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# 网络系统的协调动力学与控制

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# 网络系统的协调动力学与控制

- 一、 网络系统的协调动力学
- 二、 网络系统的脉冲一致性

# 1 网络系统的协调动力学

## 力学网络系统的协调动力学与控制

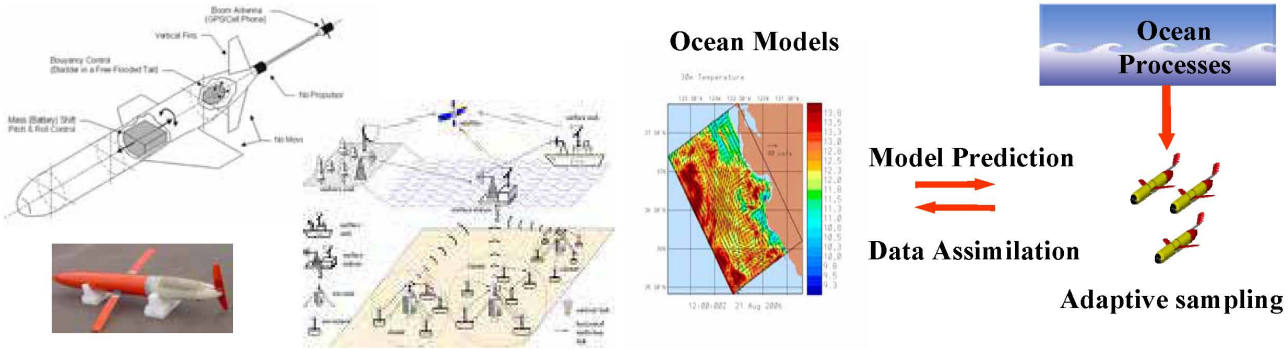
近年来面向实际工程技术领域中系统个体动力学特性的力学网络系统协调动力学与控制已引起人们的不断关注。

- **Lagrangian** 力学网络系统一致性问题（如多个无人航天器网络）
- 多刚体网络系统镇定同步问题（如多个旋转式探测卫星或水下探测器系统）
- 粒子耦合振子动力学（*PCOD*）模型协调控制（如环境抽样试验，分布采样观测等）

# 一、背景及意义

## 力学网络系统的协调动力学与控制

由于对这些力学网络系统的研究考虑了个体的物理和力学特性及其相互作用过程中的力学运动规律，因而所获结果在诸如多个移动机器人、无人航天器、传感器网络、勘测机械等实际工程机械系统关键控制技术的设计和实现是至关重要的。




目前，国际上关于这方面研究还处于起步阶段，其主要的工作是以美国普林斯顿大学力学与航空航天工程系**Leonard**等人代表性的工作。

# 一、背景及意义

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# 二、我们的工作

## 网络型质量-弹簧振子系统的同步动力学

### INTRODUCTION

The network is regarded as a good tool to deal with a set of interactional systems. In recent year, there are several research works focused on synchronization dynamics of networked mechanical oscillator systems derived from some practical engineering problems, for example, Ren studied the coupled harmonic oscillators with local interaction, Lu et al.,investigated linearly coupled networks of deterministic ratchets, Barrón and Sen detected coupled self-excited elastic beams and so on.

The mass-spring-damper (MSD) oscillator has become a typical and popular model for analysis of vibrational phenomena in engineering and technology.The motion equations of the (MSD) oscillator can be given with the following equation using Newton's law:

$$\ddot{x}(t) + b\dot{x}(t) + h(x) = F\cos(\omega t), \tag{1}$$

where $x(t)$, $\dot{x}(t)$ denote the displacement and velocity of the oscillator system, respectively, $h(x) = kx(1 + \epsilon x^2)$ $(\epsilon > 0)$ is restoring force.

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### MODEL FORMULATIONS

We consider the networked (MSD) oscillator systems in presence of a chain coupling configuration with nonlinear spring interaction. The network is described by the following form.

$$\begin{aligned}\ddot{x}_i(t) + b\dot{x}_i(t) &= \sum_{j=1, j\neq i}^{N} k_{ij}\,(x_j(t) - x_i(t))[1 + \epsilon(x_j(t) - x_i(t))^2] \\ &\quad + F\cos(\omega t), \qquad i = 1, 2, \cdots, N, \end{aligned} \tag{2}$$

where $x_i(t)$, $\dot{x}_i(t)$ denote the displacement and velocity of the $ith$ oscillator, respectively. The corresponding adjacency matrix of network topologies can be written as follows:

$$K = \begin{bmatrix} -k_{12} & k_{12} & 0 & \cdots & 0 \\ k_{21} & -k_{21} - k_{23} & k_{23} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & k_{N,N-1} & -k_{N,N-1} \end{bmatrix}.$$

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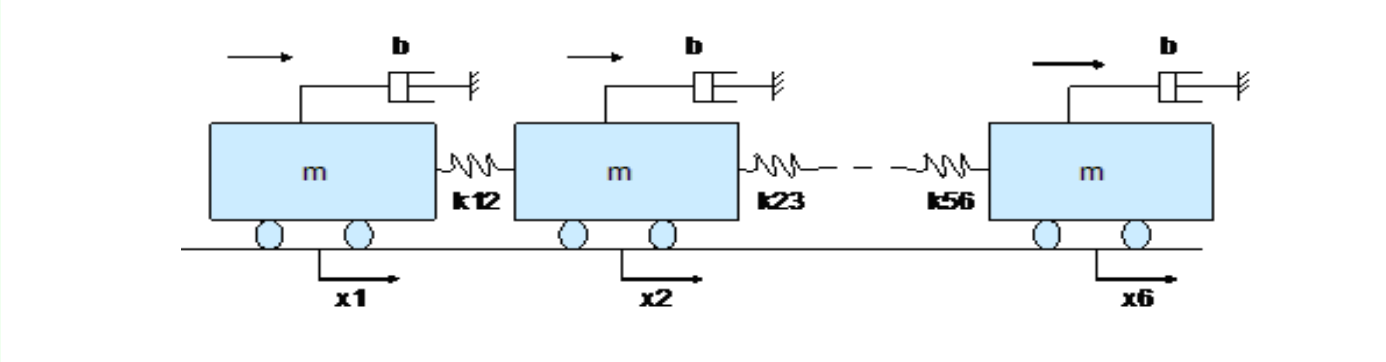
### MODEL FORMULATIONS

Obviously, a distinctive features of the networked oscillator system (2) is that its coupling configuration is nonlinear connectivity. Therefore, the network model (2) can well formulate practical architectures of some networked mechanical systems. For example, the network model (2) can be used to describe a basic vibration model of the networked oscillator system consisting of $N$ carts with cosinoidal excitation through nonlinear spring interaction, see Fig.1.



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### MODEL FORMULATIONS

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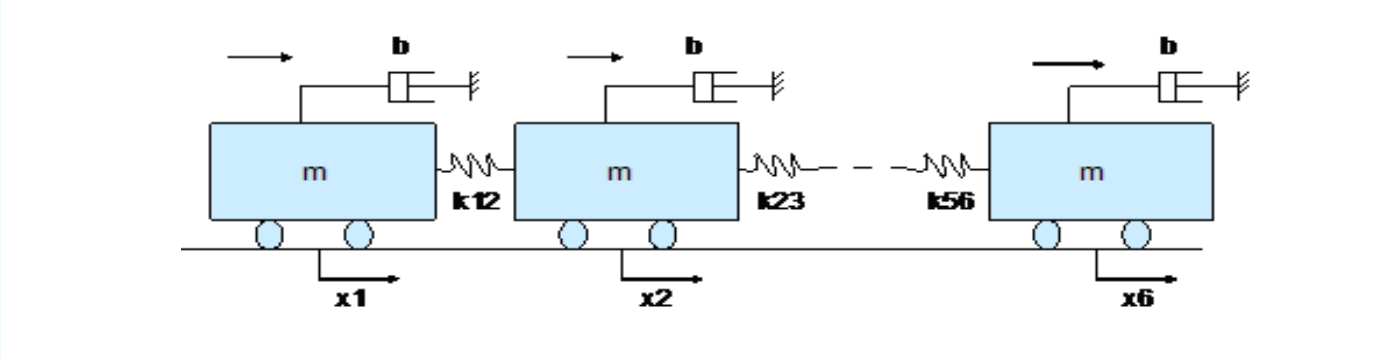

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### MODEL FORMULATIONS

To analyze the coordinated behavior for the networked (MSD) oscillator system (2), we first define the state variables of the $ith$ oscillator as

$$s_i(t) = (u_i(t),\, v_i(t))^{\top} \in R^2,$$

where $u_i(t) = x_i(t)$ and $v_i(t) = \dot{x}_i(t)(i = 1, 2, \cdots, N)$ denote the displacement and velocity of the $ith$ oscillator system, respectively. Thus, system (2) can be written as

$$\begin{cases} \dot{u}_i(t) = v_i(t), \\ \dot{v}_i(t) = -bv_i(t) + \sum\limits_{j=1, j\neq i}^{N} k_{ij}\,(u_j(t) - u_i(t))[1 + \epsilon(u_j(t) - u_i(t))^2] + F\cos(\omega t). \end{cases} \quad (3)$$

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### DEFINITION OF SYNCHRONIZATION STATES

**Definition 1.** Let $s_i(t) = (u_i(t), v_i(t))^\top$ $(i = 1, 2, \cdots, N)$ be a solution of the $ith$ (MSD) oscillator with an given initial conditions $w_{i0} = (u_{i0}, v_{i0})^\top \in R^2$ and $t_0 \in R^+$, then $s^*(t) = (u^*(t), v^*(t))^\top$ is said to be the synchronization state of the networked oscillator system (10), if

$$\lim_{t\to+\infty} \|s_i(t) - s^*(t)\| = 0, \qquad i = 1, 2 \cdots, N. \tag{4}$$

In general, the synchronization state $s^*(t)$ is dependent on the given initial conditions for system (2). Obviously, if an exact solution of synchronization state is explicitly given, then the networked oscillator system (2) will be globally asymptotically synchronized for any given initial conditions, i.e.,

$$\lim_{t\to+\infty} \|s_i(t) - s_j(t)\| = 0, \qquad i, j = 1, 2 \cdots, N, \tag{5}$$

for any $w_{i0} \in R^2$ and $t_0 \in R^+$.

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### AN EXACT SOLUTION OF SYNCHRONIZATION STATES

**Theorem 1.** Let $s_i(t) = (u_i(t), v_i(t))^\top (i = 1, 2, \cdots, N)$ be a solution of the $ith$ oscillator in system (2) under given initial conditions $w_{i0} = (u_{i0}, v_{i0})^\top \in R^2$ and $t_0 \in R^+$, then all the states $s_i(t)\,(i = 1, 2, \cdots, N)$ of the networked oscillator systems (2) will globally asymptotically convergence to the synchronization state $s^*(t)$ as $t \to +\infty$, where $s^*(t) = (u^*(t), v^*(t))^\top$ can be explicitly given by

$$\begin{cases} u^*(t) = \beta - \dfrac{F}{\omega b}\sin(\omega t_0) + \dfrac{F}{\omega\sqrt{b^2+\omega^2}}\sin(\omega t - \varphi), \\ v^*(t) = \dfrac{F}{\sqrt{b^2+\omega^2}}\cos(\omega t - \varphi), \end{cases} \tag{6}$$

where

$$\beta = \frac{1}{N}\sum_{j=1}^{N} u_{j\,0} + \frac{1}{bN}\sum_{j=1}^{N} v_{j\,0} \qquad \text{and} \qquad \varphi = \arctan(\frac{\omega}{b}). \tag{7}$$

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### REMARKS

**Remark 1.** For given initial values with respect to system (2), it is known from Theorem 1 that an exact solution of synchronization state $s^*(t)$ of system (2) can be explicitly given, which is the simplest type of harmonic motion.Accordingly, an interesting conclusion is reached that the networked oscillator system can be synchronized to a simple harmonic motion by mild nonlinear network connectivity, even if the isolated (MSD) oscillator is unstable, chaotic or others complex dynamics itself. It is obvious that the results is indeed coincident with the practical situation related to coordinated behavior of the networked controlled systems with unstable dynamics of the agents.

**Remark 2.** It is important to emphasize that, a distinctive feature of Theorem 1 is to give an exact solution of synchronization state of the networked oscillator systems. However, it is still a difficult problem to estimate an explicit solution of synchronization state for more general coupled oscillator systems, especially for nonlinearly coupled oscillator systems.

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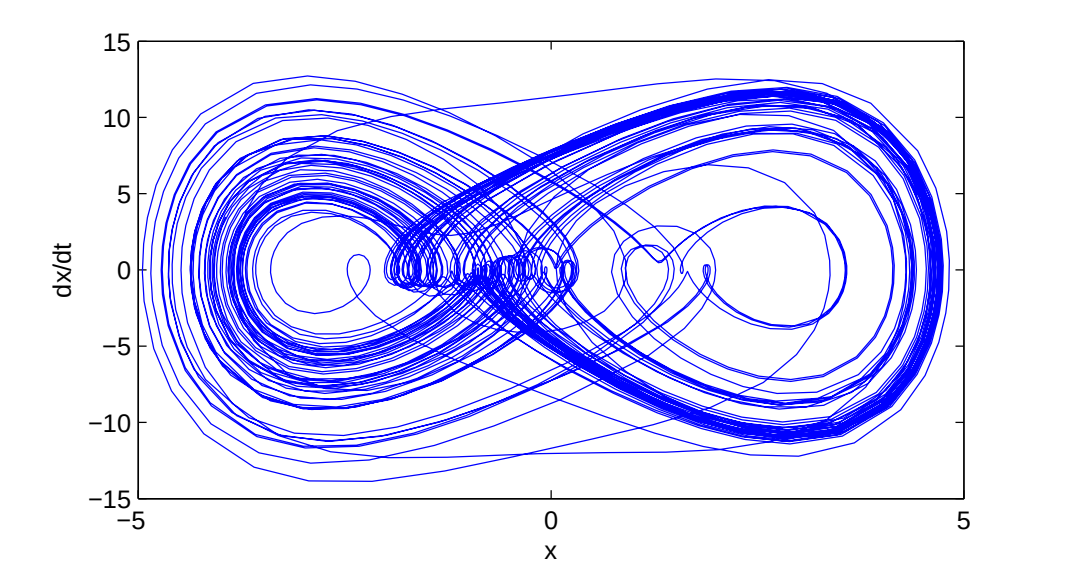
### APPLICATIONS AND SIMULATIONS

We here consider a simple model of the chain networked mechanical systems composing of six (MSD) oscillators/carts with cosinoidal excitation (Fig.1).

As is well known, the isolated (MSD) oscillator can exhibit rich behavior including quasi-period, period and chaos. It is known that with the parameters $b = 0.2$, $F = 27$ and $\omega = 1.43$, the solution trajectory of Eq. (1) approaches a chaotic attractor [**?**], as shown in Fig. 2.



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### APPLICATIONS AND SIMULATIONS

Let the stiffness matrix $K$ be selected as the following two diagonal matrices, respectively,

$$K_1 = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 1 & -2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix} \text{ and } K_2 = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 1 & -3 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 5 & -5 \end{bmatrix},$$

and the initial conditions of system (2) are taken as follows

$$\begin{aligned} w_{10} &= (9,-1)^\top, \qquad w_{20} = (-8,2)^\top, \qquad w_{30} = (-7,-3)^\top, \\ w_{40} &= (-5,-9)^\top, \qquad w_{50} = (7,-6)^\top, \qquad w_{60} = (8,4)^\top. \end{aligned} \tag{8}$$

for the initial time $t_0 = 0$. Therefore, it follows Theorem 1 that the synchronization state $s^*(t) = (u^*(t), v^*(t))^\top$ for such networked (MSD) oscillator system is explicitly given by

$$\begin{cases} u^*(t) \approx -10.17 + 13.07 \sin\big(1.43t - \arctan(7.15)\big), \\ v^*(t) \approx 18.69 \cos\big(1.43t - \arctan(7.15)\big). \end{cases} \tag{9}$$

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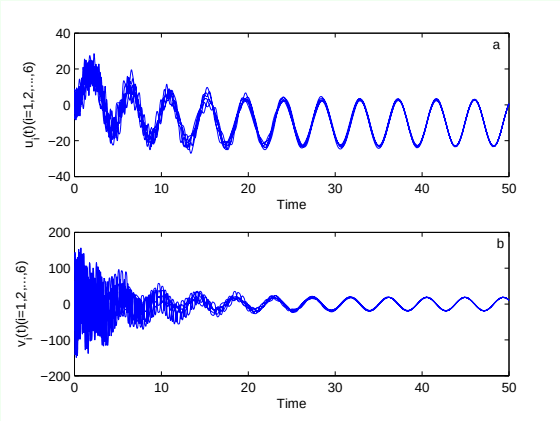
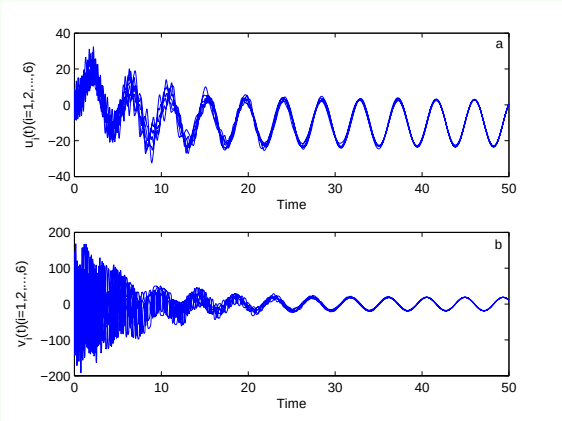
### APPLICATIONS AND SIMULATIONS

Figs. 3-4 are the simulation results corresponding to this situations, which clearly show the evolution process of the synchronization state of system (2) with two kind of the stiffness matrix in time interval $[0,\ 50]$, respectively. It is explicitly shown that the networked (MSD) oscillator system (2) can be synchronized to a simple harmonic motion by mild nonlinear network connectivity, even if the isolated (MSD) oscillator is chaotic or others, and so this conclusion is indeed consistent with theoretical results.



# 二、我们的工作

## 网络型质量-弹簧振子系统的同步动力学

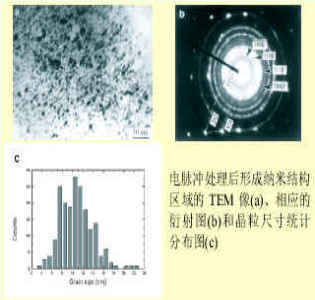
### CONCLUSIONS AND SIMULATIONS

In this work, we have investigated the synchronization dynamics for a class of networked mass-spring-damper (MSD) oscillator systems with nonlinear spring interaction. The primary contribution of this work is to give an exact solution of synchronization state for such networked oscillator systems. Accordingly, an interesting conclusion is reached that the networked oscillator systems can be synchronized to a simple harmonic motion by mild nonlinear network connectivity, even if the isolated (MSD) oscillator has complex dynamics itself. Numerical examples are given to demonstrate the effectiveness of the theoretical results, too. It is believed that the obtained results should provide some practical guidelines for the designs and implementations of coordinated control of networked dynamical systems in engineering applications.

# 2 网络系统的脉冲一致性

## 网络系统中脉冲现象

脉冲：作为表征自然界中一种瞬时突变行为，脉冲现象往往更能深刻的,精确地反映动态系统的演化规律



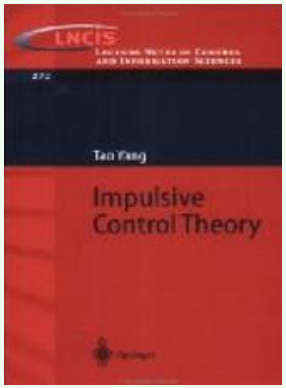
脉冲动力系统已成功地应用于大型航天器的减振装置、卫星轨道的转换、移动通讯中传输信号切换等. 生物神经实验表明：在生物的神经系统中神经元之间的传递方式是以电脉冲的过程进行的, 从而实现神经信息的产生、整合和传递.

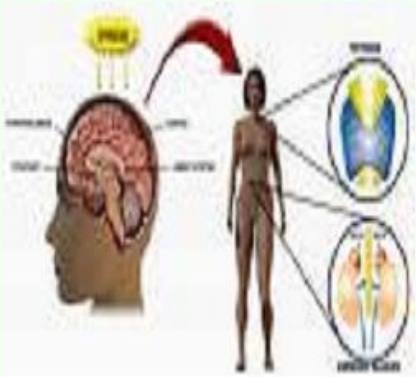
# 一、背景及意义

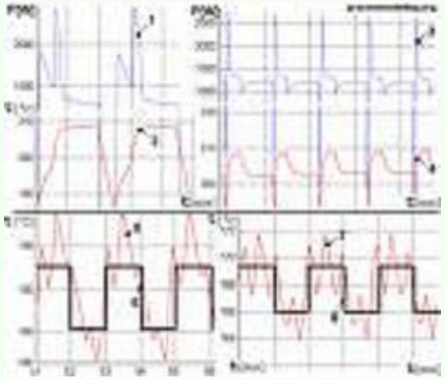
## 网络系统中脉冲一致性

脉冲控制技术：一种优于一些连续控制方案的易于实现的混沌控制技术

脉冲控制特征：简单性，有效性与鲁棒性





研究多个体网络系统脉冲一致性的动力学与控制，尤其是具有时滞与脉冲效应的大规模的复杂混杂网络系统(Complex Hybrid Network Systems)一致性问题的动力学与控制，有助于深刻理解真实世界中大多数复杂多个体网络的动力学行为及其演化规律，具有重要的科学意义及广阔的应用前景.

# 二、我们的工作

## 时滞动力系统的脉冲稳定性

### 一、脉冲时滞线性微分方程的稳定性

1. Jin Zhou, Quanjun Wu.: Exponential Stability of Impulsive Delayed Linear Differential Equations. IEEE Trans. Circ. Syst. II. **56** (2009) 744-748

考虑一般脉冲时滞线性方程:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau), \ t \neq t_k, \ t \geq t_0 \\ \Delta x(t) = C_k x(t^-), \qquad t = t_k, \ k \in N \end{cases}$$

我们证明如下主要结果：一、如果脉冲时滞线性方程是(局部)渐进稳定的，则它一定是(全局)指数稳定的; 二、脉冲能够使得一个脉冲时滞线性方程全局指数稳定，即使它本身可能是不稳定的.

### 二、具有任意时滞动力系统脉冲稳定性

2. Quanjun Wu., Jin Zhou., Lan Xiang.: Global exponential stability of impulsive differential equations with any time delays. Applied Mathematics Letters. **22** (2009) doi:10.1016/j.aml.2009.09.001

# 二、我们的工作

## 时滞动力系统的脉冲稳定性

考虑脉冲非线性时滞动力系统:

$$\begin{cases} \dot{x}(t) = f(t, x_t), & t \neq t_k, t \geq t_0 \\ \Delta x(t_k) = I_k(t_k, x_{t_k^-}), & k \in \mathbb{N} \\ x_{t_0} = \phi. \end{cases}$$

相对于目前同类问题的研究结果,我们主要创新点: 一、给出了具有任意时滞的非线性动力系统脉冲全局指数稳定的充分条件; 二、所获结果能够设计一种脉冲控制策略来稳定和同步任意一个时滞非线性动力系统,即使它本身可能是不稳定的，甚至是混沌的.

三、应用

网络控制系统的一致性 (Consensus of networked control systems);

耦合振子同步( Synchronizations of coupled oscillators);

神经网络的设计及应用(Designs and applications of neural networks);

Lagrange力学网络协调控制(Coordinated control of Lagrange mechanical networks)

# 二、我们的工作

## 混沌神经网络的脉冲稳定性

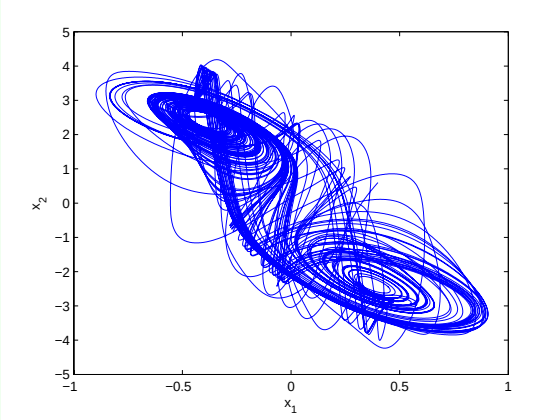
考虑一个具有两个神经元的时滞Hopfield神经网络

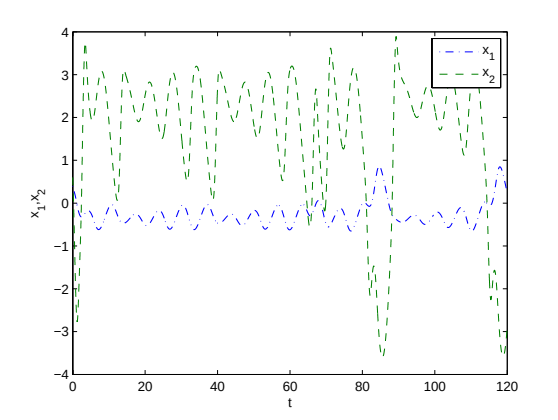
$$\dot{x}(t) = f(t, x(t), x(t-\tau)) = -Cx(t) + Ag(x(t)) + Bg(x(t-\tau(t))),$$

这里$x(t) = (x_1(t), x_2(t))^\top, g(x(t)) = (\tanh(x_1(t)), \tanh(x_2(t)))^\top, \tau(t) = 1$, 和

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{bmatrix} \quad \text{with} \quad B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{bmatrix}.$$

这个网络是一个混沌时滞的Hopfield神经网络.




A fully developed double-scroll-like chaotic attractors of the isolate delayed Hopfield neural network.

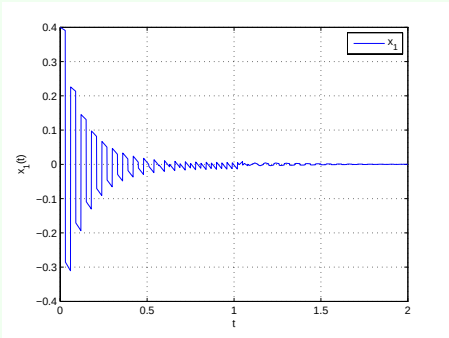
# 二、我们的工作

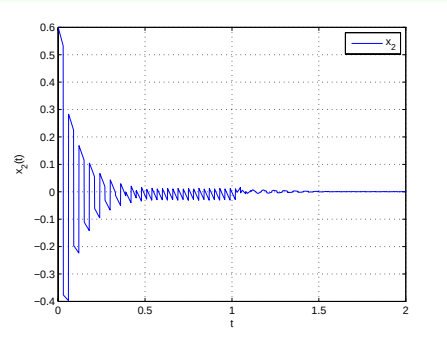
## 混沌神经网络的脉冲稳定性

对以上混沌时滞的Hopfield神经网络神经元的连接引入脉冲效应有

$$\begin{cases} Dx_1(t) = -x_1(t) + 2f_1(x_1(t))Du_{11} - 0.1f_2(x_2(t))Du_{12} \\ \qquad -1.5g_1(x_1(t-\tau(t)))Dw_{11} - 0.1g_2(x_2(t-\tau(t)))Dw_{12}, \\ Dx_2(t) = -x_2(t) - 5f_1(x_1(t))Du_{21} + 3f_2(x_2(t))Du_{22} \\ \qquad -0.2g_1(x_1(t-\tau(t)))Dw_{21} - 2.5g_2(x_2(t-\tau(t)))Dw_{22}, \end{cases}$$

取脉冲间隔$\Delta t = t_m - t_{m-1} = 0.02$, 脉冲增益$\alpha_{11}^{(k)} = -0.9$, $\alpha_{22}^{(k)} = -0.6$, $\alpha_{12}^{(k)} = \alpha_{21}^{(k)} = \beta_{ij}^{(k)} = 0.01$, $i, j = 1, 2$, 和$\lambda = 0.05$, 我们可以得到这个具有脉冲神经元连接的Hopfield神经网络是全局指数稳定的.




Stability process of the state variables in chaotic delayed neural networks with impulses in time interval $[0, 2]$.

# 二、我们的工作

## 网络系统的一致性协议

### 一、一致性问题的描述

所谓多个体动态网络系统的一致性问题(Consensus Problems)是指随着时间的演化, 所有的多个体的最终状态趋于一致. 一致性协议（算法）是多个体之间相互作用的规则,它描述了多个体之间的信息交换过程.

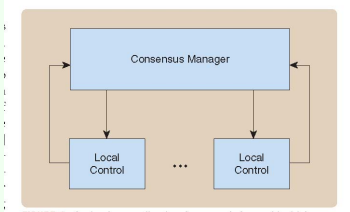
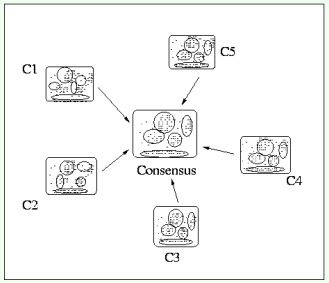



FIGURE 3 A simple coordination framework for multivehicle rendezvous. The consensus manager applies distributed consensus algorithms to guarantee that the team reaches consensus on a rendezvous objective. Based on the output of the consensus manager, each vehicle applies a local control law to achieve the rendezvous objective.




假设每个主体的动力学如下:

$$\dot{x}_i(t) = u_i(t), \qquad (x_i(t+1) = u_i(t)), \tag{10}$$

其中$x_i$表示第$i$个主体的状态,$u_i(t)$为时刻$t$的控制输入(协议).复杂多个体网络系统分布式协同控制的一个关键问题在于设计合适的协议和算法,使得系统中所有的个体可以达到一致.

# 二、我们的工作

## 网络系统的一致性协议

二、已有的一致性协议

时滞1: $\dot{x}_i(t) = \sum_{j\in N_i} a_{ij}(x_j(t-\tau_{ij}) - x_i(t-\tau_{ij}))$,

R. Olfati-Saber, R. M. Murray.: Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. on Autom. Contr., 49, 1520-1533, 2004.

时滞2: $\dot{x}_i(t) = \sum_{j\in N_i} a_{ij}(x_j(t-\tau_{ij}) - x_i(t))$,

L. Moreau.: Stability of multiagent systems with time-dependent communication links. IEEE Trans. on Autom. Contr., 50, 169-182, 2005.

时滞3: $\dot{x}_i(t) = \sum_{j\in N_i} a_{ij}(x_j(t-\tau_{ij}(t)) - x_i(t-\tau_{ij}(t)))$,

Y. G. Sun, L. Wang, G.M. Xie.: Average consensus in networks of dynamic agents with switching topologies and multiple time-varying delays. Systems & Control Letters, 57, 175-183, 2008.

上述协议仅适用于具有较小的通讯时滞的多主体网络系统, 而对于具有较大的通讯时滞的网络系统上述协议失效.

# 二、我们的工作

## 一个典型的一致性协议

1520 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 9, SEPTEMBER 2004

# Consensus Problems in Networks of Agents With Switching Topology and Time-Delays

Reza Olfati-Saber, *Member, IEEE,* and Richard M. Murray, *Member, IEEE*

$$u_i(t) = \sum_{v_j \in N_i} a_{ij}\left[x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})\right] \qquad \text{(A2)}$$

*Theorem 10:* Consider a network of integrator agents with equal communication time-delay $\tau > 0$ in all links. Assume the network topology $G$ is fixed, undirected, and connected. Then, protocol (A2) with $\tau_{ij} = \tau$ globally asymptotically solves the average-consensus problem if and only if either of the following equivalent conditions are satisfied.

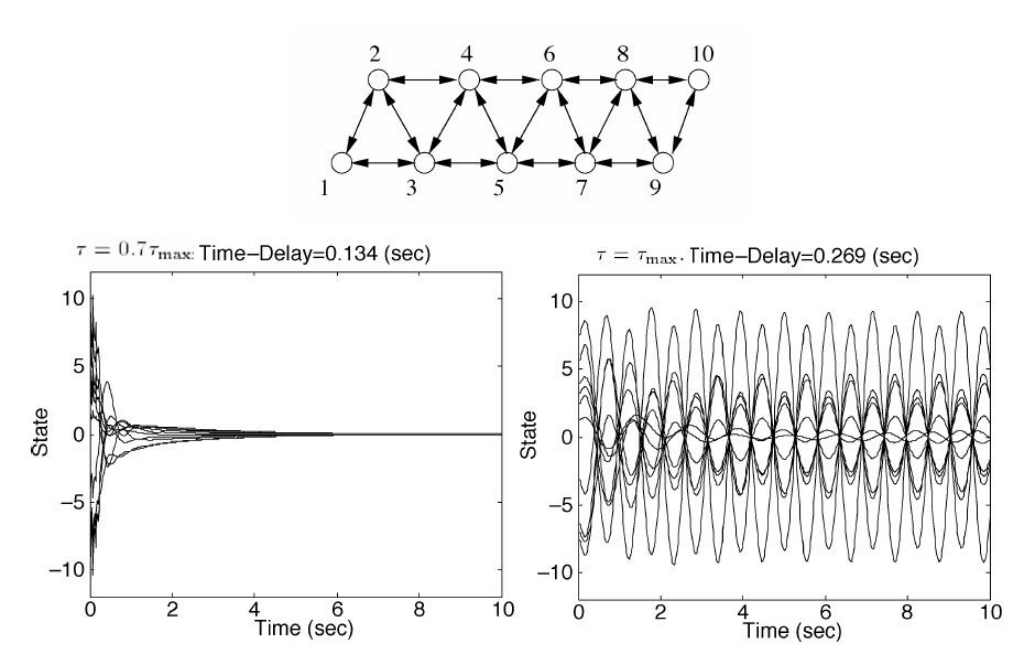
i) $\tau \in (0, \tau^*)$ with $\tau^* = \pi/2\lambda_n$, $\lambda_n = \lambda_{\max}(L)$.

ii) The Nyquist plot of $\Gamma(s) = e^{-\tau s}/s$ has a zero encirclement around $-1/\lambda_k, \forall k > 1$.

Moreover, for $\tau = \tau^*$ the system has a globally asymptotically stable oscillatory solution with frequency $\omega = \lambda_n$.

# 二、我们的工作

## 一个典型的一致性协议



# 二、我们的工作

## 网络系统脉冲一致性协议

### 一、具有任意通讯时滞的脉冲一致性

#### 1、脉冲一致性协议

我们提出下面的脉冲一致协议:

$$\begin{aligned}\dot{x}_i(t) &= \sum_{j\in N_i} a_{ij}(x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})) \\ &+ \sum_{m=1}^{+\infty}\sum_{j=1,j\neq i}^{n} b_{ij}(x_j(t_m^-) - x_i(t_m^-))\delta(t-t_m),\end{aligned} \tag{11}$$

其中$b_{ij}$为脉冲控制增益, $\delta(t)$是狄拉克函数.

如果$b_{ij}=0$, 则协议(11)变成一个基于节点$v_i$邻居节点的一致协议,

$$\dot{x}_i(t) = \sum_{j\in N_i} a_{ij}(x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})),$$

显然它是由Olfati-Saber和Murray提出的一致性协议的推广.

# 二、我们的工作

## 网络系统脉冲一致性协议

### 2、网络具有固定拓扑

在协议(11)下，系统(10)具有下面的形式:

$$\begin{cases} \dot{x}(t) = -Lx(t-\tau(t)), & t \neq t_m, t \geq t_0, \\ \Delta x(t) = -Mx(t^-), & t = t_m, m \in N, \end{cases} \tag{12}$$

其中$M = (-b_{ij})_{n\times n}$ 为一Laplacian矩阵, 它的特征值为$0 = \lambda_1(M) < \lambda_2(M) \leq \cdots \leq \lambda_n(M)$.

**定理 1** 考虑时滞动力网络(12)，对于$m \in N$, 如果下列条件成立:

$(A_1)$ $\|M\|^2 < 2\lambda_2(M^s)$;

$(A_2)$ 存在一个正常数$\alpha > 0$, 使得

$$\tau\|L\|^2 + \kappa\|L\|^2 + \frac{\tau\|L\|^2 + \kappa\|M\|^2}{1 - 2\lambda_2(M^s) + \|M\|^2} - 2\lambda_2(L^s) \leq \alpha,$$

其中$\kappa = \left[\dfrac{\tau}{\min_{m\in Z^+}\{t_m - t_{m-1}\}}\right] + 1$;

$(A_3)$ $\ln\{1 - 2\lambda_2(M^s) + \|M\|^2\} + \alpha(t_m - t_{m-1}) \leq \beta < 0$.

则时滞动力网络(12)可一致渐进地达到平均一致.

# 二、我们的工作

## 网络系统脉冲一致性协议

注释 1　定理1表明时滞动力网络(12)的平均一致性不仅依赖于整个网络的拓扑结构$L$, 还依赖于脉冲增益矩阵$M$ 和脉冲间隔$t_m - t_{m-1}$.

注释 2　脉冲一致协议(2)具有如下两个作用: 其一，如果$\tau(t) < \tau^*$,它可以加速网络系统(3)的平均一致；其二，如果$\tau(t) > \tau^*$, 则协议(11)对于任意的通讯时滞能使网络系统(12)达到平均一致.

### 3、网络具有切换拓扑

由于网络中的节点不是静态的,不难想象由于某些节点之间存在障碍物而使得这些节点之间的通讯边失效,同样可产生相反的情况, 原来无连接的某些节点可能会生成新的边.考虑下面的拓扑切换网络:

$$\begin{cases} \dot{x}(t) = L_k x(t-\tau(t)), \ t \neq t_k, \ t \geq t_0, \ k = s(t) \in I_\Gamma, \\ \Delta x(t) = -Mx(t^-), \qquad t = t_k, \ k \in N \end{cases} \tag{13}$$

其中$L_k = L(G_k)$是图$G_k = (V, \varepsilon_k, A_k)$ 的拉普拉斯矩阵, 且属于集合$\Gamma$. 集合$I_\Gamma = \{1, 2, \cdot, N\}$($N$ 表示所有可能的有向图总数). 映射$s(t) : R \to I_\Gamma$ 是一个切换信号,它决定了网络的拓扑. 这里假设通讯拓扑是强连通且平衡的. 如果$s(t)$是常数，则对应的拓扑是固定的.

# 二、我们的工作

## 网络系统脉冲一致性协议

### 3、网络具有切换拓扑

我们设矩阵$L_k^s$的特征值

$$0=\lambda_1(L_k^s)<\lambda_2(L_k^s)\leq\cdots\leq\lambda_n(L_k^s),$$

其中$\overline{\lambda}_2=\min_{k\in I_\Gamma}\lambda_2(L_k^s),\quad \|\overline{L}\|=\max_{k\in I_\Gamma}\|L_k\|.$

定理 2 考虑时滞动力网络(13)，如果下如果定理1中的$(A_1)(A_3)$和下面条件成立：：

$(A_2^{'})$ 存在一个正常数$\alpha>0$, 使得

$$\tau\|\overline{L}\|^2+\kappa\|\overline{L}\|^2+\frac{\tau\|\overline{L}\|^2+\kappa\|M\|^2}{1-2\lambda_2(M^s)+\|M\|^2}-2\overline{\lambda}_2\leq\alpha,$$

则时滞动力网络(13)可一致渐进地达到平均一致.

# 二、我们的工作

## 网络系统脉冲一致性协议

### 4、数值计算与模拟

考虑含有100个节点且具有固定拓扑的有向网络,如图1所示,容易看出它是一个强连通图. 图2表示时滞$\tau(t)=\tau^*=\dfrac{\pi}{2\lambda_n}=\dfrac{\pi}{8}$时, 动力网络(12)在不加脉冲时在时间$[0,20]$ 内的状态变量的变化过程，这说明当不加脉冲时网络是无法达到平均一致的.

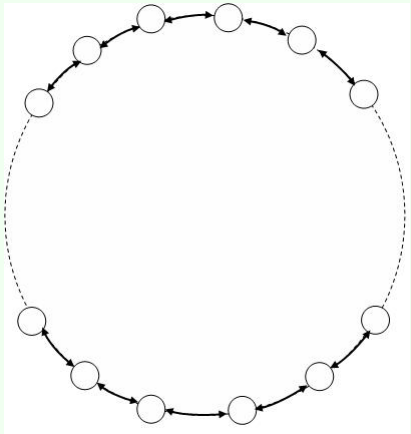


图1

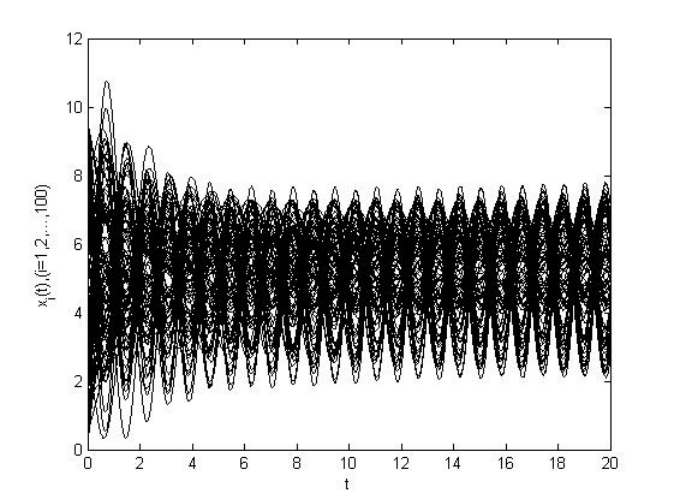



图2

# 二、我们的工作

## 网络系统脉冲一致性协议

### 4、数值计算与模拟

令脉冲间隔$\Delta t = t_m - t_{m-1} = 0.02$, 时滞$\tau(t) = 1$, 图3、图4分别为时滞动力网络(12)随不同的脉冲增益矩阵$m_{ij} = -0.015, i \neq j, m_{ij} = 1.485, i = j$; $m_{ij} = -0.018, i \neq j, m_{ij} = 1.782, i = j$在时间$[0, 2]$ 内的状态变量的变化过程. 他们都表明网络能够很快达到平均一致，且脉冲增益影响着网络的一致性.

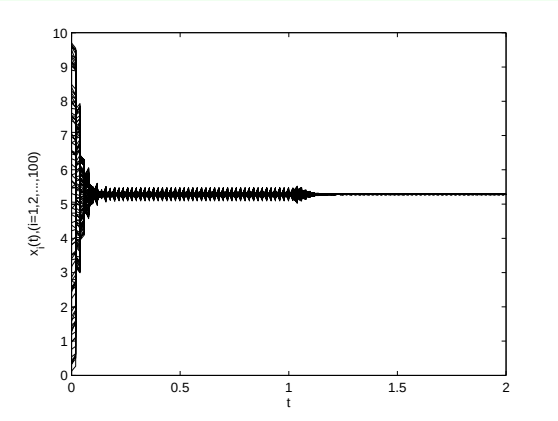



图3

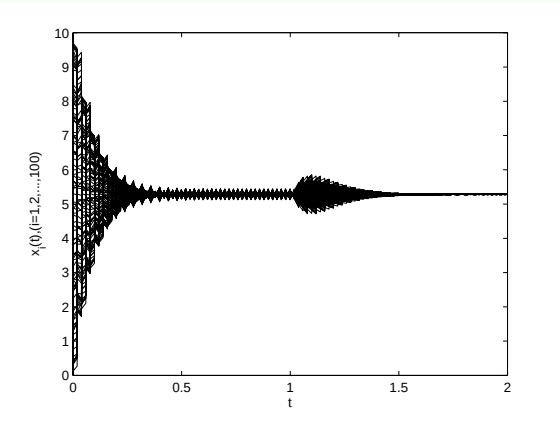



图4

# 二、我们的工作

## 网络系统脉冲一致性协议

二、具有脉冲拓扑切换的一致性

Quanjun Wu, Lan Xiang, Jin Zhou.: Average consensus in delayed networks of dynamic agents with impulsive effects, Lecture Notes in CSSITE, Complex 2009, Berlin: Springer, Part I, LNICST 4, 1124 – 1138, 2009.

### 1、脉冲一致性协议

考虑网络结构上具有脉冲效应, 我们提出下面的一致协议:

$$Dx_i(t) = \sum_{j\in N_i} a_{ij}(x_j(t-\tau_{ij}) - x_i(t-\tau_{ij}))Dw_j(t), \tag{14}$$

这里算子$D$表示分布导数，$w_i : [t_0, +\infty) \to R$是在$[t_0, +\infty)$中任意紧的子区间有界变差函数, $Dw_i$描述在网络的拓扑结构中脉冲的效应. 假设

$$Dw_j = 1 + \sum_{m=1}^{+\infty} u_m \delta(t - t_m), \qquad j = 1, 2 \cdots, N.$$

这里对每一固定的时刻$t_k$满足$t_{k-1} < t_k$和$\lim_{k\to\infty} t_k = +\infty$, $u_k$是常数.

如果$u_m = 0$, 则协议(1)变成一个基于节点$v_i$邻居节点具有时滞线性一致协议, 显然它是由Olfati-Saber和Murray提出的一致性协议的一个一般化.

# 二、我们的工作

## 网络系统脉冲一致性协议

### 2、网络具有固定拓扑

在协议(14)下，系统(10)具有下面的形式:

$$Dx(t) = Lx(t-\tau(t))Dw(t), \tag{15}$$

**定理 3** 考虑时滞动力网络(15)，对于$m \in N$, 如果下列条件成立:

$(A_1)$ $p = 2\lambda_2(L^s) - \varepsilon_1\|L\|^2\tau - \varepsilon_2\kappa\mu_m\|L\|^2 > q = \varepsilon_1^{-1}\|L\|^2\tau + \varepsilon_2^{-1}\kappa\mu_m\|L\|^2$,

其中$\varepsilon_1, \varepsilon_2 \geq 0$, $\kappa$见定理1.

$(A_2)$ 让$\lambda > 0$ 和$\varepsilon > 0$满足$\lambda - p + qe^{2\lambda\tau} \leq 0$, 和

$$\theta_m = 1 + \varepsilon + (1+\varepsilon^{-1})\mu_m^2\|L\|^2 e^{2\lambda\tau}, \quad \theta = \sup_{m\in Z^+}\left\{\frac{\ln\theta_m}{t_m - t_{m-1}}\right\}$$

使得$\theta < \lambda$.

则时滞动力网络(15)可以指数地达到平均一致.

**注释 3** 定理1表明时滞动力网络(15)的平均一致性不仅依赖于整个网络的拓扑结构$L$, 还依赖于脉冲增益$d_{im}$ 和脉冲间隔$t_m - t_{m-1}$.

# 二、我们的工作

## 网络系统脉冲一致性协议

### 3、网络具有切换拓扑

在协议(14)下，系统(10)具有下面的形式:

$$Dx(t) = L_k x(t-\tau(t))Dw(t),\ k = s(t) \in I_\Gamma, \tag{16}$$

**定理 3** 考虑时滞动力网络(16)，对于$m \in N$, 如果下列条件成立:

$(A_1)$ $p = 2\lambda_2(L^s) - \varepsilon_1\|\overline{L}\|^2\tau - \varepsilon_2\kappa\mu_m\|\overline{L}\|^2 > q = \varepsilon_1^{-1}\|\overline{L}\|^2\tau + \varepsilon_2^{-1}\kappa\mu_m\|\overline{L}\|^2$,

$(A_2)$ 让$\lambda > 0$ 和$\varepsilon > 0$满足$\lambda - p + qe^{2\lambda\tau} \le 0$, 和

$$\theta_m = 1 + \varepsilon + (1+\varepsilon^{-1})\mu_m^2\|\overline{L}\|^2 e^{2\lambda\tau}, \quad \theta = \sup_{m\in Z^+}\left\{\frac{\ln\theta_m}{t_m - t_{m-1}}\right\}$$

使得$\theta < \lambda$.

则时滞动力网络(16)可以指数地达到平均一致.

# 二、我们的工作

## 网络系统脉冲一致性协议

### 4、数值计算与模拟

考虑具有10个节点切换拓扑$\left\{G_a, G_b, G_c, G_d\right\}$的有向网络，如图5所示。图6表示具有切换拓扑的时滞系统四个状态$\left\{G_a, G_b, G_c, G_d\right\}$之间的变化。系统从$G_a$开始，每个$0.1s$跳到下一个状态. 在这种情况下, 随着演化一些边失效, 而另外一些边则生成.

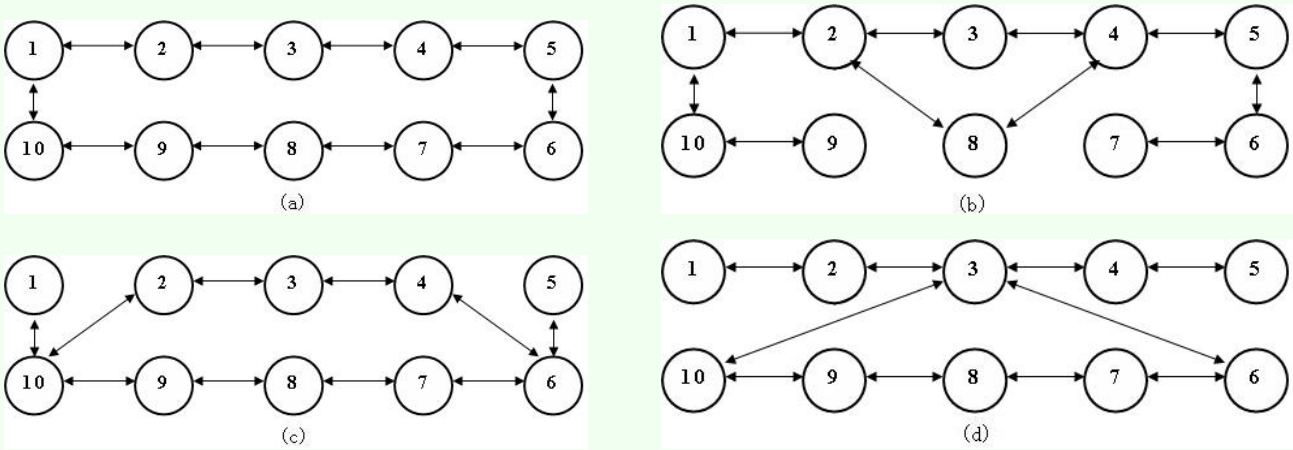


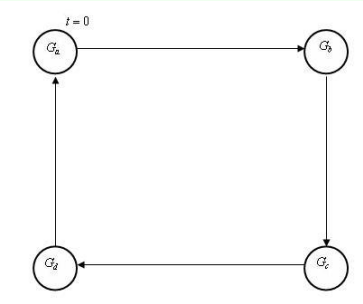

图5

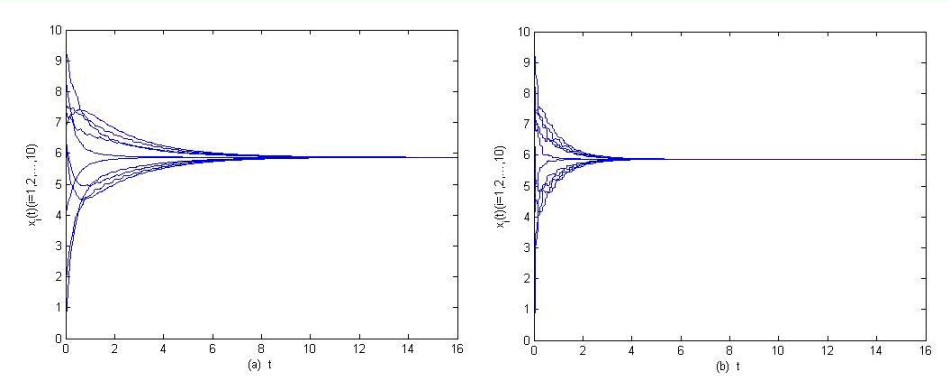
# 二、我们的工作

## 网络系统脉冲一致性协议

### 4、数值计算与模拟

令时滞$\tau(t)=0.005$, 脉冲增益$\mu=0.15$, 图7显示了时滞动力网络系统(16)在时间$[0,16]$分别为取脉冲间隔$\Delta t=0.5$和$\Delta t=0.05$的状态变量的变化过程.




$(a)\Delta t=0.5$ $(b)\Delta t=0.05$

图6和图7

# 二、我们的工作

## 总结

从动力学与控制的角度考虑具有固定与切换拓扑有向多个体时滞网络系统的平均一致性问题, 提出了在通讯时滞环境中两个网络的脉冲一致性协议, 并给出了寻求相应平均一致性问题的两个简单而又一般的判据. 本文工作最显著的特征表现如下:

一、提出的第一个一致性协议对于任意的通讯时滞的网络系统是有效的, 给出的第二个一致性协议能使网络系统指数地达到平均一致

二、给出的网络脉冲一致性协议具有简单性、有效性和鲁棒性等重要特征.

三、所获得结果表明，网络的一致性协议中的脉冲增益在寻求平均一致性问题中具有相当重要的作用.

# 二、我们的工作

## 复杂脉冲网络的同步动力学与控制

关于复杂脉冲网络同步动力学与控制的工作,可参见我们其它如下工作:


1. Zhou, J., Xiang, L., Liu, R.: Synchronization in Complex Delayed Dynamical Networks with Impulsive Effects. Physica A. **384** (2007) 684-692.

2. Cai, S., Zhou, J., Xiang, L., Liu, R.: Robust Impulsive Synchronization of Complex Delayed Dynamical Networks. Physica Letters A. **372** (2008) 4990-4995.

3. Zhou, J., Xiang, L., Liu, Z.: Robust Impulsive Synchronization of Coupled Delayed Neural Networks. Lecture Notes in Computer Science, Vol. 4492. Springer-Verlag, Berlin Heidelberg New York (2007) 16-23.

4. Xiang, L., Zhou, J., Liu, Z.: Global Synchronization of Impulsive Coupled Delayed Neural Networks, Lecture Notes in Computer Science, Vol. 3971. Springer-Verlag, Berlin Heidelberg New York (2006) 303-308.

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