Chapter 10 Doppler Processing

In this chapter Doppler processing is analyzed in the context of continuous wave (CW) radars and pulsed Doppler radars. Continuous wave radars utilize CW waveforms, which may be considered to be a pure sinewave of the form $\cos 2\pi f_0 t$. Spectra of the radar echo from stationary targets and clutter will be concentrated at f_0. The center frequency for the echoes from moving targets will be shifted by f_d, the Doppler frequency. Thus, by measuring this frequency difference CW, radars can very accurately extract target radial velocity. Because of the continuous nature of CW emission, range measurement is not possible without some modifications to the radar operations and waveforms, which will be discussed later.

Alternatively, pulsed radars utilize a stream of pulses with a specific PRI (or PRF) to generate what is known as range-Doppler maps. Each map is divided into resolution cells. The dimensions of these resolution cells are range resolution along the time axis and Doppler resolution along the frequency axis.

10.1. CW Radar Functional Block Diagram

In order to avoid interruption of the continuous radar energy emission, two antennas are used in CW radars, one for transmission and one for reception. Figure 10.1 shows a simplified CW radar block diagram. The appropriate values of the signal frequency at different locations are noted on the diagram. The individual Narrow Band Filters (NBF) must be as narrow as possible in bandwidth in order to allow accurate Doppler measurements and minimize the amount of noise power. In theory, the operating bandwidth of a CW radar is infinitesimal (since it corresponds to an infinite duration continuous sinewave). However, systems with infinitesimal bandwidths cannot physically exist, and thus, the bandwidth of CW radars is assumed to correspond to that of a gated CW waveform.

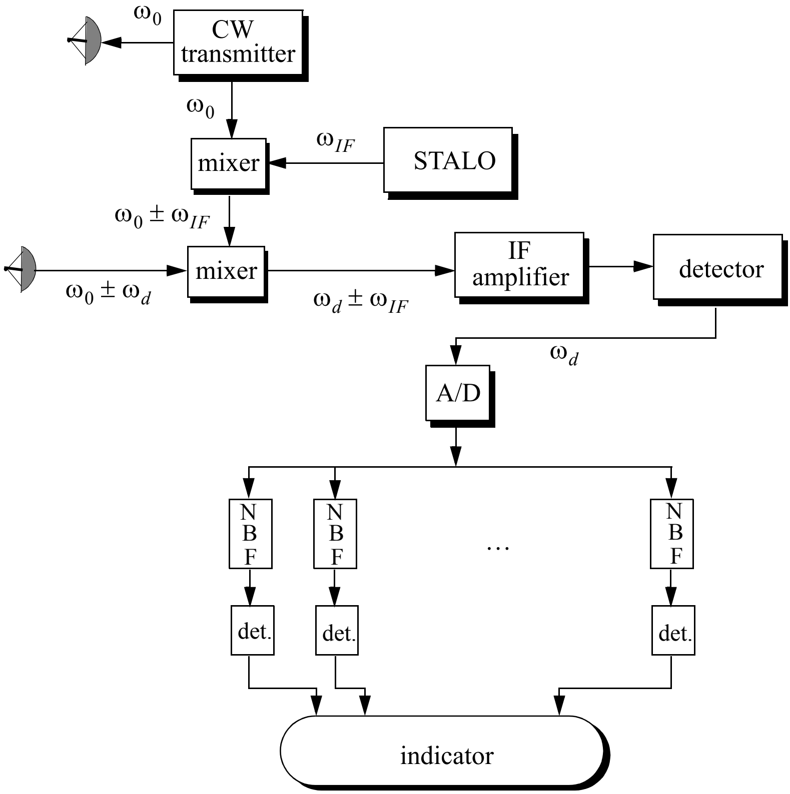


Figure 10.1. CW radar block diagram.

The NBF bank (Doppler filter bank) can be implemented using a Fast Fourier Transform (FFT). If the Doppler filter bank is implemented using an FFT of size N_{FFT}, and if the individual NBF bandwidth (FFT bin) is Δf, then the effective radar Doppler bandwidth is $N_{FFT}\Delta f/2$. The reason for the one-half factor is to account for both negative and positive Doppler shifts. The frequency resolution Δf is proportional to the inverse of the integration time.

Since range is computed from the radar echoes by measuring a two-way time delay, single frequency CW radars cannot measure target range. In order for CW radars to be able to measure target range, the transmit and receive waveforms must have some sort of timing marks. By comparing the timing marks at transmit and receive, CW radars can extract target range.

The timing mark can be implemented by modulating the transmit waveform, and one commonly used technique is Linear Frequency Modulation (LFM). Before we discuss LFM signals, we will first introduce the CW radar equation and briefly address the general Frequency Modulated (FM) waveforms using sinusoidal modulating signals.

10.1.1. CW Radar Equation

As indicated by Fig. 10.1, the CW radar receiver declares detection at the output of a particular Doppler bin if that output value passes the detection threshold within the detector box. Since the NBF bank is implemented by an FFT, only finite length data sets can be processed at a time. The length of such blocks is normally referred to as the dwell interval, integration time, or coherent processing interval. The dwell interval determines the frequency resolution or the bandwidth of the individual NBFs. More precisely,

$$\Delta f = 1/T_{Dwell} \tag{10.1}$$

T_{Dwell} is the dwell interval. Therefore, once the maximum resolvable frequency by the NBF bank is chosen the size of the NBF bank is computed as

$$N_{FFT} = 2B/\Delta f \tag{10.2}$$

B is the maximum resolvable frequency by the FFT. The factor 2 is needed to account for both positive and negative Doppler shifts. It follows that

$$T_{Dwell} = N_{FFT}/2B \tag{10.3}$$

The CW radar equation can now be derived. Consider the radar equation developed in Chapter 1. That is

$$SNR = \frac{P_{av}TG^2\lambda^2\sigma}{(4\pi)^3R^4kT_oFL} \tag{10.4}$$

where $P_{av} = (\tau/T)P_t$, τ/T, and P_t is the peak transmitted power. In CW radars the average transmitted power over the dwell interval P_{CW}, and T must be replaced by T_{Dwell}. Thus, the CW radar equation can be written as

$$SNR = \frac{P_{CW}T_{Dwell}G_tG_r\lambda^2\sigma}{(4\pi)^3R^4kT_oFLL_{win}} \tag{10.5}$$

where G_t and G_r are the transmit and receive antenna gains, respectively. The factor L_{win} is a loss term associated with the type of window (weighting) used in computing the FFT.

10.1.2. Linear Frequency Modulated CW Radar

CW radars may use LFM waveforms so that both range and Doppler information can be measured. In practical CW radars, the LFM waveform cannot be continually changed in one direction, and thus, periodicity in the modulation is normally utilized. Figure 10.2 shows a sketch of a triangular LFM waveform. The modulation does not need to be triangular; it may be sinusoidal, saw-tooth, or some other form. The dashed line in Fig. 10.2 represents the return waveform from a stationary target at range R. The beat frequency f_b is also sketched in Fig. 10.2. It is defined as the difference (due to heterodyning) between the transmitted and received signals. The time delay Δt is a measure of target range; that is,

$$\Delta t = \frac{2R}{c} \tag{10.6}$$

In practice, the modulating frequency f_m is selected such that

$$f_m = \frac{1}{2t_0} \tag{10.7}$$

The rate of frequency change, $\dot{f}$, is

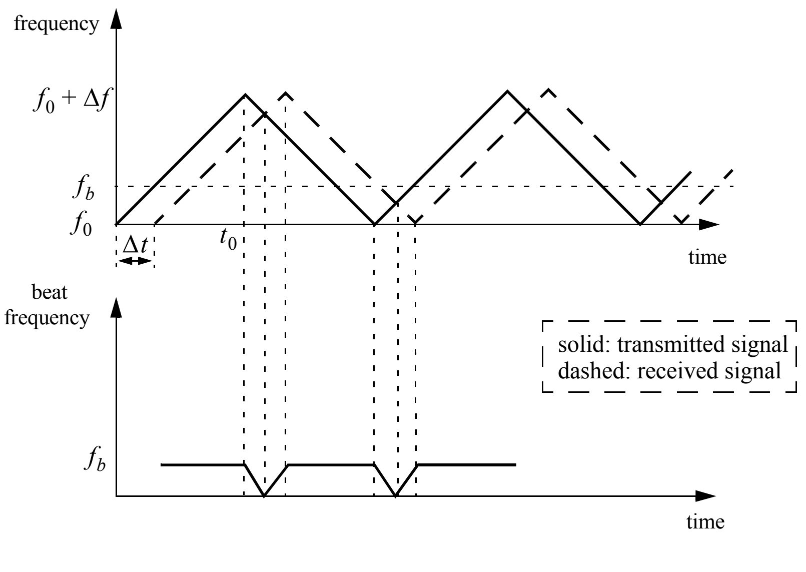


Figure 10.2. Transmitted and received triangular LFM signals and beat frequency for stationary target.

$$\dot{f} = \frac{\Delta f}{t_0} = \frac{\Delta f}{(1/2f_m)} = 2f_m \Delta f \tag{10.8}$$

where Δf is the peak frequency deviation. The beat frequency f_b is given by

$$f_b = \Delta t \dot{f} = \frac{2R}{c} \dot{f} \tag{10.9}$$

Equation (10.9) can be rearranged as

$$\dot{f} = \frac{c}{2R} f_b \tag{10.10}$$

Equating Eqs. (10.8) and (10.10) and solving for f_b yield

$$f_b = \frac{4Rf_m \Delta f}{c} \tag{10.11}$$

Now consider the case when Doppler is present (i.e., nonstationary target). The corresponding triangular LFM transmitted and received waveforms are sketched in Fig. 10.3, along with the corresponding beat frequency. As previously noted the beat frequency is defined as

$$f_b = f_{received} - f_{transmitted} \tag{10.12}$$

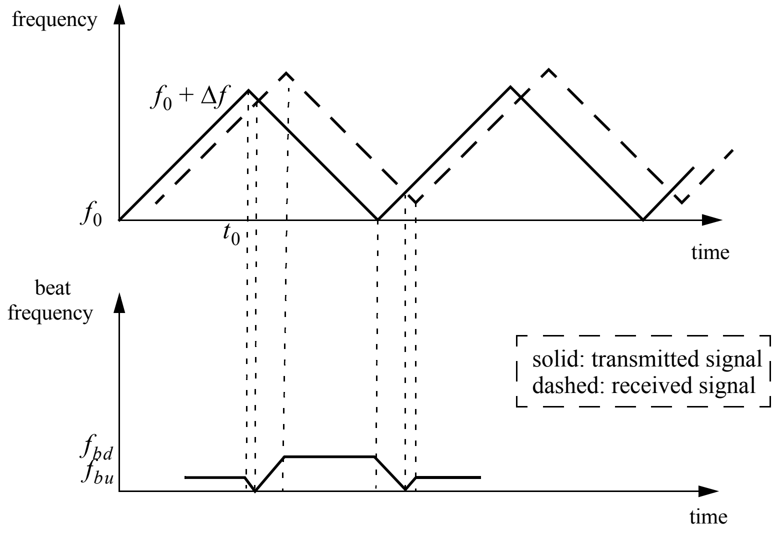


Figure 10.3. Transmitted and received LFM signals and beat frequency, for a moving target.

When the target is not stationary the received signal will contain a Doppler shift term in addition to the frequency shift due to the time delay Δt. In this case, the Doppler shift term subtracts from the beat frequency during the positive portion of the slope. Alternatively, the two terms add up during the negative portion of the slope. Denote the beat frequency during the positive (up) and negative (down) portions of the slope, respectively, as f_{bu} and f_{bd}. It follows that

$$f_{bu} = \frac{2R}{c}\dot{f} - \frac{2\dot{R}}{\lambda} \tag{10.13}$$

where $\dot{R}$ is the range rate or the target radial velocity as seen by the radar. The first term of the right-hand side of Eq. (10.13) is due to the range delay defined by Eq. (10.6), while the second term is due to the target Doppler. Similarly,

$$f_{bd} = \frac{2R}{c}\dot{f} + \frac{2\dot{R}}{\lambda} \tag{10.14}$$

Range is computed by adding Eq. (10.12) and Eq. (10.14). More precisely,

$$R = \frac{c}{4\dot{f}}(f_{bu} + f_{bd}) \tag{10.15}$$

The range rate is computed by subtracting Eq. (10.14) from Eq. (10.13),

$$\dot{R} = \frac{\lambda}{4}(f_{bd} - f_{bu}) \tag{10.16}$$

As indicated by Eq. (10.15) and Eq. (10.16), CW radars utilizing triangular LFM can extract both range and range rate information. In practice, the maximum time delay Δt_{max} is normally selected as

$$\Delta t_{max} = 0.1 t_0 \tag{10.17}$$

Thus, the maximum range is given by

$$R_{max} = \frac{0.1 c t_0}{2} = \frac{0.1 c}{4 f_m} \tag{10.18}$$

and the maximum unambiguous range will correspond to a shift equal to $2t_0$.

10.1.3. Multiple Frequency CW Radar

Continuous wave radars do not have to use LFM waveforms in order to obtain good range measurements. Multiple frequency schemes allow CW radars to compute very adequate range measurements without using frequency modulation. In order to illustrate this concept, first consider a CW radar with the following waveform

$$x(t) = A\sin 2\pi f_0 t \tag{10.19}$$

The received signal from a target at range R is

$$x_r(t) = A_r \sin(2\pi f_0 t - \varphi) \tag{10.20}$$

where the phase φ is equal to

$$\varphi = 2\pi f_0 (2R/c) \tag{10.21}$$

Solving for R we obtain

$$R = \frac{c\varphi}{4\pi f_0} = \frac{\lambda}{4\pi}\varphi \tag{10.22}$$

Clearly, the maximum unambiguous range occurs when φ is maximum, i.e., $\varphi = 2\pi$. Therefore, even for relatively large radar wavelengths, R is limited to impractical small values. Next, consider a radar with two CW signals, denoted by $s_1(t)$ and $s_2(t)$. More precisely,

$$x_1(t) = A_1 \sin 2\pi f_1 t \tag{10.23}$$

$$x_2(t) = A_2 \sin 2\pi f_2 t \tag{10.24}$$

The received signals from a moving target are

$$x_{1r}(t) = A_{r1} \sin(2\pi f_1 t - \varphi_1) \tag{10.25}$$

and

$$x_{2r}(t) = A_{r2} \sin(2\pi f_2 t - \varphi_2) \tag{10.26}$$

where $\varphi_1 = (4\pi f_1 R)/c$ and $\varphi_2 = (4\pi f_2 R)/c$. After heterodyning (mixing) with the carrier frequency, the phase difference between the two received signals is

$$\varphi_2 - \varphi_1 = \Delta\varphi = \frac{4\pi R}{c}(f_2 - f_1) = \frac{4\pi R}{c}\Delta f \tag{10.27}$$

Again R is maximum when $\Delta\varphi = 2\pi$; it follows that the maximum unambiguous range is now

$$R = c/2\Delta f \tag{10.28}$$

and since $\Delta f \ll c$, the range computed by Eq. (10.28) is much greater than that computed by Eq. (10.22).

10.2. Pulsed Radars

Pulsed radars transmit and receive a train of modulated pulses. Range is extracted from the two-way time delay between a transmitted and received pulse. Doppler measurements can be made in two ways. If accurate range measurements are available between consecutive pulses, then Doppler frequency can be extracted from the range rate $\dot{R} = \Delta R / \Delta t$. This approach works fine as long as the range is not changing drastically over the interval Δt. Otherwise, pulsed radars utilize a Doppler filter bank.

Pulsed radar waveforms can be completely defined by the following: (1) carrier frequency which may vary depending on the design requirements and radar mission; (2) pulse width, which is closely related to the bandwidth and defines the range resolution; (3) modulation; and finally (4) the pulse repetition frequency. Different modulation techniques are usually utilized to enhance the radar performance, or to add more capabilities to the radar that otherwise would not have been possible. The PRF must be chosen to avoid Doppler and range ambiguities as well as maximize the average transmitted power.

Radar systems employ low, medium, and high PRF schemes. Low PRF waveforms can provide accurate, long, unambiguous range measurements, but exert severe Doppler ambiguities. Medium PRF waveforms must resolve both range and Doppler ambiguities; however, they provide adequate average transmitted power as compared to low PRFs. High PRF waveforms can provide superior average transmitted power and excellent clutter rejection capabilities. Alternatively, high PRF waveforms are extremely ambiguous in range. Radar systems utilizing high PRFs are often called Pulsed Doppler Radars (PDR). Range and Doppler ambiguities for different PRFs are summarized in Table 10.1.

Distinction of a certain PRF as low, medium, or high PRF is almost arbitrary and depends on the radar mode of operations. For example, a $3KHz$ PRF is considered low if the maximum detection range is less than $30Km$. However, the same PRF would be considered medium if the maximum detection range is well beyond $30Km$.

Radars can utilize constant and varying (agile) PRFs. For example, Moving Target Indicator (MTI) radars use PRF agility to avoid blind speeds, as discussed in Chapter 9. This kind of agility is known as PRF staggering. PRF agility is also used to avoid range and Doppler ambiguities, as will be explained in the next three sections. Additionally, PRF agility is also used to prevent jammers from locking onto the radar's PRF. These two last forms of PRF agility are sometimes referred to as PRF jitter.

Figure 10.4 shows a simplified pulsed radar block diagram. The range gates can be implemented as filters that open and close at time intervals that correspond to the detection range. The width of such an interval corresponds to the desired range resolution. The radar receiver is often implemented as a series of contiguous (in time) range gates, where the width of each gate is achieved through pulse compression. The clutter rejection can be implemented using MTI or other forms of clutter rejection techniques. The NBF bank is normally implemented using an FFT, where bandwidth of the individual filters corresponds to the FFT frequency resolution.

TABLE 10.1. PRF ambiguities.

PRF	Range Ambiguous	Doppler Ambiguous
Low PRF	No	Yes
Medium PRF	Yes	Yes
High PRF	Yes	No

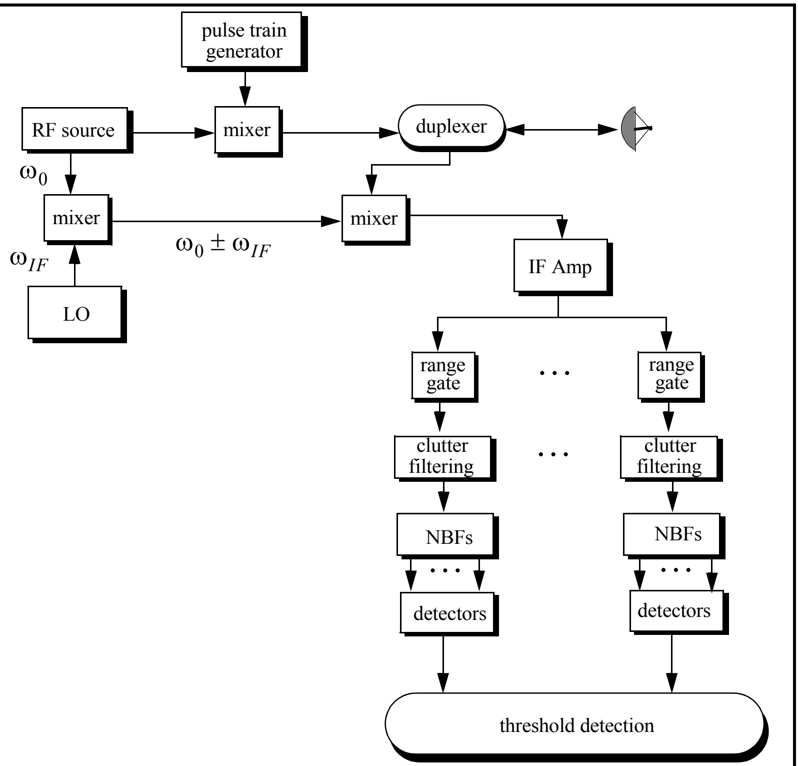


Figure 10.4. Pulsed radar block diagram.

10.2.1. Pulse Doppler Radars

In ground based radars, the amount of clutter in the radar receiver depends heavily on the radar-to-target geometry. The amount clutter is considerably higher when the radar beam has to face toward the ground. Furthermore, radars employing high PRFs have to deal with an increased amount of clutter due to folding in range. Clutter introduces additional difficulties for airborne radars when detecting ground targets and other targets flying at low altitudes. This is illustrated in Fig. 10.5. Returns from ground clutter emanate from ranges equal to the radar altitude to those which exceed the slant range along the mainbeam, with considerable clutter returns in the sidelobes and mainbeam. The presence of such large amounts of clutter interferes with radar detection capabilities and makes it extremely difficult to detect targets in the look-down mode. This difficulty in detecting ground or low altitude targets has led to the development of pulse Doppler radars where other targets, kinematics such as Doppler effects are exploited to enhance detection.

Pulse Doppler radars utilize high PRFs to increases the average transmitted power and rely on target's Doppler frequency for detection. The increase in the average transmitted power leads to an improved SNR which helps the detection process. However, using high PRFs compromise the radar's ability to detect long range target because of range ambiguities associated with high PRF applications.

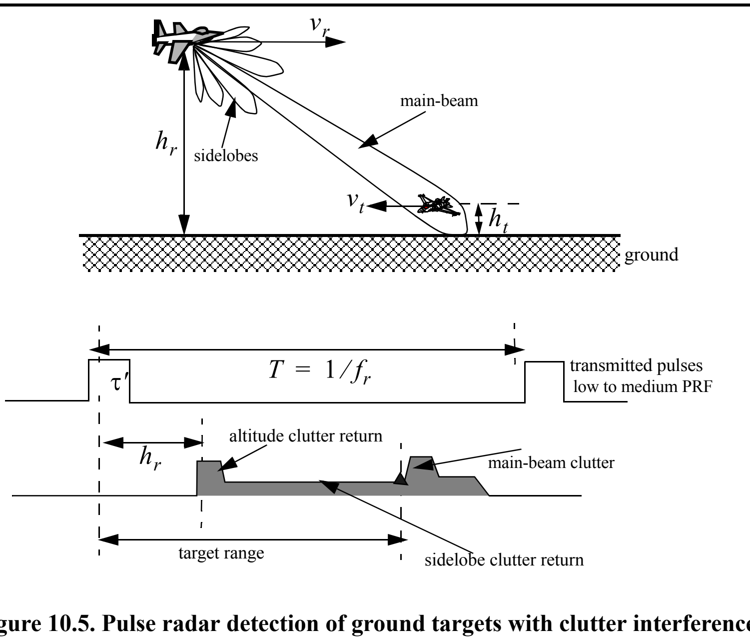


Figure 10.5. Pulse radar detection of ground targets with clutter interference.

As was explained in Chapter 9, pulse Doppler radars (or high PRF radars) have to deal with the additional increase in clutter power due to clutter folding. This has led to the development of a special class of airborne MTI filters, often referred to as AMTI. Techniques such as using specialized Doppler filters to reject clutter are very effective and are often employed by pulse Doppler radars. Pulse Doppler radars can measure target Doppler frequency (or its range rate) fairly accurately and use the fact that ground clutter typically possesses limited Doppler shift when compared with moving targets to separate the two returns. This is illustrated in Fig. 10.6. Clutter filtering (i.e., AMTI) is used to remove both main-beam and altitude clutter returns, and fast moving target detection is done effectively by exploiting its Doppler frequency. In many modern pulse Doppler radars the limiting factor in detecting slow moving targets is not clutter but rather another source of noise referred to as phase noise generated from the receiver local oscillator instabilities.

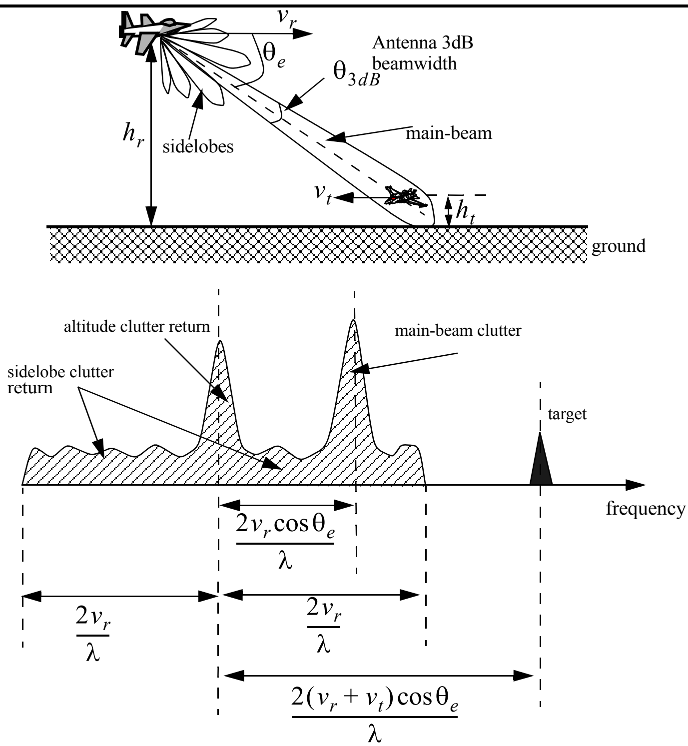


Figure 10.6. Cartoon illustrating frequency characteristics of pulse Doppler radar echoes.

10.2.2. High PRF Radar Equation

Consider a high PRF radar that uses a periodic train of very short pulses. The pulse width is τ and the period is T. This pulse train can be represented using an exponential Fourier series. The central power spectrum line (DC component) for this series contains most of the signal's power. Its value is $(\tau/T)^2$, and it is equal to the square of the transmit duty factor. Thus, the single pulse radar equation for a high PRF radar (in terms of the DC spectral power line) is

$$SNR = \frac{P_t G^2 \lambda^2 \sigma d_t^2}{(4\pi)^3 R^4 k T_o B F L d_r} \tag{10.29}$$

where, in this case, one can no longer ignore the receive duty factor since its value is comparable to the transmit duty factor. In fact, $d_r \approx d_t = \tau f_r$. Additionally, the operating radar bandwidth is now matched to the radar integration time (time on target), $B = 1/T_i$. It follows that

$$SNR = \frac{P_t \tau f_r T_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_o F L} \tag{10.30}$$

and finally,

$$SNR = \frac{P_{av} T_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_o F L} \tag{10.31}$$

where P_{av} was substituted for $P_t \tau f_r$. Note that the product $P_{av} T_i$ is a "kind of energy" product, which indicates that high PRF radars can enhance detection performance by using relatively low power and longer integration time.

Example:

Compute the single pulse SNR for a high PRF radar with the following parameters: peak power $P_t = 100KW$, antenna gain $G = 20dB$, operating frequency $f_0 = 5.6GHz$, losses $L = 8dB$, noise figure $F = 5dB$, dwell interval $T_i = 2s$, duty factor $d_t = 0.3$. The range of interest is $R = 50Km$. Assume target RCS $\sigma = 0.01m^2$.

Solution:

From Eq. (10.31) we have

$$(SNR)_{dB} = (P_{av} + G^2 + \lambda^2 + \sigma + T_i - (4\pi)^3 - R^4 - kT_o - F - L)_{dB}$$

The following table gives all parameters in dB:

P_{av}	λ^2	T_i	kT_0	$(4\pi)^3$	R^4	σ
44.771	–25.421	3.01	–23.977	32.976	187.959	–20

$$(SNR)_{dB} = 44.771 + 40 - 25.421 - 20 + 3.01 - 32.976 + 203.977 - 187.959 - 5 - 8 = 12.4dB$$

The same answer can be obtained by using the function "hprf_req.m" (see Section 10.3.2) with the following syntax:

hprf_req (100e3, 2, 20, 5.6e9, 0.01, .3, 50e3, 5, 8)

10.2.3. Pulse Doppler Radar Signal Processing

The main idea behind pulse Doppler radar signal processing is to divide the footprint (the intersection of the antenna 3dB beamwidth with the ground) into resolution cells that constitute a range Doppler map, MAP. The sides of this map are range and Doppler, as illustrated in Fig. 10.7. Fine range resolution, ΔR, is accomplished in real time by utilizing range gating and pulse compression. Frequency (Doppler) resolution is obtained from the coherent processing interval.

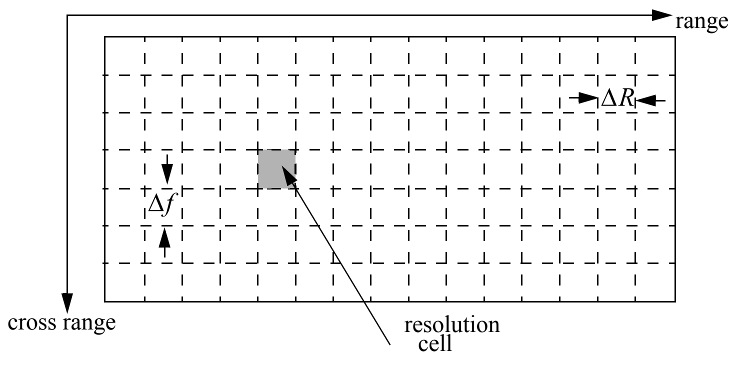


Figure 10.7. Range Doppler map.

To further illustrate this concept, consider the case where N_a is the number of azimuth (Doppler) cells, and N_r is the number of range bins. Hence, the MAP is of size $N_a \times N_r$, where the columns refer to range bins and the rows refer to azimuth cells. For each transmitted pulse within the dwell, the echoes from consecutive range bins are recorded sequentially in the first row of MAP. Once the first row is completely filled (i.e., returns from all range bins have been received), all data (in all rows) are shifted downward one row before the next pulse is transmitted. Thus, one row of MAP is generated for every transmitted pulse. Consequently, for the current observation interval, returns from the first transmitted pulse will be located in the bottom row of MAP, and returns from the last transmitted pulse will be in the top row of MAP.

Referring to Fig. 10.4, fine range resolution is achieved using the matched filter. Clutter rejection (filtering) is performed on each range bin (i.e, rows in the MAP). Then all samples from one dwell within each range bin are processed using an FFT to resolve targets in Doppler. It follows that a peak in a given resolution cell corresponds to a specific target detection at that range and Doppler frequency. Selection of the proper size FFT and its associated parameters were discussed in Chapter 2.

10.2.4. Resolving Range Ambiguities in Pulse Doppler Radars

Pulse Doppler radars exhibit serve range ambiguities because they use high PRF pulse streams. In order to resolve these ambiguities, pulse Doppler radars utilize multiple high PRFs (PRF staggering) within each processing interval (dwell). For this purpose, consider a pulse Doppler radar that uses two PRFs, f_{r1} and f_{r2}, on transmit to resolve range ambiguity, as shown in Fig. 10.8. Denote R_{u1} and R_{u2} as the unambiguous ranges for the two PRFs, respectively. Normally, these unambiguous ranges are relatively small and are short of the desired radar unambiguous range R_u (where $R_u \gg R_{u1}, R_{u2}$). Denote the radar desired PRF that corresponds to R_u as f_{rd}.

The choice of f_{r1} and f_{r2} is such that they are relatively prime with respect to one another. One choice is to select $f_{r1} = Nf_{rd}$ and $f_{r2} = (N+1)f_{rd}$ for some integer N. Within one period of the desired PRI ($T_d = 1/f_{rd}$) the two PRFs f_{r1} and f_{r2} coincide only at one location, which is the true unambiguous target position. The time delay T_d establishes the desired unambiguous range. The time delays t_1 and t_2 correspond to the time between the transmit of a pulse on each PRF and receipt of a target return due to the same pulse.

Let M_1 be the number of PRF1 intervals between transmit of a pulse and receipt of the true target return. The quantity M_2 is similar to M_1 except it is for PRF2. It follows that over the interval 0 to T_d, the only possible results are $M_1 = M_2 = M$ or $M_1 + 1 = M_2$. The radar needs only to measure t_1 and t_2. First, consider the case when $t_1 < t_2$. In this case,

$$t_1 + \frac{M}{f_{r1}} = t_2 + \frac{M}{f_{r2}} \tag{10.32}$$

for which we get

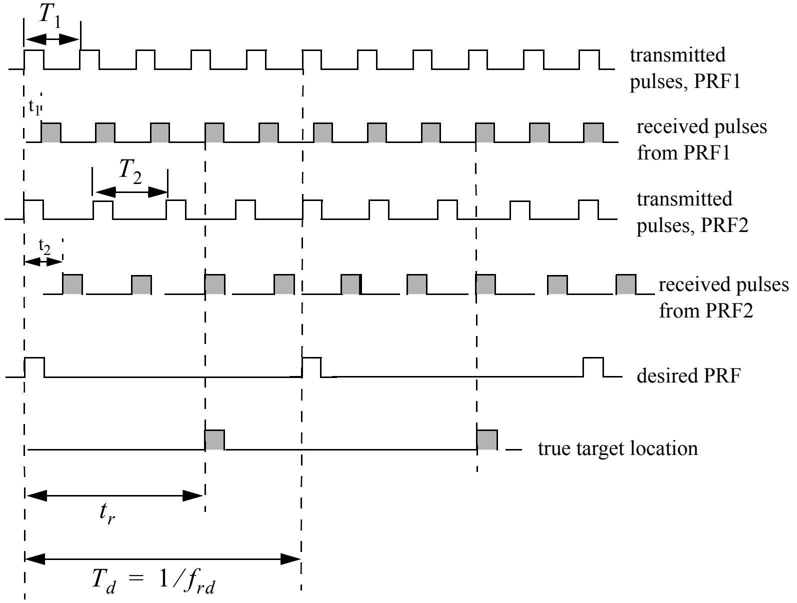


Figure 10.8. Resolving range ambiguity.

$$M = \frac{t_2 - t_1}{T_1 - T_2} \tag{10.33}$$

where $T_1 = 1/f_{r1}$ and $T_2 = 1/f_{r2}$. It follows that the round-trip time to the true target location is

$$t_r = MT_1 + t_1$$
$$t_r = MT_2 + t_2 \tag{10.34}$$

and the true target range is

$$R = ct_r/2 \tag{10.35}$$

Now, if $t_1 > t_2$, then

$$t_1 + \frac{M}{f_{r1}} = t_2 + \frac{M+1}{f_{r2}} \tag{10.36}$$

Solving for M we get

$$M = \frac{(t_2 - t_1) + T_2}{T_1 - T_2} \tag{10.37}$$

and the round-trip time to the true target location is

$$t_{r1} = MT_1 + t_1 \tag{10.38}$$

and in this case, the true target range is

$$R = \frac{ct_{r1}}{2} \tag{10.39}$$

Finally, if $t_1 = t_2$, then the target is in the first ambiguity. It follows that

$$t_{r2} = t_1 = t_2 \tag{10.40}$$

and

$$R = ct_{r2}/2 \tag{10.41}$$

Since a pulse cannot be received while the following pulse is being transmitted, these times correspond to blind ranges. This problem can be resolved by using a third PRF. In this case, once an integer N is selected, then in order to guarantee that the three PRFs are relatively prime with respect to one another, we may choose $f_{r1} = N(N+1)f_{rd}$, $f_{r2} = N(N+2)f_{rd}$, and $f_{r3} = (N+1)(N+2)f_{rd}$.

10.2.5. Resolving Doppler Ambiguity

In the case where the pulse Doppler radar is utilizing medium PRFs, it will be ambiguous in both range and Doppler. Resolving range ambiguities was discussed in the previous section. In this section Doppler ambiguity is addressed. Remember that the line spectrum of a train of pulses has $\sin x/x$ envelope (see Chapter 2), and the line spectra are separated by the PRF, f_r, as illustrated in Fig. 10.9. The Doppler filter bank is capable of resolving target Doppler as long as the anticipated Doppler shift is less than one half the bandwidth of the individual filters (i.e., one half the width of an FFT bin). Thus, pulsed radars are designed such that

$$f_r = 2f_{dmax} = (2v_{rmax})/\lambda \tag{10.42}$$

where f_{dmax} is the maximum anticipated target Doppler frequency, v_{rmax} is the maximum anticipated target radial velocity, and λ is the radar wavelength.

If the Doppler frequency of the target is high enough to make an adjacent spectral line move inside the Doppler band of interest, the radar can be Doppler ambiguous. Therefore, in order to avoid Doppler ambiguities, radar systems require high PRF rates when detecting high speed targets. When a long-range radar is required to detect a high speed target, it may not be possible to be both range and Doppler unambiguous. This problem can be resolved by using multiple PRFs. Multiple PRF schemes can be incorporated sequentially within each dwell interval (scan or integration frame) or the radar can use a single PRF in one scan and resolve ambiguity in the next. The latter technique, however, may have problems due to changing target dynamics from one scan to the next.

The Doppler ambiguity problem is analogous to that of range ambiguity. Therefore, the same methodology can be used to resolve Doppler ambiguity. In this case, we measure the Doppler frequencies f_{d1} and f_{d2} instead of t_1 and t_2.

If $f_{d1} > f_{d2}$, then we have

$$M = \frac{(f_{d2} - f_{d1}) + f_{r2}}{f_{r1} - f_{r2}} \tag{10.43}$$

And if $f_{d1} < f_{d2}$,

$$M = \frac{f_{d2} - f_{d1}}{f_{r1} - f_{r2}} \tag{10.44}$$

and the true Doppler is

$$f_d = Mf_{r1} + f_{d1} \qquad ; \; f_d = Mf_{r2} + f_{d2} \tag{10.45}$$

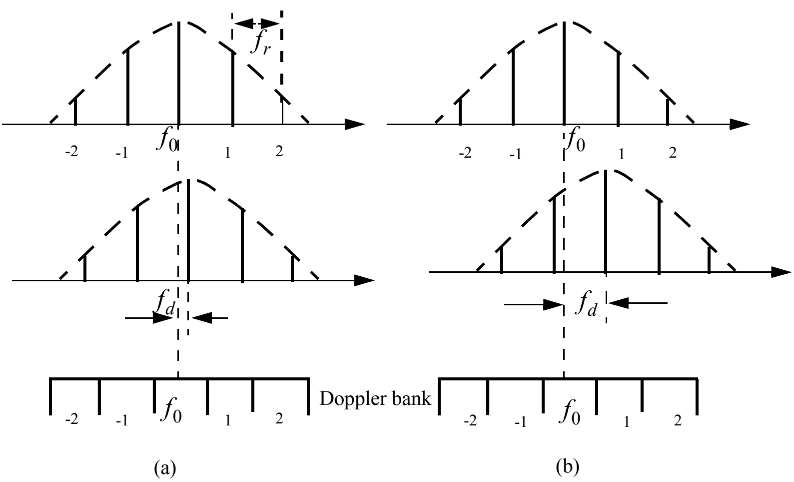


Figure 10.9. Spectra of transmitted and received waveforms, and Doppler bank. (a) Doppler is resolved. (b) Spectral lines have moved into the next Doppler filter. This results in an ambiguous Doppler measurement.

Finally, if $f_{d1} = f_{d2}$, then

$$f_d = f_{d1} = f_{d2} \tag{10.46}$$

Again, blind Dopplers can occur, which can be resolved using a third PRF.

Example:

A certain radar uses two PRFs to resolve range ambiguities. The desired unambiguous range is $R_u = 100Km$. Choose $N = 59$. Compute f_{r1}, f_{r2}, R_{u1}, and R_{u2}.

Solution:

First let us compute the desired PRF, f_{rd}

$$f_{rd} = \frac{c}{2R_u} = \frac{3 \times 10^8}{200 \times 10^3} = 1.5KHz$$

It follows that

$$f_{r1} = Nf_{rd} = (59)(1500) = 88.5KHz$$

$$f_{r2} = (N+1)f_{rd} = (59+1)(1500) = 90KHz$$

$$R_{u1} = \frac{c}{2f_{r1}} = \frac{3 \times 10^8}{2 \times 88.5 \times 10^3} = 1.695Km$$

$$R_{u2} = \frac{c}{2f_{r2}} = \frac{3 \times 10^8}{2 \times 90 \times 10^3} = 1.667Km.$$

Example:

Consider a radar with three PRFs; $f_{r1} = 15KHz$, $f_{r2} = 18KHz$, and $f_{r3} = 21KHz$. Assume $f_0 = 9GHz$. Calculate the frequency position of each PRF for a target whose velocity is $550m/s$. Calculate f_d (Doppler frequency) for another target appearing at $8KHz$, $2KHz$, and $17KHz$ for each PRF.

Solution:

The Doppler frequency is

$$f_d = 2\frac{vf_0}{c} = \frac{2 \times 550 \times 9 \times 10^9}{3 \times 10^8} = 33KHz$$

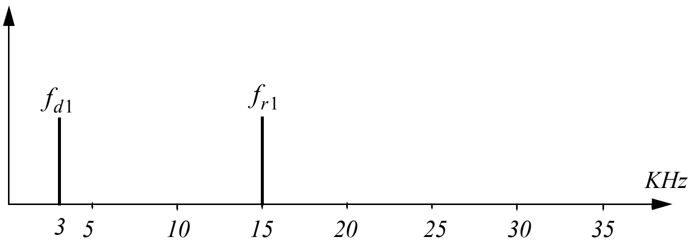
Then by using Eq. (10.42) $n_i f_{ri} + f_{di} = f_d$ where $i = 1, 2, 3$, we can write

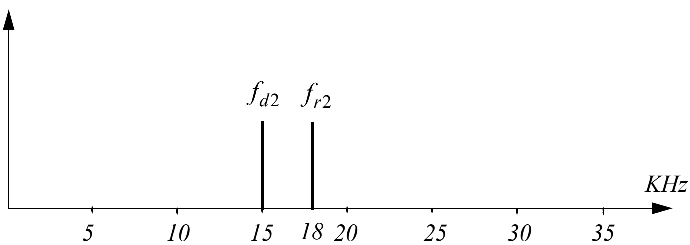
$$n_1 f_{r1} + f_{d1} = 15n_1 + f_{d1} = 33$$

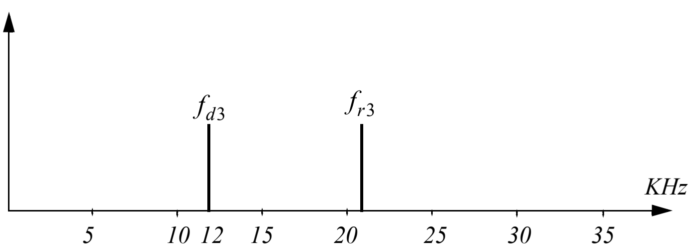
$$n_2 f_{r2} + f_{d2} = 18 n_2 + f_{d2} = 33$$

$$n_3 f_{r3} + f_{d3} = 21 n_3 + f_{d3} = 33$$

We will show here how to compute n_1*, and leave the computations of* n_2 *and* n_3 *to the reader. First, if we choose* $n_1 = 0$*, that means* $f_{d1} = 33KHz$*, which cannot be true since* f_{d1} *cannot be greater than* f_{r1}*. Choosing* $n_1 = 1$ *is also invalid since* $f_{d1} = 18KHz$ *cannot be true either. Finally, if we choose* $n_1 = 2$ *we get* $f_{d1} = 3KHz$*, which is an acceptable value. It follows that the minimum* n_1, n_2, n_3 *that may satisfy the above three relations are* $n_1 = 2$*,* $n_2 = 1$*, and* $n_3 = 1$*. Thus, the apparent Doppler frequencies are* $f_{d1} = 3KHz$*,* $f_{d2} = 15KHz$*, and* $f_{d3} = 12KHz$*, as seen below.*







Now for the second part of the problem. Again by using Eq. (10.61) we have

$$n_1 f_{r1} + f_{d1} = f_d = 15n_1 + 8$$

$$n_2 f_{r2} + f_{d2} = f_d = 18n_2 + 2$$

$$n_3 f_{r3} + f_{d3} = f_d = 21n_3 + 17$$

We can now solve for the smallest integers n_1, n_2, n_3 *that satisfy the above three relations. See the table below.*

n	**0**	**1**	**2**	**3**	**4**
f_d from f_{r1}	8	23	<u>38</u>	53	68
f_d from f_{r2}	2	20	<u>38</u>	56	
f_d from f_{r3}	17	<u>38</u>	39		

Thus, $n_1 = 2 = n_2$*, and* $n_3 = 1$*, and the true target Doppler is* $f_d = 38KHz$*. It follows that*

$$v_r = 38000 \times \frac{0.0333}{2} = 632.7\frac{m}{\text{sec}}$$

10.3. MATLAB Programs and Routines

10.3.1. MATLAB Program "range_calc.m"

The program *"range_calc.m"* solves the radar range equation of the form

$$R = \left(\frac{P_t \tau f_r T_i G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_0 F L (SNR)_o}\right)^{\frac{1}{4}} \qquad \textbf{(10.47)}$$

where P_t is peak transmitted power, τ is pulse width, f_r is PRF, G_t and G_r are respectively the transmitting and receiving antenna gain, λ is wavelength, σ is target cross section, k is Boltzman's constant, T_0 is 290 kelvin, F is system noise figure, L is total system losses, and $(SNR)_o$ is the minimum SNR required for detection.

One can choose either CW or pulsed radars. In the case of CW radars, the terms $P_t \tau f_r$ is replaced within the code by the average CW power P_{CW}. Additionally, the term T_i refers to the dwell interval. Alternatively, in the case of pulse radars T_i denotes the time on target. The plot inside Fig. 10.10 shows an example of the SNR versus the detection range for a pulse radar using the parameters shown in the figure. A MATLAB-based Graphical User Interface

(GUI) (see Fig. 10.10) is utilized in inputting and editing all input parameters. The outputs include the maximum detection range versus minimum SNR plots. The following MATLAB function is used by this GUI to generate the desired outputs.

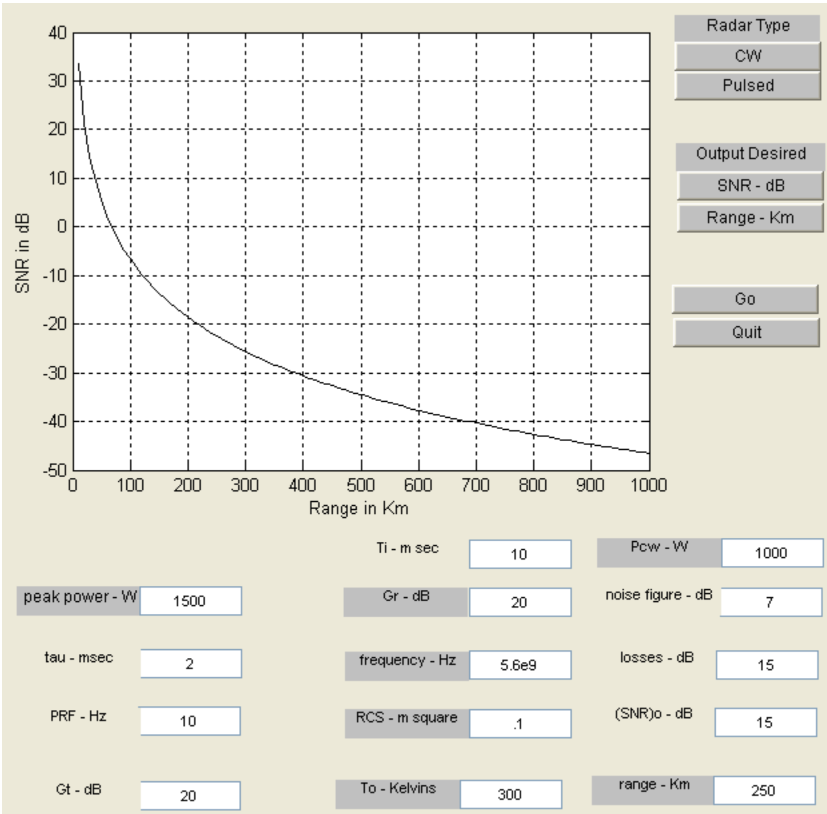


Figure 10.10. GUI work space associated with the program *"range_calc.m."*

```
function [output_par] = range_calc (pt, tau, fr, time_ti, gt, gr, freq, ...
  sigma, te, nf, loss, snro, pcw, range, radar_type, out_option)
c = 3.0e+8;
lambda = c / freq;
if (radar_type == 0)
  pav = pcw;
else
  % Compute the duty cycle
  dt = tau * 0.001 * fr;
  pav = pt * dt;
```

```
end
pav_db = 10.0 * log10(pav);
  lambda_sqdb = 10.0 * log10(lambda^2);
  sigmadb = 10.0 * log10(sigma);
  for_pi_cub = 10.0 * log10((4.0 * pi)^3);
  k_db = 10.0 * log10(1.38e-23);
  te_db = 10.0 * log10(te);
  ti_db = 10.0 * log10(time_ti);
  range_db = 10.0 * log10(range * 1000.0);
if (out_option == 0)
   %compute SNR
   snr_out = pav_db + gt + gr + lambda_sqdb + sigmadb + ti_db - ...
   for_pi_cub - k_db - te_db - nf - loss - 4.0 * range_db
  index = 0;
  for range_var = 10:10:1000
     index = index + 1;
     rangevar_db = 10.0 * log10(range_var * 1000.0);
     snr(index) = pav_db + gt + gr + lambda_sqdb + sigmadb + ti_db - ...
       for_pi_cub - k_db - te_db - nf - loss - 4.0 * rangevar_db;
  end
  var = 10:10:1000;
  plot(var,snr,'k')
  xlabel ('Range in Km');
  ylabel ('SNR in dB');
  grid
else
  range4 = pav_db + gt + gr + lambda_sqdb + sigmadb + ti_db - ...
    for_pi_cub - k_db - te_db - nf - loss - snro;
  range = 10.0^(range4/40.) / 1000.0
  index = 0;
  for snr_var = -20:1:60
    index = index + 1;
    rangedb = pav_db + gt + gr + lambda_sqdb + sigmadb + ti_db - ...
      for_pi_cub - k_db - te_db - nf - loss - snr_var;
    range(index) = 10.0^(rangedb/40.) / 1000.0;
  end
  var = -20:1:60;
  plot(var,range,'k')
  xlabel ('Minimum SNR required for detection in dB');
  ylabel ('Maximum detection range in Km');
  grid
end
return
```

10.3.2. MATLAB Function *"hprf_req.m"*

The function "*hprf_req.m*" implements the high PRF radar equation. Its syntax is as follows:

[snr] = hprf_req (pt, Ti, g, freq, sigma, dt, range, nf, loss)

where

Symbol	Description	Units	Status
pt	*peak power*	*W*	*input*
Ti	*time on target*	*seconds*	*input*
g	*antenna gain*	*dB*	*input*
freq	*frequency*	*Hz*	*input*
sigma	*target RCS*	m^2	*input*
dt	*duty cycle*	*none*	*input*
range	*target range (can be a single value or a vector)*	*m*	*input*
nf	*noise figure*	*dB*	*input*
loss	*radar losses*	*dB*	*input*
snr	*SNR (can be a single value or a vector)*	*dB*	*output*

MATLAB Function *"hprf_req.m"* Listing

```
function [snr] = hprf_req (pt, Ti, g, freq, sigma, dt, range, nf, loss)
% This program implements Eq. (10.31)
c = 3.0e+8; % speed of light
lambda = c / freq; % wavelength
pav = 10*log10(pt*dt); % compute average power in dB
Ti_db = 10*log10(Ti); % time on target in dB
lambda_sqdb = 10*log10(lambda^2); % compute wavelength square in dB
sigmadb = 10*log10(sigma); % convert sigma to dB
four_pi_cub = 10*log10((4.0 * pi)^3); % (4pi)^3 in dB
k_db = 10*log10(1.38e-23); % Boltzman's constant in dB
to_db = 10*log10(290); % noise temp. in dB
range_pwr4_db = 10*log10(range.^4); % vector of target range^4 in dB
% Implement Equation (1.72)
num = pav + Ti_db + g + g + lambda_sqdb + sigmadb;
den = four_pi_cub + k_db + to_db + nf + loss + range_pwr4_db;
snr = num - den;
return
```

Problems

10.1. In a multiple frequency CW radar, the transmitted waveform consists of two continuous sinewaves of frequencies $f_1 = 105KHz$ and $f_2 = 115KHz$. Compute the maximum unambiguous detection range.

10.2. Consider a radar system using linear frequency modulation. Compute the range that corresponds to $\dot{f} = 20, 10MHz$. Assume a beat frequency $f_b = 1200Hz$.

10.3. A certain radar using linear frequency modulation has a modulation frequency $f_m = 300Hz$ and frequency sweep $\Delta f = 50MHz$. Calculate the average beat frequency differences that correspond to range increments of 10 and 15 meters.

10.4. A CW radar uses linear frequency modulation to determine both range and range rate. The radar wavelength is $\lambda = 3cm$, and the frequency sweep is $\Delta f = 200KHz$. Let $t_0 = 20ms$. (a) Calculate the mean Doppler shift; (b) compute f_{bu} and f_{bd} corresponding to a target at range $R = 350Km$, which is approaching the radar with radial velocity of $250m/s$.

10.5. Consider a medium PRF radar on board an aircraft moving at a speed of 350 m/s with PRFs $f_{r1} = 10KHz$, $f_{r2} = 15KHz$, and $f_{r3} = 20KHz$; the radar operating frequency is $9.5GHz$. Calculate the frequency position of a nose-on target with a speed of 300 m/s. Also calculate the closing rate of a target appearing at 6, 5, and $18KHz$ away from the center line of PRF 10, 15, and $20KHz$, respectively.

10.6. A certain radar operates at two PRFs, f_{r1} and f_{r2}, where $T_{r1} = (1/f_{r1}) = T/5$ and $T_{r2} = (1/f_{r2}) = T/6$. Show that this multiple PRF scheme will give the same range ambiguity as that of a single PRF with PRI T.

10.7. Consider an X-band radar with wavelength $\lambda = 3cm$ and bandwidth $B = 10MHz$. The radar uses two PRFs, $f_{r1} = 50KHz$ and $f_{r2} = 55.55KHz$. A target is detected at range bin 46 for f_{r1} and at bin 12 for f_{r2}. Determine the actual target range.

10.8. A certain radar uses two PRFs to resolve range ambiguities. The desired unambiguous range is $R_u = 150Km$. Select a reasonable value for N. Compute the corresponding f_{r1}, f_{r2}, R_{u1}, and R_{u2}.

10.9. A certain radar uses three PRFs to resolve range ambiguities. The desired unambiguous range is $R_u = 250Km$. Select $N = 43$. Compute the corresponding f_{r1}, f_{r2}, f_{r3}, R_{u1}, R_{u2}, and R_{u3}.

10.10. In Chapter 1 we developed an expression for the Doppler shift associated with a CW radar (i.e., $f_d = \pm 2v/\lambda$, where the plus sign is used for closing targets and the negative sign is used for receding targets). CW radars can use the system shown below to determine whether the target is closing or receding. Assuming that the emitted signal is $A\cos\omega_0 t$ and the received signal is $kA\cos((\omega_0 \pm \omega_d)t + \varphi)$, show that the direction of the target can be determined by checking the phase shift difference in the outputs $y_1(t)$ and $y_2(t)$.