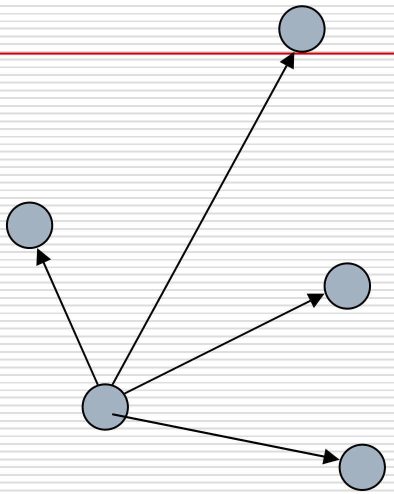
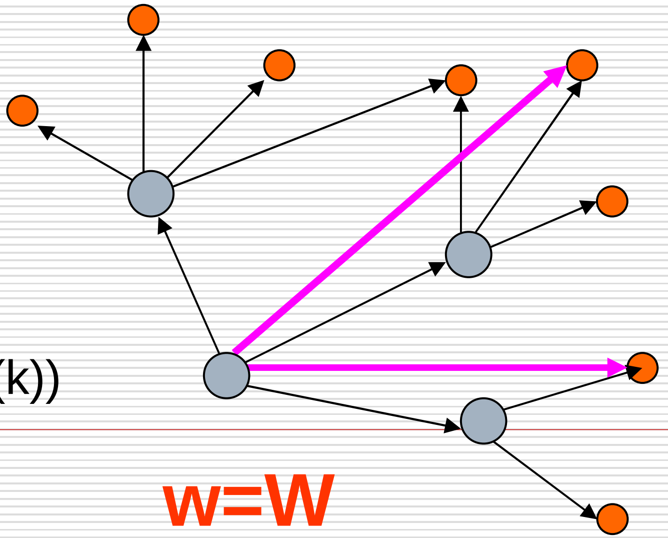
Evolutional model

1. Initialization: p(d)=c;
2. Evolution:
   a. delete friend
      rule: min{dE/dis}
   b. add friends
      rule: max{dE/dis}



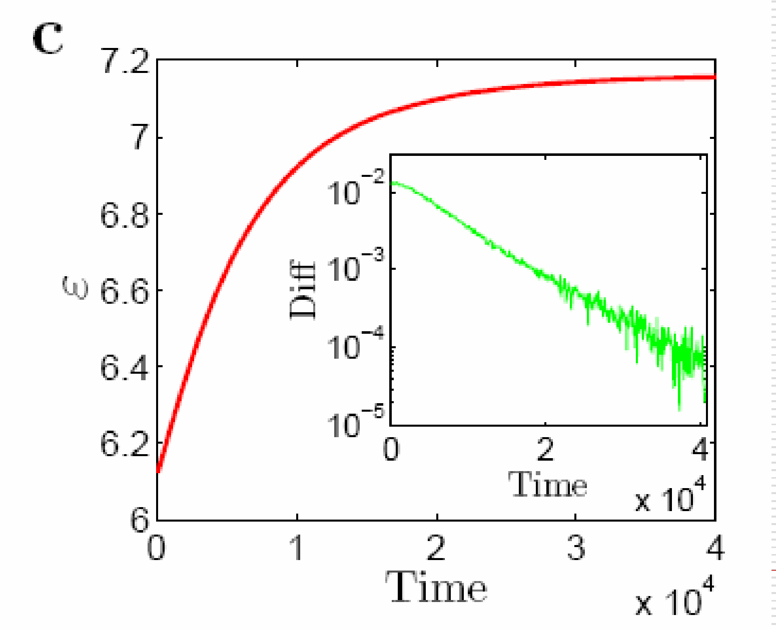


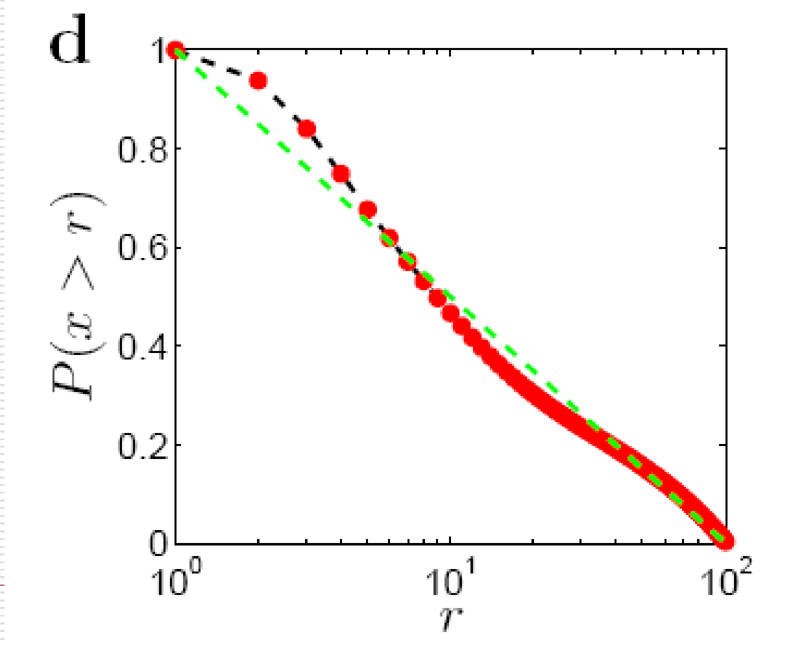
q is the rank of dE/dis, then p(dE/dis)~q^(-1-ln(k))
(2001 PRL)

Maximization of Entropy

$$\max \quad \varepsilon = -\sum_{i=1}^{n} p_i \ln(p_i)$$

$$st. \quad \sum_{j=1}^{m} d(1, j) = w$$





Theoretical Analysis

$$w = cL$$

OR

$$w = \frac{f \cdot L}{\log L}$$

$\alpha < 1$

$$\lim_{L \to \infty} E(\varepsilon_\alpha) = \log\left(\frac{c(2-a)}{1-a} + \left[\frac{c(2-a)}{1-a}\right]^2\right)$$

$\alpha > 1$

$$\lim_{L \to +\infty} E(\varepsilon_\alpha) = \frac{(a-1)(2\log 2 + \log Z(a)) + a + 1}{2(a-1)^2}$$

$P(r) \propto r^{-\alpha}$ →

$P(r) \propto r^{-1}$ is the optimal distribution

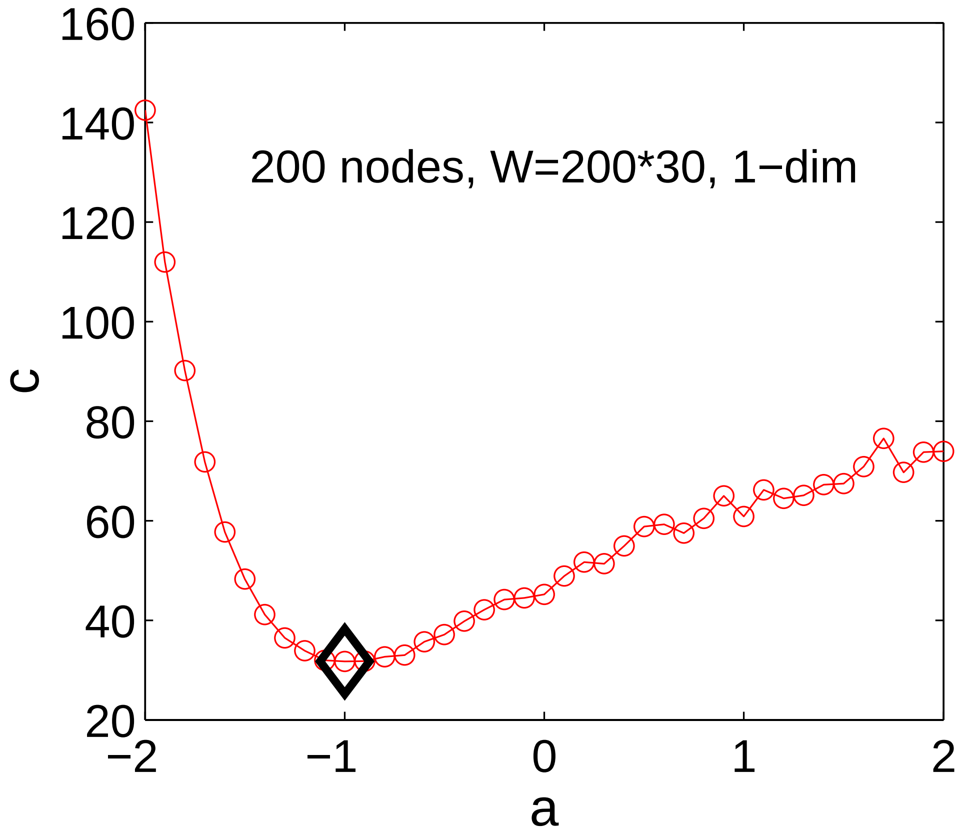
Investigating Its Effects

- Synchronization
- Traffic Dynamics
- Epidemic Process

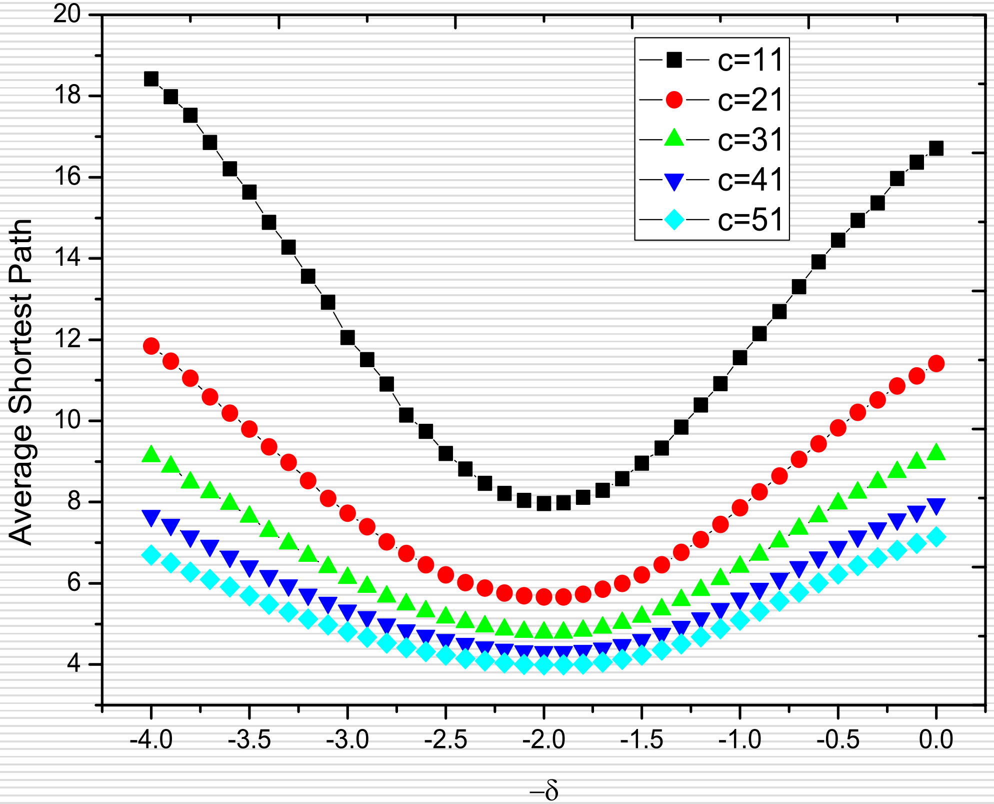
Synchronization

Synchronizability

$$R = \lambda_N / \lambda_2$$

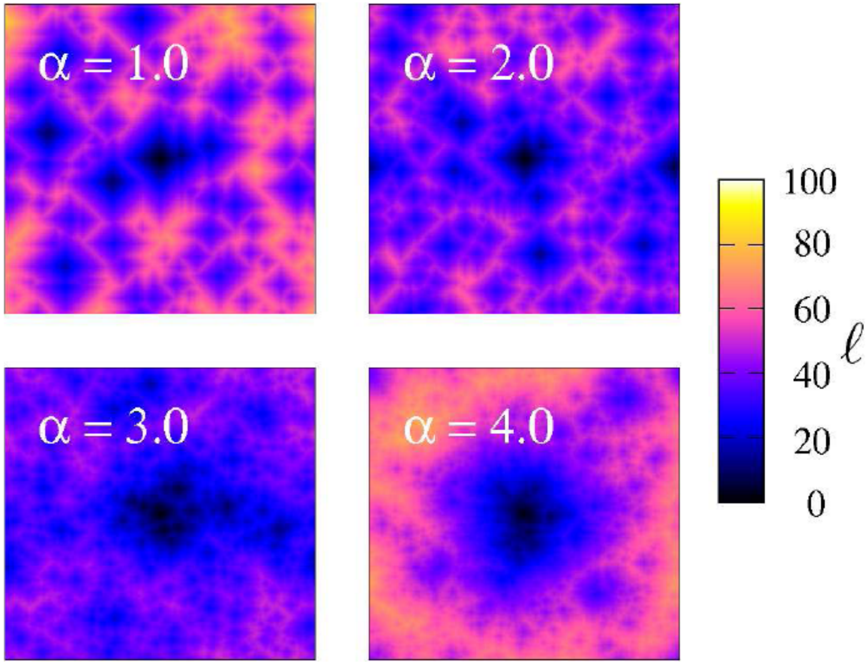


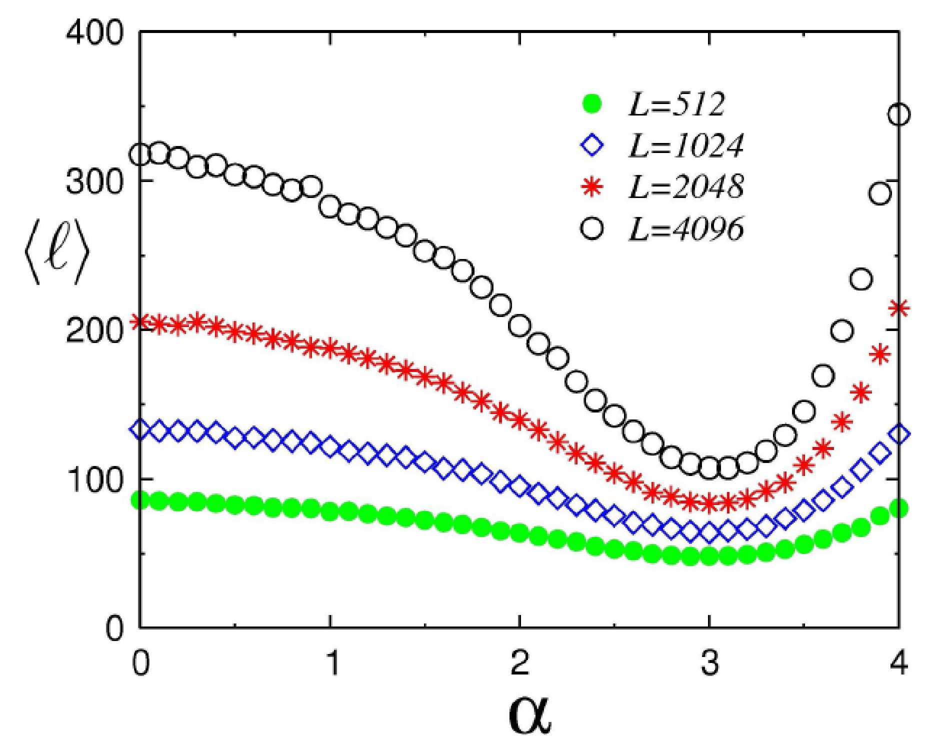
Topology and Traffic Dynamics



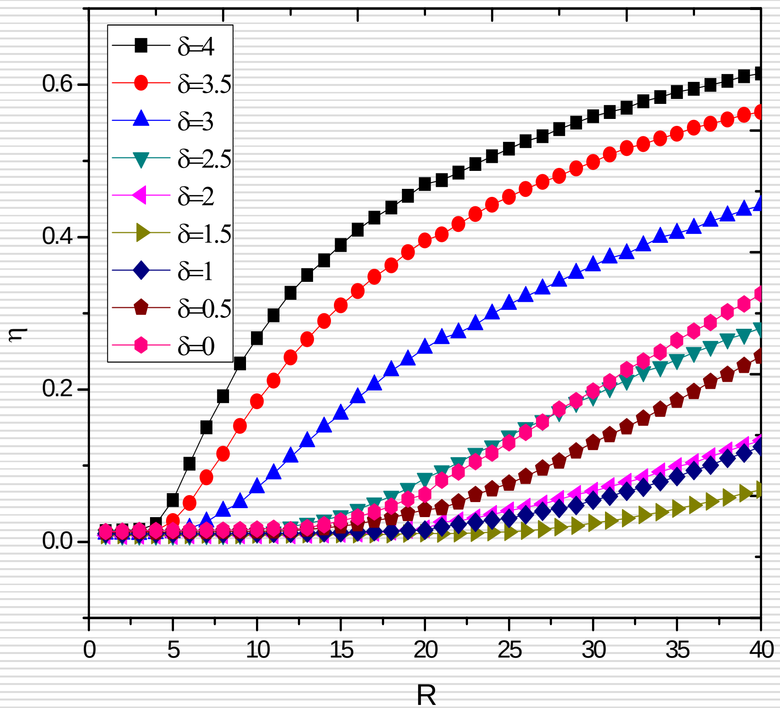
Designing optimal transport networks

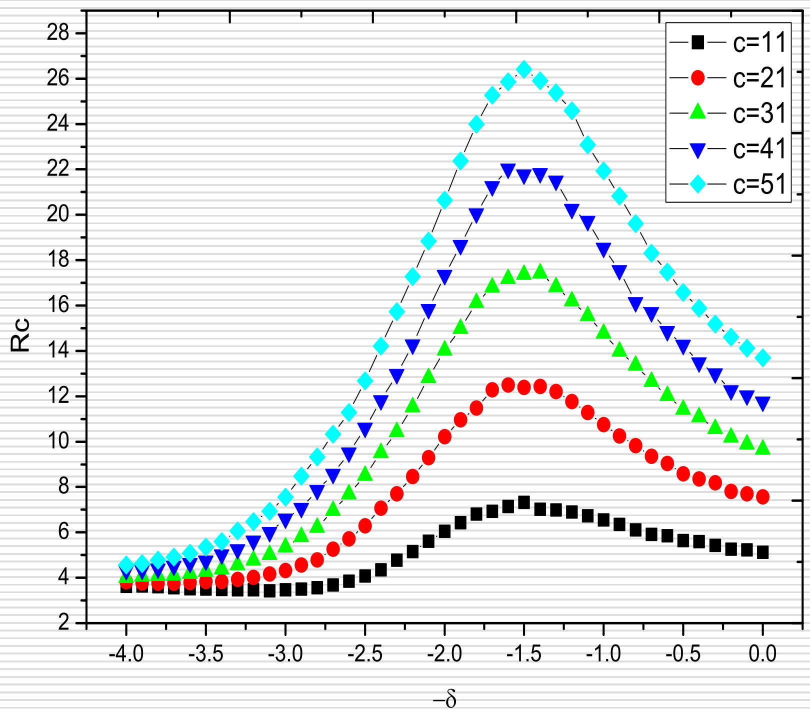
G. Li, S. D. S. Reis, A. A. Moreira, S. Havlin, H. E. Stanley, and J. S. Andrade Jr., arXiv:0908.3869v1 [physics.soc-ph]





Traffic Dynamics





Epidemic Process and Mobility

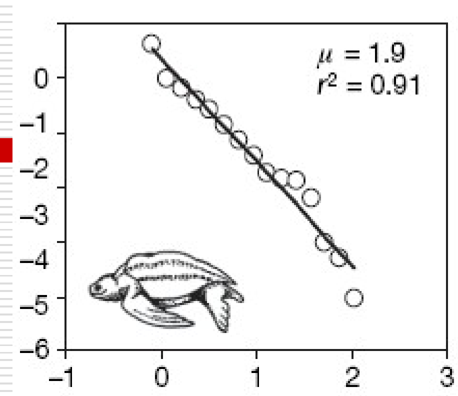
- **Viewpoint**
  - Epidemic spreading always follow the mobility of animal or human .
    - Previous works focus on the effects of topology on epidemic process
  - We want to establish a network model with a Levy flight spatial structure.

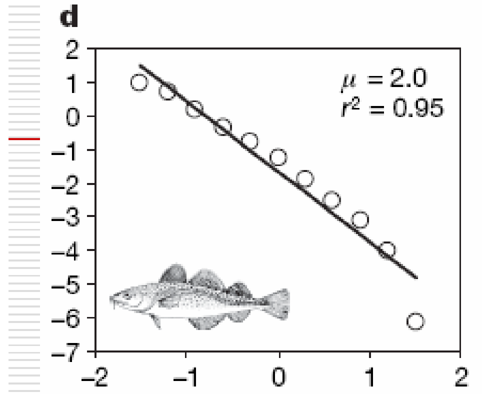
How does the mobility pattern impact on epidemic Process?

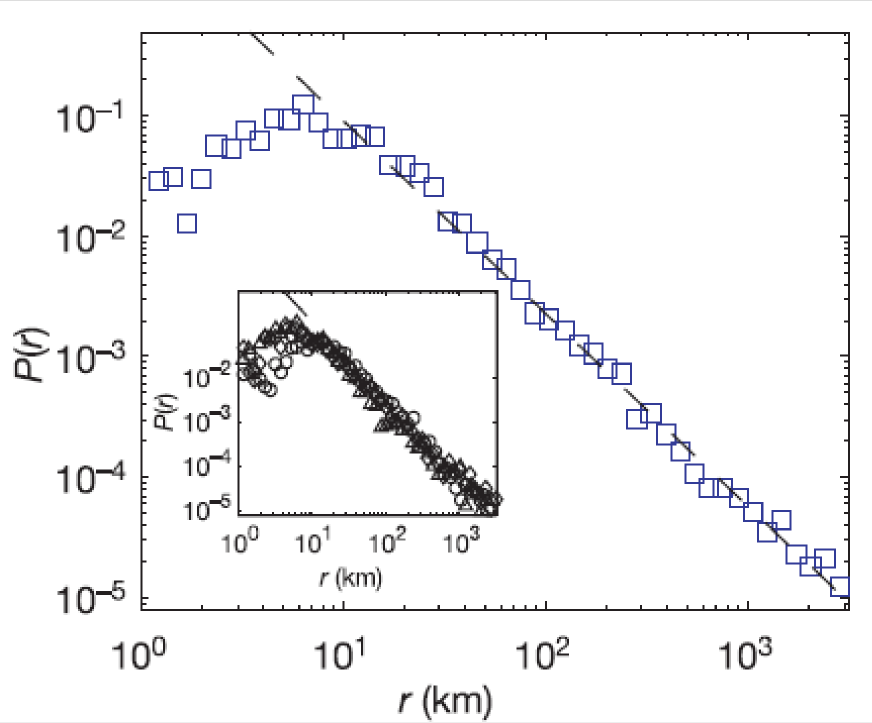
Mobility Pattern

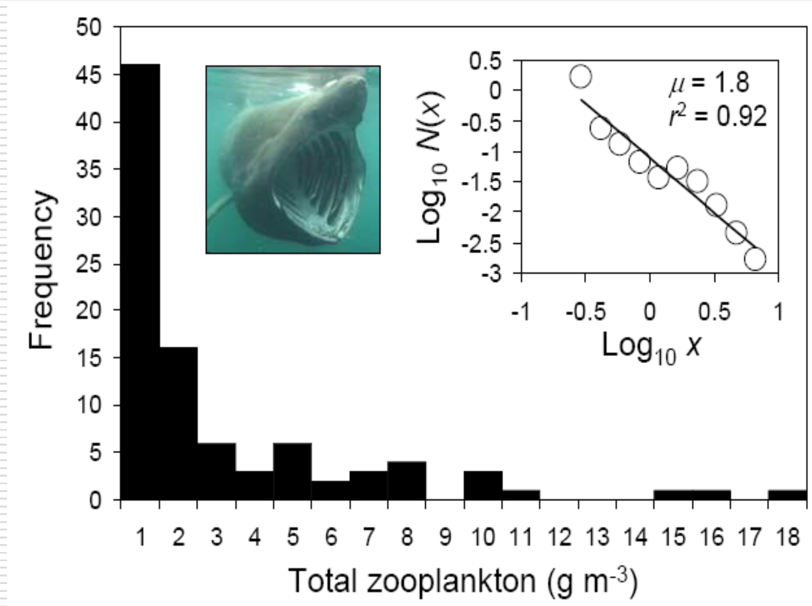
Pattern: most of the studies on animal and human mobility pattern including experimental data and theoretic analysis found that their mobility pattern follow the Levy flight:

$$\Pr(d) \propto d^{-2}$$





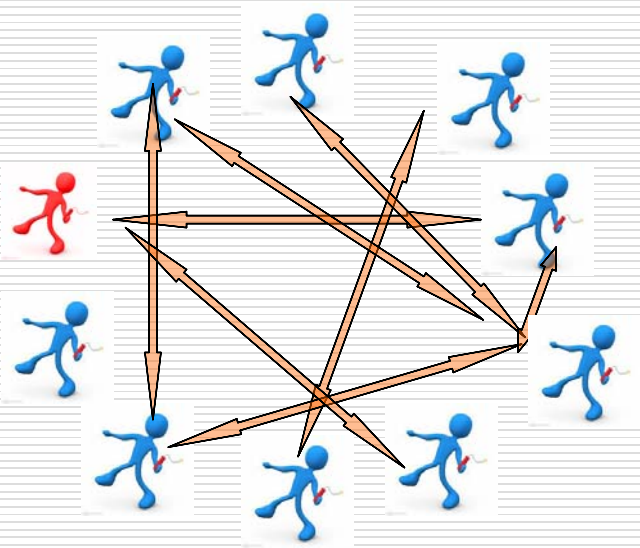




Scaling laws of marine predator search behaviour, Nature (2008)

D. Brockmann, L. Hufnagel and T. Geisel, The scaling laws of human travel, Nature, 439, 462-465, (2006).

Spatial Network with Levy Flights Properties



1. Each node denotes a small group of people.

2. Each node has limited energy.

3. The weight of edge means the expectation of contacting times.

Weighted Network with Levy Flight Spatial Structure

Step1. Given a regular ring with n nodes and a restriction on total energy $\Omega = w \times n$

Step2. For $\forall i, j \quad p(i,j) \sim d(i,j)^{-\alpha}$

Step3. (Normalization) $$p(i,j) = \frac{d(i,j)^{-\alpha}}{\sum_{d \in D(n)} d^{-\alpha}}$$

then we get a probability matrix P_{utm} ,.

Step4. (Weighted matrix) $m = \Omega / E(d)$

Weighted matrix $W = mP_{utm}$ and $w_{ij}\,(j > i)$

is related with the times of contacts c_j between node $\boldsymbol{i, j}$

SI Model

Step1. Randomly choose one node as an infected individual, others are susceptible.

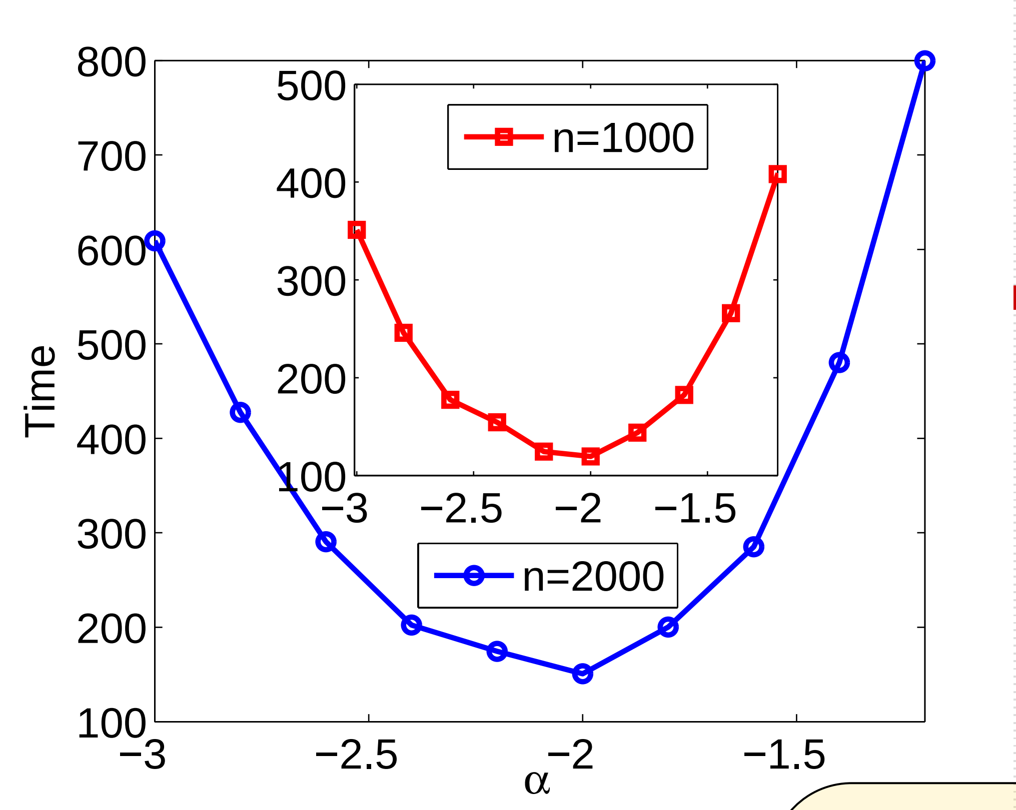
Step2. For each susceptible node, it will be infected with a probability

$$1-(1-r)^{\sum_{j\neq i} w_{ki}}$$

where j are infected nodes, and r indicated the effective infection rates .

Step3. Repeat the step 2

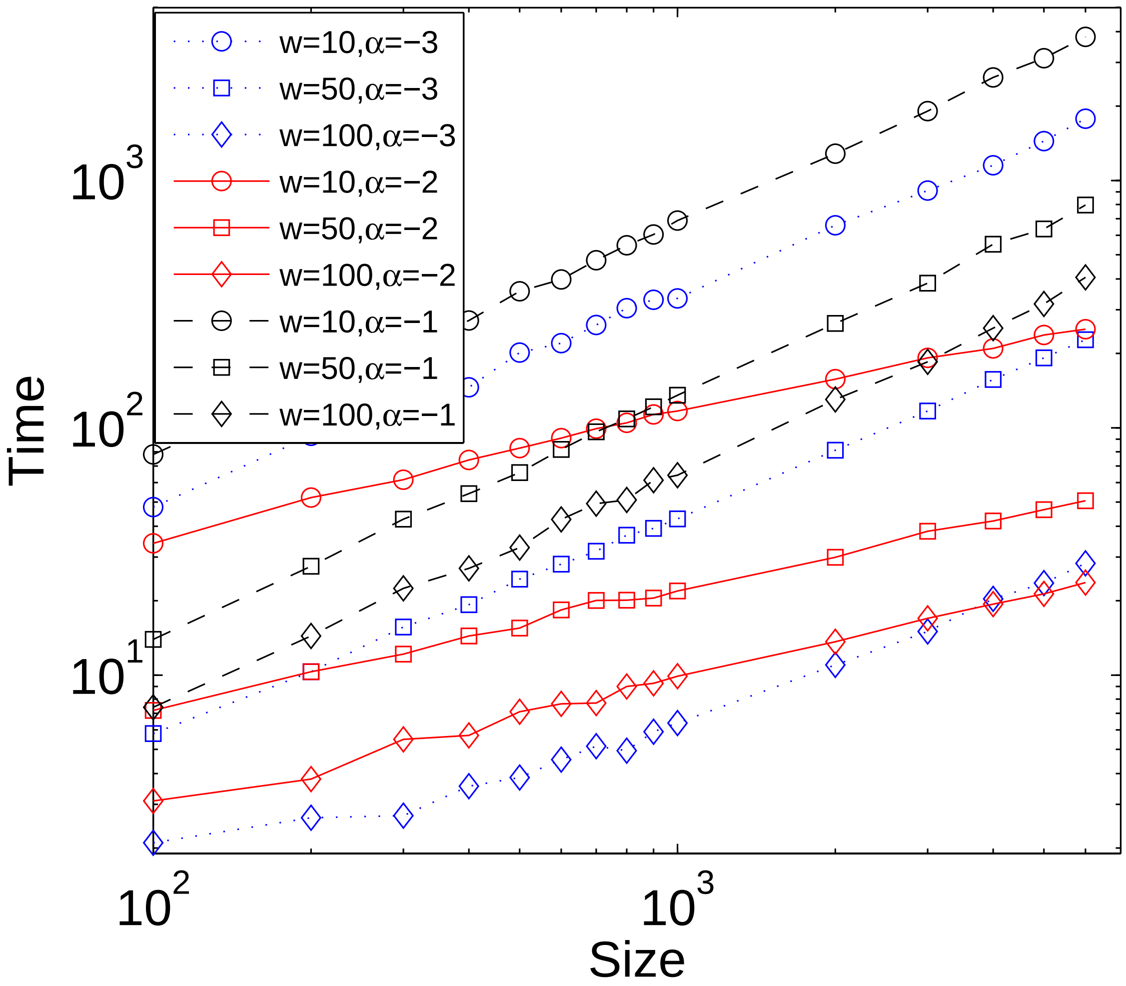
Step4. Terminate until 80% of the nodes are infected.



The curve has a lowest point when

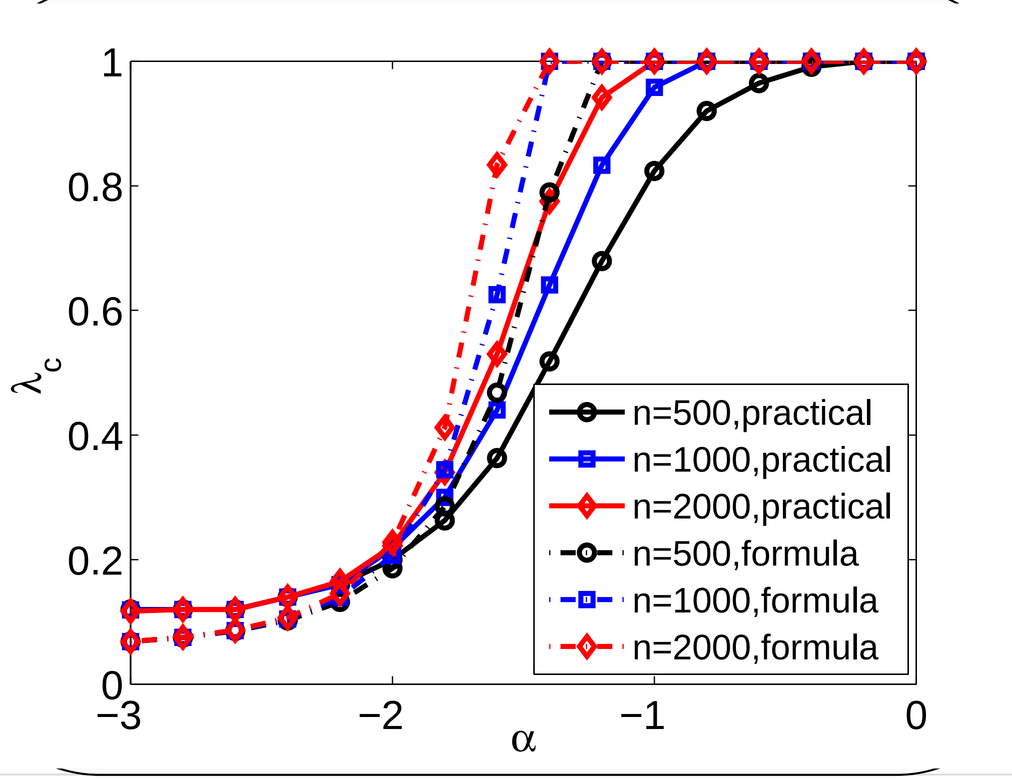
$$a \approx -2$$

Which implies that the mobility pattern
Is effective for spreading.



w=10,α=−3
w=50,α=−3
w=100,α=−3
w=10,α=−2
w=50,α=−2
w=100,α=−2
w=10,α=−1
w=50,α=−1
w=100,α=−1
Time
10^1
10^2
10^3
Size
10^2
10^3

SIS Model



1
0.8
0.6
0.4
0.2
0
−3
−2
−1
0
α
λ_c
n=500,practical
n=1000,practical
n=2000,practical
n=500,formula
n=1000,formula
n=2000,formula

Summary

1, The distance distribution between friends is scale invariant--$P(d) \sim d^{-1}$ which is an important and universal property for social networks. Some Other networks show also Scaling Law in spatial properties.

2, For Scaling Law in social networks, it may result from the maximization of entropy and can benefit individuals for collecting information.

3, Spatial Scaling properties has some important effects on dynamics, e.g. synchronization, traffic dynamics, epidemic spreading on networks.

Thank you!