_n is the n^{th} root of the Airy function and A_i' is the first derivative of the Airy function. Shatz and Polychronopoulos showed that Eq. (1.184) can be approximated by

$$F = 2\sqrt{\pi x}\sum_{n=1}^{\infty}\frac{\widehat{A_i}(a_n + ye^{j\pi/3})}{e^{j\pi/3}A_i'(a_n)}\frac{\widehat{A_i}(a_n + te^{j\pi/3})}{e^{j\pi/3}A_i'(a_n)} \tag{1.186}$$

$$\exp\left[\frac{1}{2}(\sqrt{3}+j)a_n x - \frac{2}{3}(a_n + ye^{j\pi/3})^{3/2} - \frac{2}{3}(a_n + te^{j\pi/3})^{3/2}\right]$$

where

$$\widehat{A_i}(u) = A_i(u)e^{j\frac{2}{3}u^{3/2}} \tag{1.187}$$

Shatz and Polychronopoulos showed that sum in Eq. (1.186) represents accurate computation of the propagation factor within the diffraction region. In this book, a MATLAB program called *"diffraction.m"* was written by this author to implement Eq. (1.86) where the sum is terminated at $n \le 1500$ for accurate computation. For this purpose, another MATLAB function called *"airyzo1.m"* was used to compute the roots of Airy function and the roots of its first derivative. Figure 1.39 (after Shatz) shows a typical output generated by this program for $h_t = 1000m$, $h_r = 8000m$, and $frequency = 167MHz$.

This figure can be reproduced using the following MATLAB code.

```
% Figure 1.39 or Figure 1.40
clc
clear all
close all
freq =167e6;
hr = 8000;
ht = 1000;
R = linspace(400e3,600e3,200); % range in Km
nt =1500; % number of point used in calculating infinite series
F = diffraction(freq, hr, ht, R, nt);
figure(1)
plot(R/1000,10*log10(abs(F).^2),'k','linewidth',1)
grid
xlabel('Range in Km')
ylabel('One way propagation factor in dB')
title('frequency = 167MHz; hr = 8000 m; ht = 1000m')
```

Figure 1.40 is similar to Fig. 1.39 except in this case the following parameters are used: $h_t = 3000m$, $h_r = 200m$, and $frequency = 428MHz$. Figure 1.41 shows a plot for the propagation factor using the same parameters in Fig.

1.40; however, in this figure, both intermediate and diffraction regions are shown.

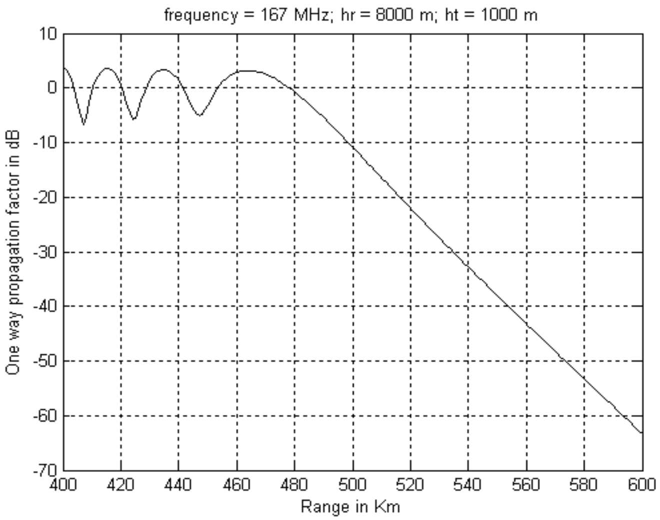


Figure 1.39. Propagation factor in the diffraction region.

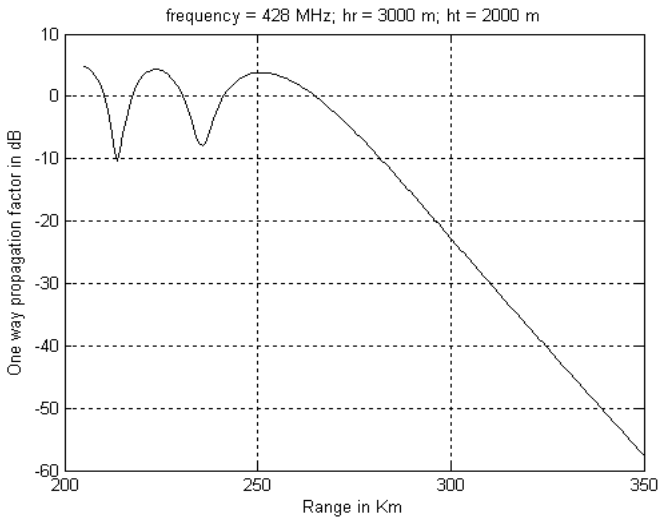


Figure 1.40. Propagation factor in the diffraction region.

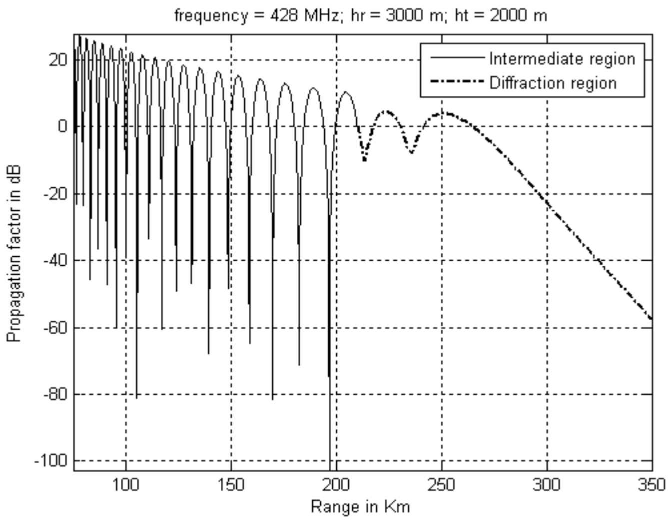


Figure 1.41. Propagation factor.

1.11. Atmospheric Attenuation

Electromagnetic waves travel in free space without suffering any energy loss. Alternatively, due to gases and water vapor in the atmosphere, radar energy suffers a loss. This loss is known as atmospheric attenuation. Atmospheric attenuation increases significantly in the presence of rain, fog, dust, and clouds. Most of the lost radar energy is normally absorbed by gases and water vapor and transformed into heat, while a small portion of this lost energy is used in molecular transformation of the atmosphere particles.

The two-way atmospheric attenuation over a range R can be expressed as

$$L_{atmosphere} = e^{-2\alpha R} \tag{1.188}$$

where α is the one-way attenuation coefficient. Water vapor attenuation peaks at about $22.3GHz$, while attenuation due to oxygen peaks at between 60 and $118GHz$. Atmospheric attenuation is severe for frequencies higher than $35GHz$. This is the reason ground-based radars rarely use frequencies higher than $35GHz$. Atmospheric attenuation is a function range, frequency, and elevation angle. Figure 1.42 shows a typical two-way atmospheric attenuation plot versus range at $3GHz$, with the elevation angle as a parameter.

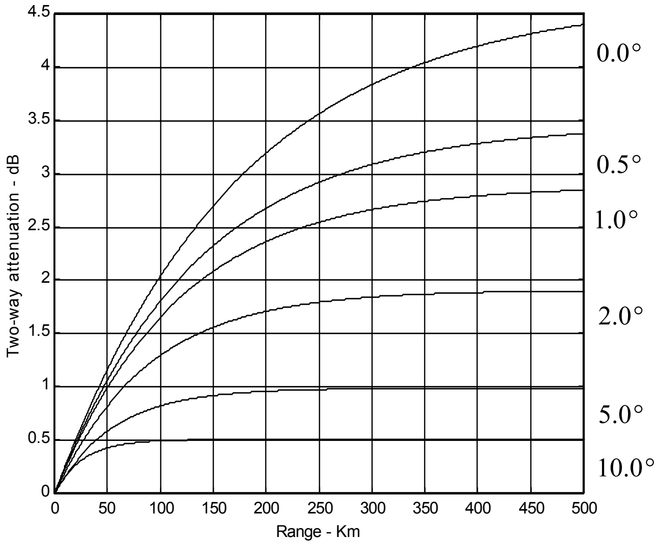


Figure 1.42. Attenuation versus range; frequency is *3 GHz*.

1.12. MATLAB Program Listings

This section presents listings for all the MATLAB programs used to produce all of the MATLAB-generated figures in this chapter. They are listed in the same order they appear in the text.

1.12.1. MATLAB Function "range_resolution.m"

The MATLAB function *"range_resolution.m"* calculates range resolution; its syntax is as follows:

[delta_R] = range_resolution(var, indicator)

where

Symbol	Description	Units	Status
var, indicator	*bandwidth, "hz"*	*Hz, none*	*inputs*
var, indicator	*pulse width, "s"*	*seconds, none*	*inputs*
delta_R	*range resolution*	*meters*	*output*

MATLAB Function "range_resolution.m" Listing

```
function [delta_R] = range_resolution(bandwidth, indicator)
% This function computes radar range resolution in meters
% the bandwidth must be in Hz ==> indicator = Hz.
% Bandwidth may be equal to (1/pulse width)==> indicator = seconds
c = 3.e+8; % speed of light
if(indicator == 'hz')
  delta_R = c / 2.0 / bandwidth;
else
  delta_R = c * bandwidth / 2.0;
end
return
```

1.12.2. MATLAB Function "radar_eq.m"

The function *"radar_eq.m"* implements Eq. (1.40); its syntax is as follows:

[snr] = radar_eq (pt, freq, g, sigma, b, nf, loss, range)

where

Symbol	Description	Units	Status
pt	peak power	Watts	input
freq	radar center frequency	Hz	input
g	antenna gain	dB	input
sigma	target cross section	m^2	input
b	bandwidth	Hz	input
nf	noise figure	dB	input
loss	radar losses	dB	input
range	target range (can be either a single value or a vector)	meters	input
snr	SNR (single value or a vector, depending on the input range)	dB	output

MATLAB Function "radar_eq.m" Listing

```
function [snr] = radar_eq(pt, freq, g, sigma, b, nf, loss, range)
% This program implements Eq. (1.40)
c = 3.0e+8; % speed of light
lambda = c / freq; % wavelength
p_peak = 10*log10(pt); % convert peak power to dB
lambda_sqdb = 10*log10(lambda^2); % compute wavelength square in dB
sigmadb = 10*log10(sigma); % convert sigma to dB
four_pi_cub = 10*log10((4.0 * pi)^3); % (4pi)^3 in dB
```

```
k_db = 10*log10(1.38e-23); % Boltzmann's constant in dB
to_db = 10*log10(290); % noise temp. in dB
b_db = 10*log10(b); % bandwidth in dB
range_pwr4_db = 10*log10(range.^4); % vector of target range^4 in dB
% Implement Equation (1.63)
num = p_peak + g + g + lambda_sqdb + sigmadb;
den = four_pi_cub + k_db + to_db + b_db + nf + loss + range_pwr4_db;
snr = num - den;
return
```

1.12.3. MATLAB Function "power_aperture.m"

The function *"power_aperture.m"* implements the search radar equation given in Eq. (1.47); its syntax is as follows:

PAP = power_aperture (snr, tsc, sigma, range, nf, loss, az_angle, el_angle)

where

Symbol	Description	Units	Status
snr	*sensitivity snr*	*dB*	*input*
tsc	*scan time*	*seconds*	*input*
sigma	*target cross section*	m^2	*input*
range	*target range*	*meters*	*input*
nf	*noise figure*	*dB*	*input*
loss	*radar losses*	*dB*	*input*
az_angle	*search volume azimuth extent*	*degrees*	*input*
el_angle	*search volume elevation extent*	*degrees*	*input*
PAP	*power aperture product*	*dB*	*output*

MATLAB Function *"power_aperture.m"* **Listing**

```
function PAP = power_aperture(snr,tsc,sigma,range,nf,loss,az_angle,el_angle)
% This program implements Eq. (1.47)
Tsc = 10*log10(tsc); % convert Tsc into dB
Sigma = 10*log10(sigma); % convert sigma to dB
four_pi = 10*log10(4.0 * pi); % (4pi) in dB
k_db = 10*log10(1.38e-23); % Boltzmann's constant in dB
To = 10*log10(290); % noise temp. in dB
range_pwr4_db = 10*log10(range.^4); % target range^4 in dB
omega = (az_angle/57.296) * (el_angle / 57.296);
% compute search volume in steraradians
Omega = 10*log10(omega); % search volume in dB
% implement Eq. (1.79)
PAP = snr + four_pi + k_db + To + nf + loss + range_pwr4_db + Omega ...
```

```
    - Sigma - Tsc;
return
```

1.12.4. MATLAB Function "range_red_factor.m"

The function *"range_red_factor.m"* implements Eqs. (1.88) and (1.89). This function generates plots of RRF versus (1) the radar operating frequency, (2) radar to jammer range, and (3) jammer power. Its syntax is as follows:

[RRF] = range_red_factor (te, pj, gj, g, freq, bj, rangej, lossj)

where

Symbol	Description	Units	Status
te	radar effective temperature	K	input
pj	jammer peak power	W	input
gj	jammer antenna gain	dB	input
g	radar antenna gain on jammer	dB	input
freq	radar operating frequency	Hz	input
bj	jammer bandwidth	Hz	input
rangej	radar to jammer range	Km	input
lossj	jammer losses	dB	input

MATLAB Function *"range_red_factor.m"* **Listing**

```
function RRF = range_red_factor (ts, pj, gj, g, freq, bj, rangej, lossj)
% This function computes the range reduction factor and produces
% plots of RRF versus wavelength, radar to jammer range, and jammer power
c = 3.0e+8;
k = 1.38e-23;
lambda = c / freq;
gj_10 = 10^( gj/10);
g_10 = 10^( g/10);
lossj_10 = 10^(lossj/10);
index = 0;
for wavelength = .01:.001:1
  index = index +1;
  jamer_temp = (pj * gj_10 * g_10 *wavelength^2) / ...
    (4.0^2 * pi^2 * k * bj * lossj_10 * (rangej * 1000.0)^2);
  delta = 10.0 * log10(1.0 + (jamer_temp / ts));
  rrf(index) = 10^(-delta /40.0);
end
w = 0.01:.001:1;
figure (1)
semilogx(w,rrf,'k')
```

```
grid
xlabel ('Wavelength in meters')
ylabel ('Range reduction factor')
index = 0;
for ran =rangej*.3:10:rangej*2
  index = index + 1;
  jamer_temp = (pj * gj_10 * g_10 *lambda^2) / ...
    (4.0^2 * pi^2 * k * bj * lossj_10 * (ran * 1000.0)^2);
  delta = 10.0 * log10(1.0 + (jamer_temp / ts));
  rrf1(index) = 10^(-delta /40.0);
end
figure(2)
ranvar = rangej*.3:10:rangej*2 ;
plot(ranvar,rrf1,'k')
grid
xlabel ('Radar to jammer range in Km')
ylabel ('Range reduction factor')
index = 0;
for pjvar = pj*.01:100:pj*2
  index = index + 1;
  jamer_temp = (pjvar * gj_10 * g_10 *lambda^2) / ...
    (4.0^2 * pi^2 * k * bj * lossj_10 * (rangej * 1000.0)^2);
  delta = 10.0 * log10(1.0 + (jamer_temp / ts));
  rrf2(index) = 10^(-delta /40.0);
end
figure(3)
pjvar = pj*.01:100:pj*2;
plot(pjvar,rrf2,'k')
grid
xlabel ('Jammer peak power in Watts')
ylabel ('Range reduction factor')
```

1.12.5. MATLAB Function *"ref_coef.m"*

The function *"ref_coef.m"* calculates the horizontal and vertical magnitude and phase response of the reflection coefficient. The syntax is as follows

[rh,rv] = ref_coef (psi, epsp, epspp)

where

Symbol	Description	Status
psi	*grazing angle in degrees (can be a vector or a scalar)*	*input*
epsp	ε'	*input*

Symbol	Description	Status
epspp	ε''	*input*
rh	*horizontal reflection coefficient complex vector*	*output*
rv	*vertical reflection coefficient complex vector*	*output*

MATLAB Function** "ref_coef.m" **Listing

```
function [rh,rv] = ref_coef (psi, epsp, epspp)
eps = epsp - i .* epspp;
psirad = psi.*(pi./180.);
arg1 = eps - (cos(psirad).^2);
arg2 = sqrt(arg1);
arg3 = sin(psirad);
arg4 = eps.*arg3;
rv = (arg4-arg2)./(arg4+arg2);
rh = (arg3-arg2)./(arg3+arg2);
```

1.12.6. MATLAB Function "divergence.m"

The MATLAB function *"divergence.m"* calculates the divergence. The syntax is as follows:

$$D = divergence\ (psi,\ r1,\ r2,\ hr,\ ht)$$

where

Symbol	Description	Status
psi	*grazing angle in degrees (can be vector or scalar)*	*input*
r1	*ground range between radar and specular point in Km*	*input*
r2	*ground range between specular point and target in Km*	*input*
hr	*radar height in meters*	*input*
ht	*target height in meters*	*input*
D	*divergence*	*output*

MATLAB Function** "divergence.m" **Listing

```
function [D] = divergence(psi, r1, r2, hr, ht)
% calculates divergence
%      inputs  %%%%%%%%%%%%%%%%%%%%%%%%%
%   r1  ground range between radar and specular point in Km
%   r2  ground range between specular point and target in Km
%   psi grazing angle in degrees
%      parameters  %%%%%%%%%%%%%%%%%%%%%%%
%   re  4/3 earth radius 4/3 * 6375 Km
```

```
% r = r1 + r2
psi = psi .* pi ./180; % psi in radians
re = (4/3) * 6375e3;
r = r1 + r2;
arg1 = re .* r . * sin(psi) ;
arg2 = ((2 .* r1 .* r2 ./ cos(psi)) + re .* r. * sin(psi)) .* (1+hr./re) .* (1+ht./re);
D = sqrt(arg1 ./ arg2);
return
```

1.12.7. MATLAB Function "surf_rough.m"

The MATLAB function *"surf_rough.m"* calculates the surface roughness reflection coefficient. The syntax is as follows:

Sr = surf_rough (hrms, freq, psi)

where

Symbol	Description	Status
hrms	*surface rms roughness value in meters*	*input*
freq	*frequency in Hz*	*input*
psi	*grazing angle in degrees*	*input*
Sr	*surface roughness coefficient*	*output*

MATLAB Function *"surf_rough.m"* **Listing**

```
function Sr = surf_rough(hrms, freq, psi)
clight = 3e8;
psi = psi .* pi ./ 180; % angle in radians
lambda = clight / freq; % wavelength
g = (2.* pi .* hrms .* sin(psi) ./ lambda).^2;
Sr = exp(-2 .* g);
return
```

1.12.8. MATLAB Program "multipath.m"

```
% This program calculates and plots the propagation factor versus
% target range with a fixed target height.
% The free space radar-to-target range is assumed to be known.
clear all;
close all;
eps = 0.01;
%%%%%%%%%%%%%% input %%%%%%%%%%%%%%%%%%
ro = 6375e3; % earth radius
re = ro * 4 /3; % 4/3 earth radius
freq = 3000e6; % frequency
```

```
lambda = 3.0e8 / freq; % wavelength
hr = 30.48; % radar height in meters
ht = 2 .* hr; % target height in meters
Rd1 = linspace(2e3, 55e3, 500); % slant range 3 to 55 Km 500 points
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% determine whether the traget is beyond the radar's line of sight
range_to_horizon = sqrt(2*re) * (sqrt(ht) + sqrt(hr)); % range to horizon
index = find(Rd1 > range_to_horizon);
if isempty(index);
   Rd = Rd1;
else
   Rd = Rd1(1:index(1)-1);
   fprintf('****** WARNING ****** \n')
   fprintf('Maximum range is beyond radar line of sight. \n')
   fprintf('Target is in diffraction region \n')
   fprintf('****** WARNING ****** \n')
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
val1 = Rd.^2 - (ht -hr).^2;
val2 = 4 .* (re + hr) .* (re + ht);
r = 2 .* re .* asin(sqrt(val1 ./ val2));
phi = r ./ re;
p = sqrt(re .* (ht + hr) + (r.^2 ./4)) .* 2 ./ sqrt(3);
exci = asin((2 .* re .* r .* (ht - hr) ./ p.^3));
r1 = (r ./ 2) - p .* sin(exci ./3);
phi1 = r1 ./ re;
r2 = r - r1;
phi2 = r2 ./ re;
R1 = sqrt( re.^2 + (re + hr).^2 - 2 .* re .* (re + hr) .* cos(phi1));
R2 = sqrt( re.^2 + (re + ht).^2 - 2 .* re .* (re + ht) .* cos(phi2));
psi = asin((2 .* re .* hr + hr^2 - R1.^2) ./ (2 .* re .* R1));
deltaR = R1 + R2 - Rd;
%%%%%%%%%%%%%% input surface roughness %%%%%%%%%%%%%%%%
hrms = 1;
psi = psi .* 180 ./ pi;
[Sr] = surf_rough(hrms, freq, psi);
%%%%%%%%%%%%%% input divergence %%%%%%%%%%%%%%%%%%
[D] = divergence(psi, r1, r2, hr, ht);
%%%%%%%%%%%%%% input smooth earth ref. coefficient %%%%%%%%%%%%
epsp = 50;
epspp = 15;
[rh,rv] = ref_coef (psi, epsp, epspp);
D = 1;
 Sr =1;
gamav = abs(rv);
phv = angle(rv);
gamah = abs(rh);
phh = angle (rh);
```

```
gamav =1;
phv = pi;
Gamma_mod = gamav .* D .* Sr;
Gamma_phase = phv; %
rho = Gamma_mod;
delta_phi = 2 .* pi .* deltaR ./ lambda;
alpha = delta_phi + phv;
F = sqrt( 1 + rho.^2 + 2 .* rho .* cos( alpha));
Ro = 185.2e3; % reference range in Km
F_free = 40 .* log10(Ro ./ Rd);
F_dbr = 40 .* log10( F .* Ro ./ Rd);
F_db = 40 .* log10( eps + F );
figure(1)
plot(Rd./1000, F_db,'k','linewidth',1)
grid
xlabel('slant range in Km')
ylabel('propagation factor in dB')
axis tight
axis([2 55 -60 20])
figure(2)
plot(Rd./1000, F_dbr,'k',Rd./1000, F_free,'k-.','linewidth',1)
grid
xlabel('slant range in Km')
ylabel('Propagation factor in dB')
axis tight
axis([2 55 -40 80])
legend('with multipath','free space')
title('frequency = 3GHz; ht = 60 m; hr = 30 m')
```

1.12.9. MATLAB Program "diffraction.m"

This function utilizes Shatz's model to calculate the propagation factor in the diffraction region. It utilizes the MATLAB function *"airy.m"* which is part of the Signal Processing Toolbox. Its syntax is as follows

F = diffraction(freq, hr, ht, R, nt);

where

% Generalized spherical earth propagation factor calculations

Symbol	Description	Status
freq	*radar operating frequency*	*Hz*
hr	*radar height*	*meters*
ht	*target height*	*meters*

Symbol	Description	Status
R	*range over which to calculate the propagation factor*	*Km*
nt	*number of data point is the series given in Eq. (1.186)*	*none*
F	*propagation factor in diffraction region*	*dB*

MATLAB Program *"diffraction.m"* Listing

```
function F = diffraction(freq, hr, ht,R,nt);
%   Generalized spherical earth propagation factor calculations
%   After Shatz: Michael P. Shatz, and George H. Polychronopoulos, An
%   Algorithm for Elevation of Radar Propagation in the Spherical Earth
%   Diffraction Region. IEEE Transactions on Antenna and Propagation,
%   VOL. 38, NO.8, August 1990.
format long
re = 6373e3 * (4/3); % 4/3 earth radius in Km
[an] = airyzo1(nt);% calculate the roots of the Airy function
c = 3.0e8; % speed of light
lambda = c/freq; % wavelength
r0 = (re*re*lambda / pi)^(1/3);
h0 = 0.5 * (re*lambda*lambda/pi/pi)^(1/3);
y = hr / h0;
z = ht / h0;
%%%%%%%%%%%%
par1 = exp(sqrt(-1)*pi/3);
pary1 = ((2/3).*(an + y .* par1).^(1.5));
   pary = exp(pary1);
   parz1 = ((2/3).*(an + z .* par1).^(1.5));
   parz = exp(parz1);
   f1n = airy(an + y * par1) .* airy(an + z * par1) .* pary .*parz ;
   f1d = par1 .* par1 .* airy(1,an) .* airy(1,an);
   f1 = f1n ./ f1d;
   index = find(f1<1e6);
%%%%%%%%%%%%
F = zeros(1,size(R,2));
for range = 1:size(R,2)
   x(range) = R(range)/r0;
   f2 = exp(0.5 .* (sqrt(3) +sqrt(-1)) .*an.*x(range) - pary1 -parz1);
   victor = f1(index) .* f2(index);
   fsum = sum(victor);
   F(range) = 2 .*sqrt(pi.*x(range)) .* fsum;
end
```

1.12.10. MATLAB Program "airyzo1.m"

The function "airyzo1.m" was developed to compute the roots of the Airy function. Its syntax is as follows:

$$[an] = airyzo1(nt)$$

where the input *nt* is the number of required roots, and the output *[an]* is the roots (zeros) vector.

MATLAB Program** "airyzo1.m" **Listing

```
function [an] = airyzo1(nt)
%   This program is a modified version of a function obtained from
%   free internet source www.mathworks.com/matlabcentral/fileexchange/
%   modified by B. Mahafza (bmahafza@dbresearch.net) in 2005
%       ================================
%   Purpose: This program computes the first nt zeros of Airy
%   functions Ai(x)
%   Input :  nt     --- Total number of zeros
%   Output:  an ---    first nt roots for Ai(x)
format long
an = zeros(1,nt);
xb = zeros(1,nt);
ii = linspace(1,nt,nt);
u = 3.0.*pi.*(4.0.*ii-1)./8.0;
u1 = 1./(u.*u);
rt0 = -(u.*u).^(1.0./3.0).*((((-15.5902.*u1+.929844).* ...
u1-.138889).*u1+.10416667).*u1+1.0);
rt = 1.0e100;
while(abs((rt-rt0)./rt)> 1.e-12);
x = rt0;
ai = airy(0,x);
ad = airy(1,x);
rt=rt0-ai./ad;
if(abs((rt-rt0)./rt)> 1.e-12);
rt0 = rt;
end;
end;
an(ii)= rt;
end
```

1.12.11. MATLAB Program "fig_31_32.m"

```
% This program produces Figs. 1.31 and 1.32
close all
clear all
```

```
psi = 0.01:0.25:90;
epsp = [2.8];
epspp = [0.032];% 0.87 2.5 4.1];
[rh1,rv1] = ref_coef(psi, epsp,epspp);
gamamodv1 = abs(rv1);
gamamodh1 = abs(rh1);
epsp = [5.8] ;
epspp = [0.87];
[rh2,rv2] = ref_coef(psi, epsp,epspp);
gamamodv2 = abs(rv2);
gamamodh2 = abs(rh2);
epsp = [10.3];
epspp = [2.5];
[rh3,rv3] = ref_coef(psi, epsp,epspp);
gamamodv3 = abs(rv3);
gamamodh3 = abs(rh3);
epsp = [15.3];
epspp = [4.1];
[rh4,rv4] = ref_coef(psi, epsp,epspp);
gamamodv4 = abs(rv4);
gamamodh4 = abs(rh4);
figure(1)
semilogx(psi,gamamodh1,'k',psi,gamamodh2,'k-.', ...
psi,gamamodh3,'k.',psi,gamamodh4,'k:','linewidth',1.5);
grid
xlabel('grazing angle - degrees');
ylabel('reflection coefficient - amplitude')
legend('moisture = 0.3%','moisture = 10%%','moisture = 20%','moisture = 30%')
title('horizontal polarization')
% legend ('Vertical Polarization','Horizontal Polarization')
% pv = -angle(rv);
% ph = angle(rh);
% figure(2)
% plot(psi,pv,'k',psi,ph,'k -.');
% grid
% xlabel('grazing angle - degrees');
% ylabel('reflection coefficient - pahse')
% legend ('Vertical Polarization','Horizontal Polarization')
```

Problems

1.1. (a) Calculate the maximum unambiguous range for a pulsed radar with PRF of $200Hz$ and $750Hz$. (b) What are the corresponding PRIs?

1.2. For the same radar in Problem 1.1, assume a duty cycle of 30% and peak power of $5KW$. Compute the average power and the amount of radiated energy during the first $20ms$.

1.3. A certain pulsed radar uses pulse width $\tau = 1\mu s$. Compute the corresponding range resolution.

1.4. An X-band radar uses PRF of $3KHz$. Compute the unambiguous range and the required bandwidth so that the range resolution is $30m$. What is the duty cycle?

1.5. Compute the Doppler shift associated with a closing target with velocity 100, 200, and 350 meters per second. In each case compute the time dilation factor. Assume that $\lambda = 0.3m$.

1.6. In reference to Fig. 1.8, compute the Doppler frequency for

$v = 150m/s$, $\theta_a = 30°$, and $\theta_e = 15°$. Assume that $\lambda = 0.1m$.

1.7. (a) Develop an expression for the minimum PRF of a pulsed radar; (b) compute $f_{r_{min}}$ for a closing target whose velocity is $400m/s$. (c) What is the unambiguous range? Assume that $\lambda = 0.2m$.

1.8. An L-band pulsed radar is designed to have an unambiguous range of $100Km$ and range resolution $\Delta R \le 100m$. The maximum resolvable Doppler frequency corresponds to $v_{target} \le 350m/\text{sec}$. Compute the maximum required pulse width, the PRF, and the average transmitted power if $P_t = 500W$.

1.9. Compute the aperture size for an X-band antenna at $f_0 = 9GHz$. Assume antenna gain $G = 10, 20, 30 \ dB$.

1.10. An L-band radar (1500 MHz) uses an antenna whose gain is $G = 30dB$. Compute the aperture size. If the radar duty cycle is $d_t = 0.2$ and the average power is $25KW$, compute the power density at range $R = 50Km$.

1.11. For the radar described in Problem 1.9, assume the minimum detectable signal is $5dBm$. Compute the radar maximum range for $\sigma = 1.0, 10.0, 20.0m^2$.

1.12. Consider an L-band radar with the following specifications: operating frequency $f_0 = 1500MHz$, bandwidth $B = 5MHz$, and antenna gain $G = 5000$. Compute the peak power, the pulse width, and the minimum detectable signal for this radar. Assume target RCS $\sigma = 10m^2$, the single pulse SNR is $15.4dB$, noise figure $F = 5dB$, temperature $T_0 = 290K$, and maximum range $R_{max} = 150Km$.

1.13. Consider a low PRF C-band radar operating at $f_0 = 5000MHz$. The antenna has a circular aperture with radius $2m$. The peak power is $P_t = 1MW$ and the pulse width is $\tau = 2\mu s$. The PRF is $f_r = 250Hz$, and the effective temperature is $T_0 = 600K$. Assume radar losses $L = 15dB$ and target RCS $\sigma = 10m^2$. (a) Calculate the radar's unambiguous range; (b) calculate the range R_0 that corresponds to $SNR = 0dB$; (c) calculate the SNR at $R = 0.75R_0$.

1.14. Repeat the second example in Section 1.6 with $\Omega = 4°$, $\sigma = 1m^2$, and $R = 400Km$.

1.15. The atmospheric attenuation can be included in the radar equation as another loss term. Consider an X-band radar whose detection range at $20Km$ includes a $0.25dB/Km$ atmospheric loss. Calculate the corresponding detection range with no atmospheric attenuation.

1.16. Let the maximum unambiguous range for a low PRF radar be R_{max}. (a) Calculate the SNR at $(1/2)R_{max}$ and $(3/4)R_{max}$. (b) If a target with $\sigma = 10m^2$ exists at $R = (1/2)R_{max}$, what should the target RCS be at $R = (3/4)R_{max}$ so that the radar has the same signal strength from both targets.

1.17. A Millie-Meter Wave (MMW) radar has the following specifications: operating frequency $f_0 = 94GHz$, PRF $f_r = 15KHz$, pulse width $\tau = 0.05ms$, peak power $P_t = 10W$, noise figure $F = 5dB$, circular antenna with diameter $D = 0.254m$, antenna gain $G = 30dB$, target RCS $\sigma = 1m^2$, system losses $L = 8dB$, radar scan time $T_{sc} = 3s$, radar angular coverage $200°$, and atmospheric attenuation $3dB/Km$. Compute the following: (a) wavelength λ, (b) range resolution ΔR, (c) bandwidth B, (d) the SNR as a function of range, (e) the range for which $SNR = 15dB$, (f) antenna beam width, (g) antenna scan rate, (h) time on target, (i) the effective maximum range when atmospheric attenuation is considered.

1.18. A radar with antenna gain G is subject to a repeater jammer whose antenna gain is G_J. The repeater illuminates the radar with three fourths of the incident power on the jammer. (a) Find an expression for the ratio between the power received by the jammer and the power received by the radar. (b) What is this ratio when $G = G_J = 200$ and $R/\lambda = 10^5$?

1.19. A radar has the following parameters: peak power $P_t = 65KW$, total losses $L = 5dB$, operating frequency $f_o = 8GHz$, PRF $f_r = 4KHz$, duty cycle $d_t = 0.3$, circular antenna with diameter $D = 1m$, effective aperture is 0.7 of physical aperture, noise figure $F = 8dB$. (a) Derive the various parameters needed in the radar equation. (b) What is the unambiguous range? (c) Plot the SNR versus range (1 Km to the radar unambiguous range) for a 5dBsm target, and (d) if the minimum SNR required for detection is 14 dB, what is the detection range for a 6 dBsm target? What is the detection range if the SNR threshold requirement is raised to 18 dB?

1.20. A radar has the following parameters: Peak power $P_t = 50KW$; total losses $L = 5dB$; operating frequency $f_o = 5.6GHz$; noise figure $F = 10dB$ pulse width $\tau = 10\mu s$; PRF $f_r = 2KHz$; antenna beamwidth $\theta_{az} = 1°$ and $\theta_{el} = 5°$. (a) What is the antenna gain? (b) What is the effective aperture if the aperture efficiency is 60%? (c) Given a 14 dB threshold detection, what is the detection range for a target whose RCS is $\sigma = 1m^2$?

1.21. A certain radar has losses of 5 dB and a receiver noise figure of 10 dB. This radar has a detection coverage requirement that extends over 3/4 of a hemisphere and must complete it in 3 second. The base line target RCS is 6 dBsm and the minimum SNR is 15 dB. The radar detection range is less than 80 Km. What is the average power aperture product for this radar so that it can satisfy its mission?

1.22. Assume a bandwidth of $150KHz$. (a) Compute the noise figure for the three cascaded amplifiers. (b) Compute the effective temperature for the three cascaded amplifiers. (c) Compute the overall system noise figure.

1.23. An exponential expression for the index of refraction is given by $n = 1 + 315 \times 10^{-6} \exp(-0.136h)$ where the altitude h is in Km. Calculate the index of refraction for a well-mixed atmosphere at 10% and 50% of the troposphere.

1.24. A source with equivalent temperature $T_0 = 290K$ is followed by three amplifiers with specifications shown in the table below.

Amplifier	F, *dB*	G, *dB*	T_e
1	You must compute	12	350
2	10	22	
3	15	35	

1.25. Reproduce Figs. 1.30 and 1.31 by using $f = 8GHz$ and (a) $\varepsilon' = 2.8$ and $\varepsilon'' = 0.032$ (dry soil); (b) $\varepsilon' = 47$ and $\varepsilon'' = 19$ (sea water at $0°C$); (c) $\varepsilon' = 50.3$ and $\varepsilon'' = 18$ (lake water at $0°C$).

1.26. In reference to Fig. 8.16, assume a radar height of $h_r = 100m$ and a target height of $h_t = 500m$. The range is $R = 20Km$. (a) Calculate the lengths of the direct and indirect paths. (b) Calculate how long it will take a pulse to reach the target via the direct and indirect paths.

1.27. A radar at altitude $h_r = 10m$ and a target at altitude $h_t = 300m$, and assuming a spherical earth, calculate r_1, r_2, and ψ_g.

1.28. In the previous problem, assuming that you may be able to use the small grazing angle approximation: (a) Calculate the ratio of the direct to the indirect signal strengths at the target. (b) If the target is closing on the radar with velocity $v = 300m/s$, calculate the Doppler shift along the direct and indirect paths. Assume $\lambda = 3cm$.

1.29. Derive an asymptotic form for Γ_h and Γ_v when the grazing angle is very small.

1.30. In reference to Fig. 1.37, assume a radar height of $h_r = 100m$ and a target height of $h_t = 500m$. The range is $R = 20Km$. (a) Calculate the lengths of the direct and indirect paths. (b) Calculate how long it will take a pulse to reach the target via the direct and indirect paths.

1.31. Using the law of cosines, derive Eq. (1.138) from (1.137).

1.32. In the previous problem, assuming that you may be able to use the small grazing angle approximation. (a) Calculate the ratio of the direct to the indirect signal strengths at the target. (b) If the target is closing on the radar with velocity $v = 300m/s$, calculate the Doppler shift along the direct and indirect paths. Assume $\lambda = 3cm$.

1.33. In the previous problem, assuming that you may be able to use the small grazing angle approximation: (a) Calculate the ratio of the direct to the indirect signal strengths at the target. (b) If the target is closing on the radar with velocity $v = 300m/s$, calculate the Doppler shift along the direct and indirect paths. Assume $\lambda = 3cm$.

1.34. Calculate the range to the horizon corresponding to a radar at $5Km$ and $10Km$ of altitude. Assume 4/3 earth.

1.35. Develop a mathematical expression that can be used to reproduce Fig. 1.42.

1.36. Modify the MATLAB program *"multipath.m"* so that it uses the true spherical ground range between the radar and the target.

1.37. Modify the MATLAB program *"multipath.m"* so that it accounts for the radar antenna.