连续系统和离散系统中广义同步研究

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内容提要

- **1.** 引言
- 2. 微分方程系统的广义同步定理
- **3.** 离散系统的广义同步定理
- **4.** 小结

1. 引 言

定义 1 设

$$\frac{d\boldsymbol{X}}{dt} = F(\boldsymbol{X}) \tag{1}$$

$$\frac{d\boldsymbol{Y}}{dt} = G(\boldsymbol{Y}, \boldsymbol{X}_m) \tag{2}$$

$\boldsymbol{X} \in \mathbb{R}^n(\mathbb{R}^{n\times n}), \boldsymbol{Y} \in \mathbb{R}^m(\mathbb{R}^{m\times m})$. 如有 $H : \mathbb{R}^m(\mathbb{R}^{m\times m}) \longrightarrow \mathbb{R}^m(\mathbb{R}^{m\times m})$, 和 $B = B_x \times B_y \subset \mathbb{R}^n \times \mathbb{R}^m(\mathbb{R}^{n\times n} \times \mathbb{R}^{m\times m})$ 使得 $(\boldsymbol{X}(0), \boldsymbol{Y}(0)) \in B$ 时

$$\lim_{t\to+\infty} \|\boldsymbol{X}_m(t) - H^{-1}(\boldsymbol{Y}(t))\| = 0 \tag{3}$$

$$\boldsymbol{X}_m(t) = (x_1(t), x_2(t), \cdots, x_m(t)). \tag{4}$$

$$(\boldsymbol{X}_m(t) = (x_{i,j}(t))_{m\times m}) \tag{5}$$

则称 (1) 和 (2) 关于 H 广义同步.

定义 1 设

$$\frac{d\mathbf{X}}{dt} = F(\mathbf{X}) \quad (1)$$
$$\frac{d\mathbf{Y}}{dt} = G(\mathbf{Y}, \mathbf{X}_m) \quad (2)$$

- 一般情形: 对于2个连续的混沌系统 (1)和(2)，如果系统 (2) 与系统(1) , 关于微分同胚变换H广义同步.
-
- 系统(2) 中的G(Y, Xm), 具体表达形式是什么呢?

反之给定一个系统(1) 和一个微分同胚H, 如何构造系统(2), 使得2个系统关于H CGS呢？

定义 2 设

$$\boldsymbol{X}(k+1) = F(\boldsymbol{X}(k)) \tag{1}$$

$$\boldsymbol{Y}(k+1) = G(\boldsymbol{Y}(k), \boldsymbol{X}_m(k)) \tag{2}$$

$\boldsymbol{X} \in \mathbb{R}^n(\mathbb{R}^{n\times n}), \boldsymbol{Y} \in \mathbb{R}^m(\mathbb{R}^{m\times m})$. 如有 $H : \mathbb{R}^m(\mathbb{R}^{m\times m}) \longrightarrow \mathbb{R}^m(\mathbb{R}^{m\times m})$, 和 $B = B_x \times B_y \subset \mathbb{R}^n \times \mathbb{R}^m(\mathbb{R}^{n\times n} \times \mathbb{R}^{m\times m})$ 使得 $(\boldsymbol{X}(0), \boldsymbol{Y}(0)) \in B$ 时

$$\lim_{t\to+\infty} \|\boldsymbol{X}_m(t) - H^{-1}(\boldsymbol{Y}(t))\| = 0 \tag{3}$$

$$\boldsymbol{X}_m(t) = (x_1(t), x_2(t), \cdots, x_m(t)). \tag{4}$$

$$(\boldsymbol{X}_m(t) = (x_{i,j}(t))_{m\times m}) \tag{5}$$

则称 *(1)* 和 *(2)* 关于 H 广义同步.

定义 2 设

$$\mathbf{X}(\boldsymbol{k}+\boldsymbol{1}) = F(\mathbf{X}(\boldsymbol{k})) \tag{1}$$

$$\mathbf{Y}(\boldsymbol{k}+\boldsymbol{1}) = G(\mathbf{Y}(k), \mathbf{X}_m(k)) \tag{2}$$

- 一般情形: 对于2个连续的混沌系统 (1)和(2)， 如果系统 (2) 与系统(1) , 关于微分同胚变换H广义同步.
-
- 系统(2) 中的G(Y, Xm), 具体表达形式是什么呢?

反之给定一个系统(1) 和一个微分同胚H, 如何构造系统(2), 使得2个系统关于H CGS呢？

2. 微分方程系统的广义同步(GS)定理

2.1. 矢量微分方程的GS定理

定理 1 设 $\mathbf{X}, \mathbf{X}_m, \mathbf{Y}, G(\mathbf{Y}, \mathbf{X}_m)$ 同定义 1.H 是 C^1 同胚. 记 $\mathbf{X}_m = V(\mathbf{Y}) = H^{-1}(\mathbf{Y})$. 如果系统 (1) 和 (2)关于变换 H 是广义同步的，则 $G(\mathbf{Y}, \mathbf{X}_m)$ 可写成以下形式：

$$G(\mathbf{Y}, \mathbf{X}_m) = [\dot{V}(\mathbf{Y})]^{-1}[F_m(\mathbf{X}) - q(\mathbf{X}_m, \mathbf{Y})]$$

$$\dot{V}(\mathbf{Y}) = \begin{bmatrix} \frac{\partial V_1}{\partial y_1} & \frac{\partial V_1}{\partial y_2} & \cdots & \frac{\partial V_1}{\partial y_m} \\ \frac{\partial V_2}{\partial y_2} & \frac{\partial V_2}{\partial y_2} & \cdots & \frac{\partial V_2}{\partial y_m} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial V_m}{\partial y_m} & \frac{\partial V_m}{\partial y_2} & \cdots & \frac{\partial V_m}{\partial y_m} \end{bmatrix}$$

$$F_m(\mathbf{X}) = (f_1(\mathbf{X}), f_2(X), \ldots, f_m(\mathbf{X}))^T$$

且函数 $q(\mathbf{X}_m, \mathbf{Y}) = (q_1(\mathbf{X}_m, \mathbf{Y}), q_2(\mathbf{X}_m, \mathbf{Y}), \ldots, q_m(\mathbf{X}_m, \mathbf{Y}))^T$ 使得误差方程

$$\dot{\mathbf{e}} = \dot{\mathbf{X}}_m - \dot{V}(\mathbf{Y})\dot{\mathbf{Y}} = q(\mathbf{X}_m, \mathbf{Y})$$

零解渐进稳定.

Zhang X., Min L., USTB J, (3)2000.

2.1. 非自治微分方程的GS定理

定义 1 设

$$\frac{d\mathbf{X}}{dt} = F(\mathbf{X}, t) \quad (1)$$

$$\frac{d\mathbf{Y}}{dt} = G(\mathbf{Y}, \mathbf{X}, t) \quad (2)$$

$\mathbf{X} \in \mathbb{R}^n$, $\mathbf{Y} \in \mathbb{R}^m$. 如果存在 $H: \mathbb{R}^n \times \mathbb{R}^+ \longrightarrow \mathbb{R}^m$ 和一个子集 $B = B_x \times B_y \subset \mathbb{R}^n \times \mathbb{R}^m$ 使得当初始条件 $(\mathbf{X}(0), \mathbf{Y}(0)) \in B$ 时，下式成立

$$\lim_{t \to +\infty} \|H(\mathbf{X}(t), t) - \mathbf{Y}(t)\| = 0 \quad (3)$$

则称系统 (1) 和 (2) 关于 H 广义同步.

Liu T, Ji Ye & Min L. Proceedings Part B First Chinese Forum on Chaos Appl. Aug. 2007

定理 2 如果由定义 *1*给出的非自治系统 *(1)* 和 *(2)*关于 $H(\mathbf{X},t)$ 广义同步. 则 *(2)* 中 $G(\mathbf{Y},\mathbf{X},t)$ 可以写成如下形式

$$G(\mathbf{Y},\mathbf{X},t)=\frac{\partial H}{\partial \mathbf{X}}F(\mathbf{X},t)+\frac{\partial H}{\partial t}+Q(\mathbf{X},\mathbf{Y},t)$$

$$\frac{\partial H}{\partial \mathbf{X}}=\begin{pmatrix}\frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_n}\\ \vdots & \ddots & \vdots\\ \frac{\partial h_m}{\partial x_1} & \cdots & \frac{\partial h_m}{\partial x_n}\end{pmatrix},\quad \frac{\partial H}{\partial t}=(\frac{\partial h_1}{\partial t},\cdots,\frac{\partial h_m}{\partial t})^T$$

且函数

$$Q(\mathbf{X},\mathbf{Y},t)=(q_1(\mathbf{X},\mathbf{Y},t),q_2(\mathbf{X},\mathbf{Y},t)),\cdots,q_m(\mathbf{X},\mathbf{Y},t)))^T$$

保证误差方程

$$\dot{\mathbf{E}}=\frac{d(\mathbf{Y}-H(\mathbf{X},t))}{dt}=Q(\mathbf{X},\mathbf{Y},t)$$

零解渐近稳定.

2.3. 双定向微分方程GS定理

定理 3 设

$$\frac{d\boldsymbol{X}}{dt} = F(\boldsymbol{X},\boldsymbol{Y}), \quad (1)$$

$$\frac{d\boldsymbol{Y}}{dt} = G(\boldsymbol{X},\boldsymbol{Y}), \quad (2)$$

$$\boldsymbol{X} \in \mathbb{R}^n, \boldsymbol{Y} \in \mathbb{R}^m$$

(1) 和 (2) 关于 $H:\mathbb{R}^m \to \mathbb{R}^m$ GS. 则 $G(\boldsymbol{X},\boldsymbol{Y})$ 有形式

$$G(\boldsymbol{X},\boldsymbol{Y}) = \frac{\partial H}{\partial \boldsymbol{X}} F(\boldsymbol{X},\boldsymbol{Y}) + Q(\boldsymbol{X},\boldsymbol{Y}) \quad (3)$$

$$\frac{\partial H}{\partial \boldsymbol{X}} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_2} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \frac{\partial h_m}{\partial x_2} & \cdots & \frac{\partial h_m}{\partial x_n} \end{bmatrix},$$

$Q(\boldsymbol{X},\boldsymbol{Y})$ 使得

$$\dot{\boldsymbol{E}} = \frac{d(\boldsymbol{Y} - H(\boldsymbol{X}))}{dt} = Q(\boldsymbol{X},\boldsymbol{Y}) = (q_1(\boldsymbol{X},\boldsymbol{Y}),\cdots,q_m(\boldsymbol{X},\boldsymbol{Y}))^T \quad (4)$$

零解渐进稳定，其中 $\boldsymbol{E} = \boldsymbol{Y} - H(\boldsymbol{X})$.**Ye J, Liu T, Min L. Phy Lett A.372,2008**

2.4. 阵列微分方程的GS定理

$$
\begin{cases}
\dot{x}_{1\,i,j} & = & f_{1\,i,j}(\mathbf{X}) \\
\dot{x}_{2\,i,j} & = & f_{2\,i,j}(\mathbf{X}) \\
\vdots & & \\
\dot{x}_{m\,i,j} & = & f_{m\,i,j}(\mathbf{X}) \\
\vdots & & \\
\dot{x}_{n\,i,j} & = & f_{n\,i,j}(\mathbf{X})
\end{cases}
\qquad (1)
$$

$$
\left\{
\begin{array}{lcl}
\dot{y}_{1\,i,j} & = & g_{1\,i,j}(\mathbf{Y},\mathbf{X}) \\
\dot{y}_{2\,i,j} & = & g_{2\,i,j}(\mathbf{Y},\mathbf{X}) \\
\quad\vdots & & \\
\dot{y}_{m\,i,j} & = & g_{m\,i,j}(\mathbf{Y},\mathbf{X})
\end{array}
\right.
\tag{2}
$$

$$
\mathbf{X} = (x_{1\,i,j}, x_{2\,i,j}, \ldots, x_{n\,i,j})^T, \tag{3}
$$

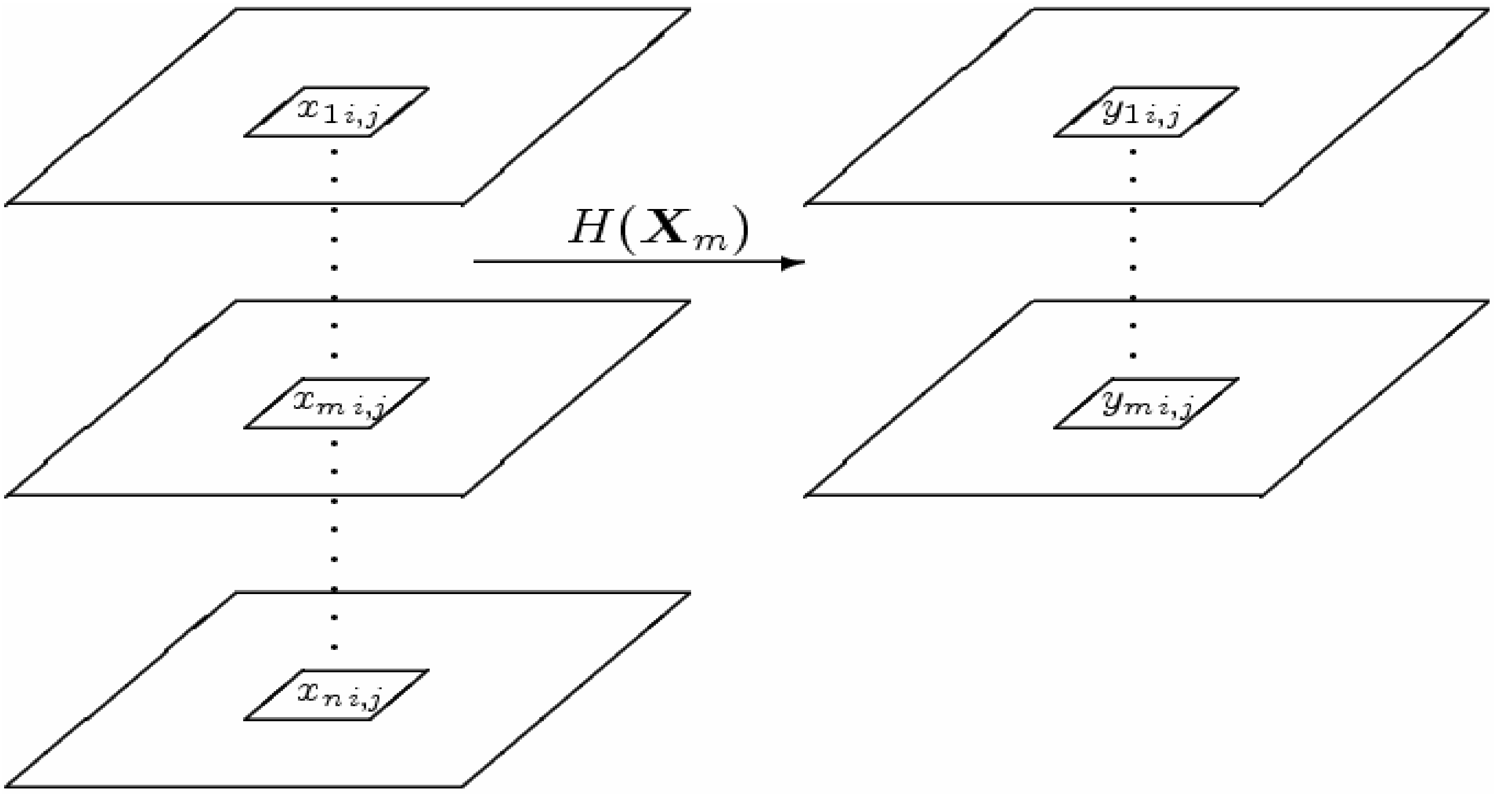
$$
\mathbf{Y} = (y_{1\,i,j}, y_{2\,i,j}, \ldots, y_{m\,i,j})^T, \tag{4}
$$

$$
i = 1, 2, \ldots, M, \quad j = 1, 2, \ldots, N, m \leq n.
$$

Compact form

$$\dot{\mathbf{X}} = F(\mathbf{X}), \tag{3}$$

$$\dot{\mathbf{Y}} = G(\mathbf{Y}, \mathbf{X}_m) \tag{4}$$



Min L. Zang H. Proc. 2009 ICCCS.

定理 4 $H:\mathbb{R}^{mM\times N}\longrightarrow\mathbb{R}^{mM\times N}$,

$$\begin{aligned} H(\mathbf{X}_m) &= (h_{l\,i,j}(\mathbf{X}_m)) \\ &\triangleq (y_{l\,i,j}), \\ &= \mathbf{Y} \end{aligned} \qquad (1)$$

连续可微. 则 (3) 和 (4) 关于 $\mathbf{Y}=H(\mathbf{X}_m)$ GS, 当且仅当 $G(\mathbf{Y},\mathbf{X}_m)=(G_{l\,i,j}(\mathbf{Y},\mathbf{X}_m))$ 具有形式

$$
\begin{aligned}
G_{l\,i,j}(\mathbf{Y},\mathbf{X}_m) &= \sum_{l'=1}^{m}\sum_{i'=1}^{M}\sum_{j'=1}^{N}\frac{\partial h_{l\,i,j}(\mathbf{X}_\mathrm{m})}{\partial x_{l'\,i',j'}} f_{l'\,i',j'}(\mathbf{X}) \\
&\quad - q_{l\,i,j}(\mathbf{Y},\mathbf{X}_m), \\
&\quad l=1,\cdots,m, i=1,\cdots,M; j=1,\cdots,N
\end{aligned}
$$

$q_{l\,i,j}(\mathbf{X}_m,\mathbf{Y})'s$ 使得误差方程

$$
\begin{aligned}
\frac{d\mathbf{e}}{dt} &= \frac{dH(\mathbf{X}_m)}{dt} - \frac{d\mathbf{Y}}{dt} \\
&= (q_{l\,i,j}(\mathbf{Y},\mathbf{X}_m)). \qquad (1)
\end{aligned}
$$

零解渐进稳定.

3. 离散系统的广义同步(GS)定理

3.1 离散矢量映射的GS定理

定义 2 设

$$\boldsymbol{X}(\boldsymbol{k}+1) = F(\boldsymbol{X}(\boldsymbol{k})) \quad (1)$$

$$\boldsymbol{Y}(\boldsymbol{k}+1) = G(\mathbf{Y}(k), \boldsymbol{X}_m(k)) \quad (2)$$

$$\mathbf{X}(\mathbf{k})=(x_1(k), x_2(k), \ldots, x_n(k))^T$$

$$\mathbf{X_m}(\mathbf{k})=(x_1(k), x_2(k), \ldots, x_m(k))^T, m \le n$$

$$F(\mathbf{X}(k))=(f_1(\mathbf{X}(k)), f_2(\mathbf{X}(k)), \ldots f_n(\mathbf{X}(k)))^T$$

$$G(\mathbf{Y}(k), \mathbf{X}(k))=(g_1(\mathbf{Y}(k), \mathbf{X}(k)), g_2(\mathbf{Y}(k), \mathbf{X}(k)), \ldots, g_m(\mathbf{Y}(k), \mathbf{X}(k))^T$$

臧鸿雁，闵乐泉,北京科技大学学报，(1) (2007)

定理 5 设 $H:\mathbb{R}^m \longrightarrow \mathbb{R}^m$ $H(x_1,x_2,\ldots,x_m)=(y_1,y_2,\ldots,y_m)$ 可逆且 $V(\mathbf{Y})=\mathbf{X}_m$ 是 H 的逆. 如果 *(1)* 和 *(2)* 关于 $\mathbf{Y}=H(\mathbf{X}_m)GS$, 则当且仅当 $G(\mathbf{Y},\mathbf{X})$ 具有形式

$$G(\mathbf{Y},\mathbf{X})=H[F_m(\mathbf{X})-q(\mathbf{X}_m,\mathbf{Y})] \tag{1}$$

$$F_m(\mathbf{X}) = (f_1(\mathbf{X}),f_2(\mathbf{X}),\ldots,f_m(\mathbf{X}))^T$$

$$q(\mathbf{X}_m,\mathbf{Y})=(q_1(\mathbf{X}_m,\mathbf{Y}),q_2(\mathbf{X}_m,\mathbf{Y}),\ldots,q_m(\mathbf{X}_m,\mathbf{Y}))^T$$

使得误差方程零解渐进稳定:

$$\begin{aligned} \mathrm{e}(k+1) &= \mathbf{X}_m(k+1)-V(\mathbf{Y}(k+1)) \\ &= q(\mathbf{X}_m,\mathbf{Y}). \end{aligned} \tag{2}$$

3.2 非自治离散映射的GS定理

$$\mathbf{X}(k+1) = F(\mathbf{X}(k), k), \tag{1}$$

$$\mathbf{Y}(k+1) = G(\mathbf{X}(k), \mathbf{Y}(k), k), \tag{2}$$

$$\mathbf{X} \in \mathbb{R}^n, \mathbf{Y} \in \mathbb{R}^m,$$

$$F(\mathbf{X}(k), k) = (f_1(\mathbf{X}(k), k), f_2(\mathbf{X}(k), k), \cdots, f_n(\mathbf{X}(k), k))^T \in \mathbb{R}^n,$$

$$G(\mathbf{X}(k), \mathbf{Y}(k), k) = (g_1(\mathbf{X}(k), \mathbf{Y}(k), k), g_2(\mathbf{X}(k), \mathbf{Y}(k), k), \cdots, g_m(\mathbf{X}(k), \mathbf{Y}(k), k))^T \in \mathbb{R}^m.$$

Liu T, Ye J, Min L.

定理 6 如果 (1) 和 (2) 关于变换 H： $H:\mathbb{R}^n\times\mathbb{N}\to\mathbb{R}^m$ GS. 则当且仅当 $G(\mathbf{X}(k),\mathbf{Y}(k),k)$ 具有形式

$$G(\mathbf{X}(k),\mathbf{Y}(k),k)=H(F(\mathbf{X}(k),k),k+1)+q(\mathbf{X}(k),\mathbf{Y}(k),k)$$

$$q(\mathbf{X}(k),\mathbf{Y}(k),k)=(q_1(\mathbf{X}(k),\mathbf{Y}(k),k),q_2(\mathbf{X}(k),\mathbf{Y}(k),k),\cdots,q_m(\mathbf{X}(k),\mathbf{Y}(k),k))^T,$$

使得误差方程零解渐进稳定

$$\mathbf{e}(k+1)=\mathbf{Y}(k+1)-H(\mathbf{X}(k+1),k+1)=q(\mathbf{X}(k),\mathbf{Y}(k),k).$$

2.3. 双定向离散映射的GS定理

定义 1

$$\mathbf{X}(i+1) = F(\mathbf{X}(i), \mathbf{Y}(i)), \quad (1)$$

$$\mathbf{Y}(i+1) = G(\mathbf{X}(i), \mathbf{Y}(i)), \quad (2)$$

where

$$\mathbf{X}(i) \in \mathbb{R}^n, \mathbf{Y}(i) \in \mathbb{R}^m,$$

$$F(\mathbf{X}(i), \mathbf{Y}(i)) = (f_1(\mathbf{X}(i), \mathbf{Y}(i)), \cdots, f_n(\mathbf{X}(i), \mathbf{Y}(i)))^T \in \mathbb{R}^n,$$

$$G(\mathbf{X}(i), \mathbf{Y}(i)) = (g_1(\mathbf{X}(i), \mathbf{Y}(i)), \cdots, g_m(\mathbf{X}(i), \mathbf{Y}(i)))^T \in \mathbb{R}^m.$$

If there exists a transformation

$$H : \quad \mathbb{R}^n \to \mathbb{R}^m \quad (3)$$

$$H(\mathbf{X}(i)) = (h_1(\mathbf{X}(i)), \cdots, h_m(\mathbf{X}(i)))^T \quad (4)$$

and a subset $B = B_{\mathbf{X}} \times B_{\mathbf{Y}} \subset \mathbb{R}^n \times \mathbb{R}^m$ such that all trajectories of (1) and (2) with initial conditions in B satisfy

$$\lim_{i \to +\infty} \|\mathbf{Y}(i) - H(\mathbf{X}(i))\| = 0.$$

Then the systems given by (1) and (2) are said to be in GS with respect to H.

Ye J, Liu T, Min L. Phy Lett A.372, 2008

定理 7 如果 (1) 和 (2) 关于变换 $H:\mathbb{R}^n\times\mathbb{N}\to\mathbb{R}^m$ GS. 则当且仅当 $G(\mathbf{X}(k),\mathbf{Y}(k),k)$ 具有形式

$$G(\mathbf{X}(k),\mathbf{Y}(k),k)=H(F(\mathbf{X}(k),k),k+1)+q(\mathbf{X}(k),\mathbf{Y}(k),k)$$

$$q(\mathbf{X}(k),\mathbf{Y}(k),k)=(q_1(\mathbf{X}(k),\mathbf{Y}(k),k),q_2(\mathbf{X}(k),\mathbf{Y}(k),k),\cdots,q_m(\mathbf{X}(k),\mathbf{Y}(k),k))^T,$$

使得误差方程零解渐进稳定

$$\mathbf{e}(k+1)=\mathbf{Y}(k+1)-H(\mathbf{X}(k+1),k+1)=q(\mathbf{X}(k),\mathbf{Y}(k),k).$$

3.4 离散阵列映射的GS定理

$$\begin{cases} x_{1\,i,j}(k+1) &= f_{1\,i,j}(\mathbf{X}(k)) \\ x_{2\,i,j}(k+1) &= f_{2\,i,j}(\mathbf{X}(k)) \\ \vdots & \\ x_{m\,i,j}(k+1) &= f_{m\,i,j}(\mathbf{X}(k)) \\ \vdots & \\ x_{n\,i,j}(k+1) &= f_{n\,i,j}(\mathbf{X}(k)) \end{cases} \tag{1}$$

$$\begin{cases} y_{1\,i,j}(k+1) &= g_{1\,i,j}(\mathbf{Y}(k),\mathbf{X}(k)) \\ y_{2\,i,j}(k+1) &= g_{2\,i,j}(\mathbf{Y}(k),\mathbf{X}(k)) \\ \vdots & \\ y_{m\,i,j}(k+1) &= g_{m\,i,j}(\mathbf{Y}(k),\mathbf{X}(k)) \end{cases} \tag{2}$$

where

$$\begin{aligned} i &= 1,2,\ldots,M, \quad j = 1,2,\ldots,N, \\ \mathbf{X}(k) &= (x_{1\,i,j}(k), x_{2\,i,j}(k), \ldots, x_{n\,i,j}(k))^T, \\ \mathbf{Y}(k) &= (y_{1\,i,j}(k), y_{2\,i,j}(k), \ldots, y_{m\,i,j}(k))^T. \end{aligned}$$

In a compact form, systems (1) and (2) can be written as

$$\mathbf{X}(k+1) = F(\mathbf{X}(k)) \quad (3)$$

$$\mathbf{Y}(k+1) = G(\mathbf{Y}(k), \mathbf{X}(k)) \quad (4)$$

where

$$F(\mathbf{X}(k)) = (f_{1\,i,j}(\mathbf{X}(k)), f_{2\,i,j}(\mathbf{X}(k)), \ldots, f_{n\,i,j}(\mathbf{X}(k)))^T \quad (5)$$

$$G(\mathbf{Y}(k), \mathbf{X}(k)) = (g_{1\,i,j}(\mathbf{Y}(k), \mathbf{X}(k)), g_{2\,i,j}(\mathbf{Y}(k), \mathbf{X}(k)), \ldots, g_{m\,i,j}(\mathbf{Y}(k), \mathbf{X}(k)))^T \quad (6)$$

Definition The DTAE systems defined by (3) and (4) are said to be in GS with respect to an invertible transformation $H : \mathbb{R}^{mM\times N} \longrightarrow \mathbb{R}^{mM\times N}$, if there exists an open subset $B \subset \mathbb{R}^{mM\times N} \times \mathbb{R}^{mM\times N}$, such that for any trajectory $(\mathbf{X}(k), \mathbf{Y}(k))$ of systems (3) and (4), with initial condition $(\mathbf{X}(0), \mathbf{Y}(0)) \in B$, the following property holds:

$$\lim_{k\to+\infty} \|\mathbf{X}_m(k) - H^{-1}(\mathbf{Y}(k))\| = 0 \qquad (3)$$

where

$$\mathbf{X}_m(k) \quad = \quad (x_{1\,i,j}(k), x_{2\,i,j}(k), \ldots, x_{m\,i,j}(k))^T \qquad (4)$$

Zang H, Min L. Proc. 3rd IEEE Conf. On IEA 2008.

Theorem 8 Let $\mathbf{X}(k), \mathbf{Y}(k), \mathbf{X}_m(k), F(\mathbf{X}(k))$ and $G(\mathbf{Y}(k), \mathbf{X}(k))$ be defined above. Suppose that $H : \mathbb{R}^{mM\times N} \longrightarrow \mathbb{R}^{mM\times N}$ defined by

$$H(x_{1\,i,j}(k), x_{2\,i,j}(k), \ldots, x_{m\,i,j}(k)) \\ = (y_{1\,i,j}(k), y_{2\,i,j}(k), \ldots, y_{m\,i,j}(k))$$

is an invertible transformation and $V(\mathbf{Y}(k)) = \mathbf{X}_m(k)$ is the inverse function of H. If the two systems (3) and (4) are in GS via the transformation $\mathbf{Y}(k) = H(\mathbf{X}_m(k))$, then the function $G(\mathbf{Y}(k), \mathbf{X}(k))$ given in (4) will have the following form:

$$G(\mathbf{Y}(k), \mathbf{X}(k)) = H[F_m(\mathbf{X}(k)) - q(\mathbf{X}_m(k), \mathbf{Y}(k))]$$

$$F_m(\mathbf{X}(k)) = (f_{1\,i,j}(\mathbf{X}(k)), f_{2\,i,j}(\mathbf{X}(k)), \ldots, f_{m\,i,j}(\mathbf{X}(k)))^T$$

and the function

$$\begin{aligned} q(\mathbf{X}_m(k), \mathbf{Y}(k)) &= (q_1(\mathbf{X}_m(k), \mathbf{Y}(k)), q_2(\mathbf{X}_m(k), \mathbf{Y}(k)), \\ &\quad \ldots, q_m(\mathbf{X}_m(k), \mathbf{Y}(k)))^T \end{aligned}$$

guarantees that the zero solution of the following error equation is asymptotically stable:

$$\begin{aligned} \mathbf{e}(k+1) &= \mathbf{X}_m(k+1) - V(\mathbf{Y}(k+1)) \\ &= q(\mathbf{X}_m(k), \mathbf{Y}(k)). \end{aligned}$$

3. Application

a Henon CNN with three state variables and one port is introduced as follows.

$$
\begin{aligned}
\mathbf{X}(k+1) &= \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{pmatrix} \\
&= F(\mathbf{X}(k)) \\
&= (f_{1\,i,j}(\mathbf{X}(k)), f_{2\,i,j}(\mathbf{X}(k)), f_{3\,i,j}(\mathbf{X}(k)))^T \\
&= \begin{pmatrix} 1 + x_{2i,j}(k) - a x_{1i,j}(k)^2 \\ b x_{1i,j}(k) + x_{3i,j}(k) \\ -b x_{1i,j}(k) + D[x_{3i+1,j}(k) + x_{3i-1,j}(k) \\ + x_{3i,j+1}(k) + x_{3i,j-1}(k) \\ -4 x_{3i,j}(k)] \end{pmatrix} \quad (14)
\end{aligned}
$$

$$H : \mathbb{R}^{3\,M \times N} \longrightarrow \mathbb{R}^{3\,M \times N}$$

is defined by

$$
\begin{aligned}
& H(x_{1\,i,j}, x_{2\,i,j}, x_{3\,i,j}) \\
=& (H_1(x_{1\,i,j}, x_{2\,i,j}, x_{3\,i,j}), H_2(x_{1\,i,j}, x_{2\,i,j}, x_{3\,i,j}), \\
& H_3(x_{1\,i,j}, x_{2\,i,j}, x_{3\,i,j})) \\
=& (\ln(\frac{x_{2\,i,j}+x_{3\,i,j}-x_{1\,i,j}}{2}+\sqrt{(\frac{x_{2\,i,j}+x_{3\,i,j}-x_{1\,i,j}}{2})^2+1}, \\
& (\ln(\frac{x_{1\,i,j}+x_{3\,i,j}-x_{2\,i,j}}{2}+\sqrt{(\frac{x_{1\,i,j}+x_{3\,i,j}-x_{2\,i,j}}{2})^2+1}, \\
& (\ln(\frac{x_{1\,i,j}+x_{2\,i,j}-x_{3\,i,j}}{2}+\sqrt{(\frac{x_{1\,i,j}+x_{2\,i,j}-x_{3\,i,j}}{2})^2+1}) \\
=& (y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}). \qquad (15)
\end{aligned}
$$

Therefore the inverse function V of H has the form:

$$
\begin{aligned}
V(\mathbf{Y}(k)) &= H^{-1}(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}) \\
&= (V_1(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}), \\
&\quad V_2(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}), V_3(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j})) \\
&= (\frac{1}{2}(e^{y_{2\,i,j}} - e^{-y_{2\,i,j}} + e^{y_{3\,i,j}} - e^{-y_{3\,i,j}}), \\
&\quad \frac{1}{2}(e^{y_{1\,i,j}} - e^{-y_{1\,i,j}} + e^{y_{3\,i,j}} - e^{-y_{3\,i,j}}), \\
&\quad \frac{1}{2}(e^{y_{1\,i,j}} - e^{-y_{1\,i,j}} + e^{y_{2\,i,j}} - e^{-y_{2\,i,j}})) \\
&= (x_{1\,i,j}, x_{2\,i,j}, x_{3\,i,j}).
\end{aligned}
$$

Now let

$$
\begin{aligned}
& q(\mathbf{X}_m(k), \mathbf{Y}(\mathbf{k})) \\
= \quad & \frac{1}{8} \times (e(k)) = \frac{1}{8} \times (\mathbf{X}_m(k) - V(\mathbf{Y}(\mathbf{k}))) \\
= \quad & \frac{1}{8} \times \begin{pmatrix} x_{1\,i,j} - V_1(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}) \\ x_{2\,i,j} - V_2(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}) \\ x_{3\,i,j} - V_3(y_{1\,i,j}, y_{2\,i,j}, y_{3\,i,j}) \end{pmatrix} \\
\triangleq \quad & \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}
\end{aligned}
$$

then (13) be the zero solution asymptotically stable.

$$
\begin{aligned}
& F_m(\mathbf{X}) - q(\mathbf{X}_m, \mathbf{Y}) \\
= & \begin{pmatrix} f_{1\,i,j}(\mathbf{X}(k)) - q_1 \\ f_{2\,i,j}(\mathbf{X}(k)) - q_2 \\ f_{3\,i,j}(\mathbf{X}(k)) - q_3 \end{pmatrix} \\
\triangleq & \begin{pmatrix} A \\ B \\ C \end{pmatrix}
\end{aligned}
$$

Hence the driven system has the form:

$$
\begin{aligned}
& Y(k+1) = G(\mathbf{Y}(k), \mathbf{X}(k)) \\
= \quad & H[F_m(\mathbf{X}(k)) - q(\mathbf{X}_m(k), \mathbf{Y}(k))] \\
= & \begin{pmatrix}
\ln(\frac{1}{2}(B+C-A)+\sqrt{(\frac{1}{2}(B+C-A))^2+1}) \\
\ln(\frac{1}{2}(A+C-B)+\sqrt{(\frac{1}{2}(A+C-B))^2+1}) \\
\ln(\frac{1}{2}(A+B-C)+\sqrt{(\frac{1}{2}(A+B-C))^2+1})
\end{pmatrix}
\end{aligned}
\tag{16}
$$

Now, let us choose the following initial conditions:

$$\begin{cases} [x_{1\,i,j}(0)] = 0.1 + 0.1 rand(M,N) \\ [x_{2\,i,j}(0)] = 0.1 + 0.1 rand(M,N) \\ [x_{3\,i,j}(0)] = 0.1 + 0.1 rand(M,N) \\ y_{1\,i,j}(0) = \ln(\frac{1}{2}(x_{2\,i,j}(0) + x_{3\,i,j}(0) - x_{1\,i,j}(0)) \\ \qquad + \sqrt{\frac{1}{2}((x_{2\,i,j}(0) + x_{3\,i,j}(0) - x_{1\,i,j}(0))^2 + 1}) \\ y_{2\,i,j}(0) = \ln(\frac{1}{2}(x_{1\,i,j}(0) + x_{3\,i,j}(0) - x_{2\,i,j}(0)) \\ \qquad + \sqrt{\frac{1}{2}((x_{1\,i,j}(0) + x_{3\,i,j}(0) - x_{2\,i,j}(0))^2 + 1}) \\ y_{3\,i,j}(0) = \ln(\frac{1}{2}(x_{1\,i,j}(0) + x_{2\,i,j}(0) - x_{3\,i,j}(0)) \\ \qquad + \sqrt{\frac{1}{2}((x_{1\,i,j}(0) + x_{2\,i,j}(0) - x_{3\,i,j}(0))^2 + 1}) \end{cases} \quad (17)$$

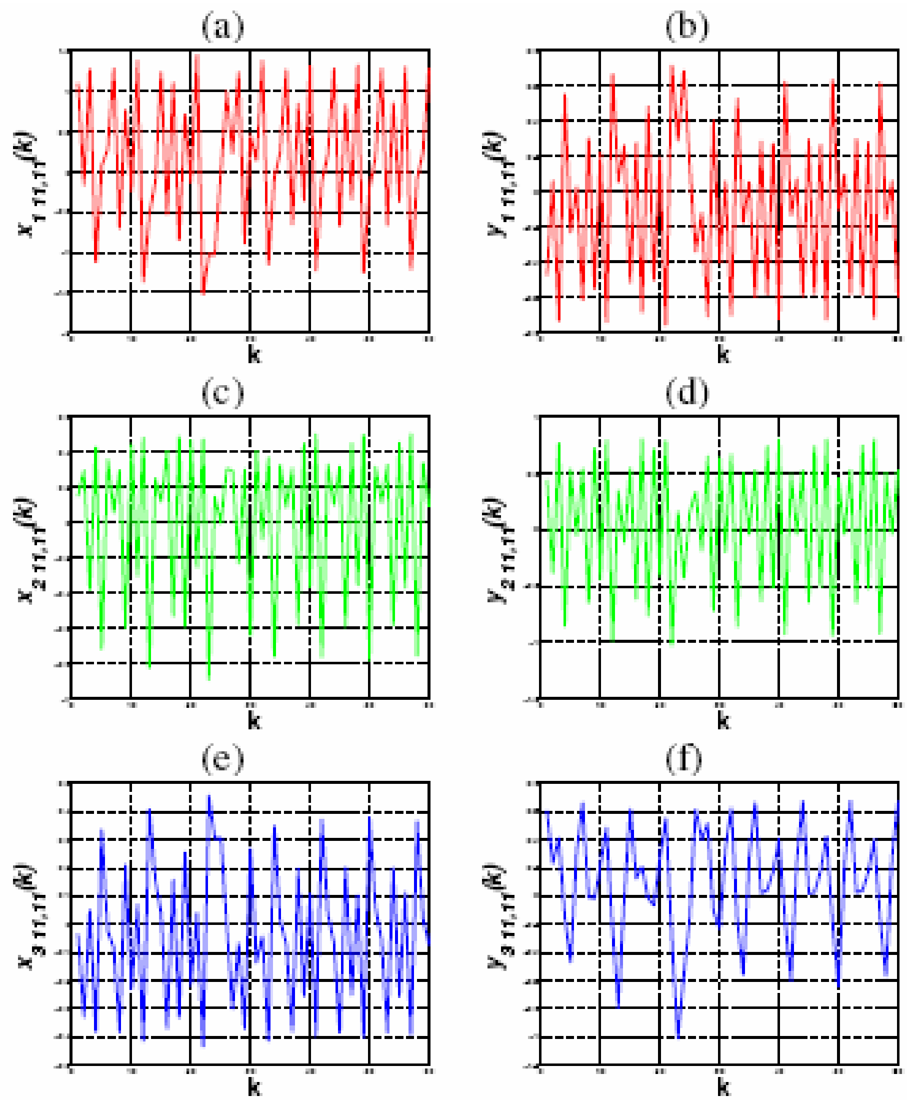
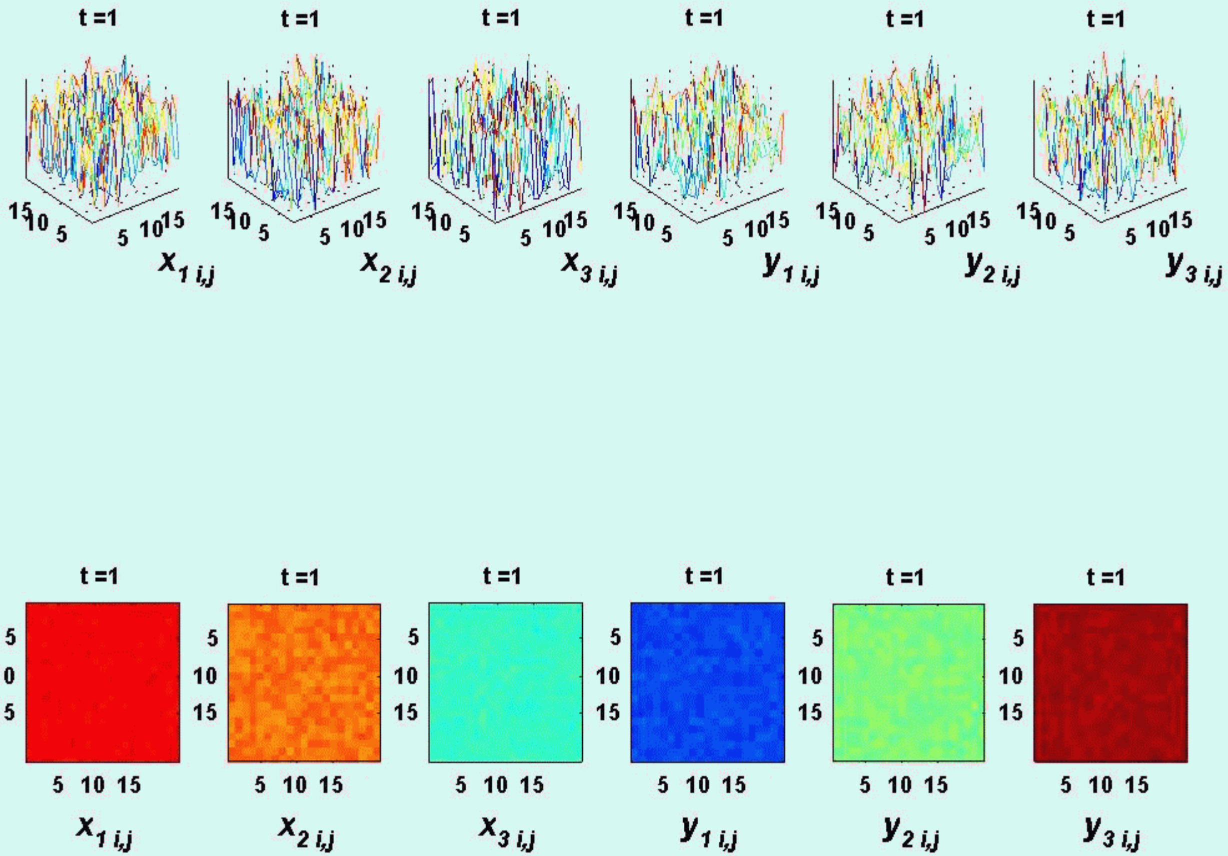


Fig. 1. Chaotic trajectories of the components of the state variables ($\mathbf{X}$(t), $\mathbf{Y}$(t)): (a) $x_{1\,11,11}$, (b) $y_{1\,11,11}$, (c) $x_{2\,11,11}$, (d) $y_{2\,11,11}$ and (e) $x_{3\,11,11}$, (f) $y_{3\,11,11}$.



t =1
5
10
15
x 1 i,j
x 2 i,j
x 3 i,j
y 1 i,j
y 2 i,j
y 3 i,j

4. 小结

- 1) 介绍了连续系统和离散系统中的8个广义
- 同步定理.
- 2) 这些定理给出1构造广义同步系统的
- 方法.
- 3) 可以预期借助于这些定理， 有助于
- 理解某些复杂系统间的广义同步现
- 象.

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