

RADAR SIGNAL ANALYSIS AND PROCESSING USING MATLAB®

RADAR SIGNAL ANALYSIS AND PROCESSING USING MATLAB[®]

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CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
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Chapman & Hall/CRC
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

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Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-1-4200-6643-2 (Hardcover)

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Library of Congress Cataloging-in-Publication Data

Mahafza, Bassem R.
Radar signal analysis and processing using MATLAB / Bassem R. Mahafza.
p. cm.
"A CRC title."
Includes bibliographical references and index.
ISBN 978-1-4200-6643-2 (hardback : alk. paper)
1. Radar cross sections. 2. Signal processing. 3. Radar targets. 4. MATLAB. I. Title.

TK6575.M267 2008
621.3848--dc22

2008014584

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<http://www.crcpress.com>

To my four sons:

Zachary,
Joseph,
Jacob, and
Jordan

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Preface

In the year 2000 my book *Radar Systems Analysis and Design Using MATLAB¹*® was published. This book very quickly turned into a bestseller which prompted the publication of its second edition in the year 2005. At the time of its publication, it was based on my years of teaching graduate level courses on radar systems analysis and design including advanced topics in radar signal processing. The motivation behind it was to introduce a college-suitable comprehensive textbook that provides hands-on experience with MATLAB® companion software. Over the years, I have also taught numerous industry courses on the subject of radar systems. Based on my combined teaching experience and real-world work at deciBel Research, Inc., the following conclusion has become very evident to me: There is big appetite and demand for textbooks and reference books that are primarily focused on aspects of radar signals and signal processing. Having arrived at this conclusion, I decided to write this textbook, *Radar Signal Analysis and Processing Using MATLAB*®, which is focused on radar signal analysis and processing.

Unlike other books on the subject, the emphasis is not on signal processing per se, but on signals and signal processing in the context of radar applications. Many good textbooks are already available on signal processing but not on signal processing as it applies to radar applications. This new textbook has many desirable features that include clear and concise presentation of the theory and companion user-friendly MATLAB code. This code is reconfigurable to demonstrate the theory and perform the associated analysis/design trades as well as allow users to vary the inputs in order to better analyze their relevant and unique requirements. This new book should serve as a reference book or as a textbook for a graduate level courses on the subject. It concentrates on the fundamentals and adopts a rigorous mathematical approach of the subject. Many examples and end of chapter problems are included. Finally, a companion Instructor's Manual is also available through the publisher for professors who adopt this book as a text. The Instructor's Manual includes many other problems not listed in the text and their solutions.

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1. All MATLAB® functions and programs provided in this book were developed using MATLAB R2007b with the Signal Processing Toolbox, on a PC with Windows XP Professional operating system.
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Radar Signal Analysis and Processing Using MATLAB[®] is written so that it can be used as a reference book or as a textbook for two graduate level courses with emphasis on signals and signal processing. Instructors using this book as a text may choose the following chapter breakdown for their curriculum. **Chapters 1** through **Chapter 7** can be used for the first course, while **Chapters 8** through **11** may be used for the second course. Chapter 11 (Target Tracking), Chapter 12 (Synthetic Aperture Radar), and Chapter 13 (Radar Cross Section) from my other book *Radar Systems Analysis and Design Using MATLAB*[®] may also be used to supplement both courses.

Radar Signal Analysis and Processing Using MATLAB[®] introduces numerous programs and functions of MATLAB using version R2007a. All MATLAB programs and functions provided in this book can be downloaded from the CRC Press Website. For this purpose and using your favorite Internet browser type in www.crcpress.com and hit return. Once you reach the main CRC Press home page, scroll down to the link called “*Electronic Products*” and double click on “*Downloads & Updates,*” then follow the instructions on the screen.

Chapter 1 of this book presents an overview of radar systems operation and design. The approach is to derive the radar range equation and analyze the different radar parameters in the context of this radar equation. The surveillance radar equation is derived. Special topics that affect radar signal processing are presented and analyzed in the context of the radar equation. This includes the effects of system noise, wave propagation, jamming, and target Radar Cross Section (RCS). **Chapter 2** introduces a top level review of elements of signal theory that are relevant to radar detection and radar signal processing. It is assumed that the reader has sufficient and adequate background in signals and systems as well as in the Fourier transform and its associated properties.

In **Chapter 3** a review of random variables and processes is presented. Instructors using this text may assume that students have already acquired the necessary background as a prerequisite to this course and, thus, may elect to omit this chapter from their syllabus, except for Section 3.6. **Chapter 4** is focused on the matched filter. It presents the unique characteristic of the matched filter and develops a general formula for the output of the matched filter that is valid for any waveform. **Chapters 5** and **6** analyze the output of the matched filter in the context of the ambiguity function. In Chapter 5 several analog waveforms are analyzed; this includes the single unmodulated pulse, the Linear Frequency Modulation (LFM) pulse, unmodulated pulse train, LFM pulse train, stepped frequency waveforms, and nonlinear FM waveforms. Chapter 6 is concerned with discrete coded waveforms. In this chapter, unmodulated pulse-train codes are analyzed as well as binary codes, polyphase codes, and frequency codes.

Chapter 7 introduces the subject of radar target detection and pulse integration. Swerling models are analyzed in the context of noncoherent integration and the square law detector. The topic of Constant False Alarm Rate (CFAR) is also presented in detail. Chapter 8 introduces the most common techniques in radar signal processing. The matched filter receiver as well as the stretch processor receiver are analyzed. Chapter 9 is concerned with radar clutter. Comprehensive analysis of the subject of clutter is introduced, including the Moving Target Indicator (MTI). Chapter 10 is primarily concerned with radar Doppler processing. Both continuous wave and pulsed radars are considered. Pulse Doppler radars are introduced and analyzed. Chapter 11 is focused on adaptive array processing. For this purpose, a top level overview of phased array antennas is first introduced followed by beamforming and the most common techniques in adaptive array processing.

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February 2008