SOILS AND FOUNDATIONS Vol. 30, No. 1, 55-64, Mar. 1990
Japanese Society of Soil Mechanics and Foundation Engineering

LOAD-SETTLEMENT ANALYSIS FOR BORED PILES USING HYPERBOLIC TRANSFER FUNCTIONS

HIDEKI HIRAYAMA[i)]

ABSTRACT

A practical method for analyzing the load-settlement behaviour of bored cast-in-place plies has been proposed. The method employs Kondner-type hyperbolic load-transfer functions, by which non-linear behaviour can be simulated plainly and simply, for skin resistance and end resistance. The parameters required may be determined from the results of usual in-situ and/or laboratory tests. The procedure for determining the parameters is discussed in detail : It is indicated that displacements needed to mobilize half of the ultimate skin resistance are as small as 1/100 of those of the ultimate end resistance. One-dimensional finite element formulation, which personal computers are enough to solve, for the model proposed is described. Analytical results of loading tests for large-diameter bored piles are discussed. The method proposed may be useful for the following situations : the preliminary analysis for planning an appropriate loading-test program and the back analysis of loading tests for optimizing the design.

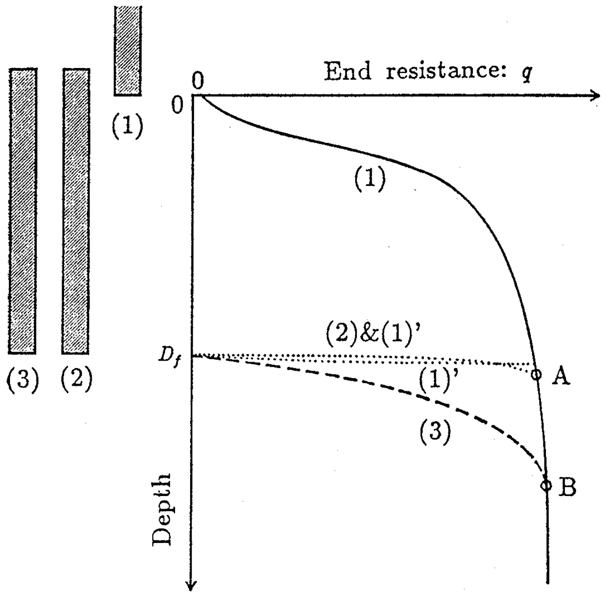
Key words : bearing capacity, cast-in-place pile, in-situ test, finite element method, friction, load test, pile, settlement, vertical load (**IGC** : E 4/E 2)

INTRODUCTION

Non-displacement piles, such as bored cast-in-place piles, carry most of the applied vertical load by shaft friction under the working load or design load. This situation arises from the large difference in required displacement between ultimate skin friction and ultimate end resistance.

It is most reasonable to define physical ultimate end resistance as the end resistance of point A for displacement piles or point B for non-displacement piles as shown in Fig. 1 (Whitaker, 1976 ; Kishida and Takano, 1977 ; De Beer, 1988). As the ultimate bearing capacity does not increase so much below a certain depth (e. g. Kerisel, 1964 ; Hirayama, 1988), the ultimate bearing capacity at the depth D_f of a non-displacement pile is practically equal to that of a displacement pile. While tip settlements required to mobilize physical ultimate end resistance are about 10% of the base diameter for displacement piles, they are more than one base-diameter for non-displacement piles (Kerisel, 1964 ; BCP Committee, 1971 ; De Beer, 1988).

i) Chief Research Engineer, Osaka Soil Test Lab., 3-1-23 Nishi-Hommachi, Nishi-ku, Osaka 550.
Manuscript was received for review on January 26, 1989.
Written discussions on this paper should be submitted before October 1, 1990, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.



(1) jacked pile from ground surface
(1)' rebound/reloading of jacked pile
(2) displacement pile (embedded depth = D_f)
(3) non-displacement pile (embedded depth = D_f)

Fig. 1. Schematic physical ultimate end resistance

This large difference results from the prestress conditions caused by pile installation in the surrounding ground of the pile tip. On the other hand, displacements required to mobilize ultimate skin friction are only 0.5%-2% of the shaft diameter (Reese, 1978 ; De Beer, 1988), and are not influenced so much by the method of pile installation (e.g. Komada, 1975).

Because of impractical tip displacements required to mobilize ultimate end resistance for non-displacement piles, various bearing capacity criteria which are empirical in nature have been proposed and used by different researchers and design organizations (Vesic, 1977). The values determined from these criteria are often called ultimate bearing capacity, though they are far from the physical ultimate state for end resistance.

It has been pointed out that end resistance of bored piles, especially long or large-diameter ones, is not so contributive as has been assumed in the conventional design where ultimate vertical resistance is calculated from the sum of so-called ultimate end resistance and ultimate skin resistance. It has been suggested that bored piles should be designed by using the allowable value of settlement taking into account the process of mobilization of skin friction and end resistance (e.g. Focht and O'Neill, 1985). A fair number of theoretical methods for analysing load-settlement behaviour have been proposed based on the load-transfer, elastic-based, or finite-element method (e.g. Poulos and Davis, 1980). Those proposals, however, seem not to be widely used in engineering practice mainly because input data required are difficult to determine from the results of common site investigation.

Among these methods, the load-transfer method is a direct extension of the conventional design. That is to say, the conception of $Q_{ult}=Q_{f,ult}+Q_{e,ult}$ is extended to $Q(S)=Q_f(S)+Q_e(S)$; where Q_f=skin resistance, Q_e=end resistance, S=settlement at the pile head, and the subscript *ult* represents ultimate value. A large number of data on the distribution of axial loads from instrumented pile tests have been obtained, and have revealed the load-transfer characteristics. Personal computers have been so widely available that any non-linear transfer curves can be easily implemented. However, the non-linearity employed should be simple in terms of input parameters.

This paper describes a practical load-transfer method employing hyperbolic transfer functions in which the constants required for calculation can be determined from usual in-situ tests, such as cone penetration tests (CPT) and standard penetration tests (SPT), and/or laboratory compression tests. Loading tests of large-diameter bored piles are analyzed, and the results are discussed from the viewpoint of application to disign.

TRANSFER FUNCTIONS AND PARAMETERS REQUIRED

Hyperbolic Load-Transfer Curves

Two-constant hyperbolas are preferable to other two-constant curves such as bi-linear curves for friction resistance and end resist-

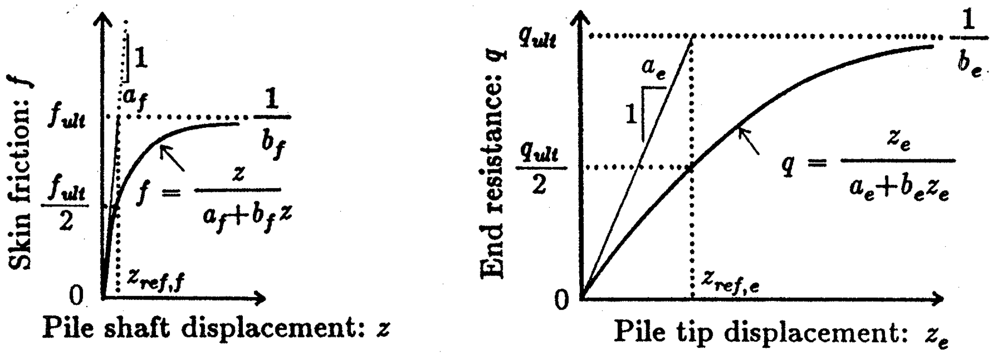


Fig. 2. Hyperbolic stress-displacement curves

ance (e. g. Clough and Duncan, 1971 ; Sabini and Sapio, 1981 ; Takahasi et al., 1981). In addition, the method for determining the constants of the hyperbola based on measured data (e. g. Clough and Duncan, 1971) is rather simpler in terms of calculation and judgement than methods of other two-constant curves.

Therefore, for the relationships between skin friction f and pile shaft displacement at a certain point z, and between end resistance q and tip displacement z_e, the following Kondner-type hyperbolic curves shown in Fig. 2 are employed :

$$f=\frac{z}{a_f+b_f z} \quad (1)$$

$$q=\frac{z_e}{a_e+b_e z_e} \quad (2)$$

where a_f, b_f, a_e and b_e are constants.

Skin friction sometimes decreases from a peak value to a residual one in the cases of sensitive clays, stiff clays and dense sands. In order to simulate this behaviour at least the other two parameters are needed : the ratio of the residual value to the peak value and the displacement required to reach the residual state. If data for some local stratum show such behaviour which is too significant to neglect for load-settlement curves, these parameters have to be introduced. In such a case, however, it may be practical to employ a reduction factor for the peak value. For example, Semple and Rigden (1984) proposed a reduction factor termed a length factor, which is a function of pile length-to-diameter ratio, for driven pipe piles in clays.

Determination of b_f and b_e

The constants b_f and b_e can be determined from

$$b_f=1/f_{ult} \quad (3)$$

$$b_e=1/q_{ult} \quad (4)$$

where the subscript *ult* represents ultimate value. Therefore, the values of f_{ult} and q_{ult} based on conventional local experience are directly available. It should be noticed, however, that q_{ult} is physical ultimate end resistance as discussed in "INTRODUCTION" and Fig. 1. Therefore, various conventional definitions of ultimate end bearing capacity for bored piles do not correspond to q_{ult} in Eq. (4). This will be discussed in detail in "*Determination of a_e*".

Determination of a_f

The constant a_f is the reciprocal of the initial tangent as shown in Fig. 2, but the initial tangent is not easy to determine since it is very sensitive to soil disturbance, etc. In order to obtain overall agreement, the displacement required to mobilize half of f_{ult} is defined as the reference displacement for skin friction $z_{ref,f}$, and a_f is determined from $a_f=z_{ref,f}/f_{ult}$ as shown in Fig. 2.

Reese (1978) summarized the relationship between skin resistance in stiff clays and the displacement of the shaft, and concluded that the displacement at which maximum skin friction is mobilized, $z_{ful,f}$, ranges about 0.5%-2% of the shaft diameter D_s ; i. e., $z_{ful,f} \simeq (0.005\text{-}0.02)D_s$. It is also mentioned that the range of $z_{ful.f}$ in sands is similar to that in clays. According to the results, the reference displacement for skin resistance, $z_{ref.f}$, is about a quarter of $z_{ful.f}$, i. e., $z_{ref.f} \simeq (0.001\text{-}0.005)D_s$. Later data (e. g., Goeke and Hustad, 1979; Anderson et al., 1985; Sharma et al., 1986; Shibata et al., 1987) support the above-mentioned ranges of $z_{ful.f}$ and $z_{ref.f}$, averaging $z_{ful.f} \simeq 0.01\,D_s$ and $z_{ref.f} \simeq 0.0025\,D_s$. According to an elastic analysis, Whitaker (1976) also suggested that $z_{ful.f} \simeq 0.01\,D_s$. Judging from the above results, a_f may be determined from

$$z_{ref.f}=0.0025D_s \quad (D_s=\text{shaft diameter}) \tag{5}$$

$$a_f=0.0025\,D_s/f_{ult} \tag{6}$$

It should be noticed, however, that the value of $z_{ref.f}$ may range $(0.001\text{-}0.005)D_s$ depending on many conditions of the interface and the stress-strain behaviour of surrounding soils: soil characteristics, stresses of the ground surrounding the pile shaft, construction methods of bored piles, time from construction to a loading test, loading procedure and speed, etc. In the back analysis of the results of loading tests, the values of $z_{ref.f}$ may have to be changed from Eq. (5).

Determination of a_e

The displacement required to mobilize half of q_{ult} is defined as the reference displacement for end resistance $z_{ref.e}$, and a_e is determined from $a_e=z_{ref.e}/q_{ult}$ as shown in Fig. 2.

Few loading tests have been carried out up to the physical ultimate state of bored piles, which allows to evaluate the reference displacement for end resistance, $z_{ref.e}$. The results of loading tests for bored precast piles in sands by BCP Committee (1971) are summarized in Table 1.

Table 1. Summary of end resistance in loading tests carried out up to the ultimate state (after BCP Committee, 1971)

Test No.	Pile length[m]	Diameter: D_e[m]	q_{ult} [kPa]	$q_{0.05}$* [kPa]	$q_{0.05}/q_{ult}$	$z_{ref,e}$**
5C	11.0	0.20	27,000	4,000	0.15	$0.27D_e$
3C	11.0	0.20	25,000	5,100	0.20	$0.22D_e$
4B	4.8	0.20	6,700	1,400	0.21	$0.21D_e$
1B	4.0	0.20	5,500	700	0.13	$0.42D_e$
				Average	0.17	$0.28D_e$

* $q_{0.05}=q(\text{for } z_e/D_e=0.05)$
** Reference displacement for end resistance

Jamiolkowski et al. (1984) summarized the relationship between $q(\text{for } z_e/D_e=0.05)$; i. e., end pressure when tip displacement z_e is 5% of the base diameter D_e; and cone bearing capacity q_c near the pile tip for eleven bored piles having the tip embedded in sands. The results are as follows:

$$q(\text{for } z_e/D_e=0.05)/q_c=0.14-0.24 \quad (\text{average}=0.17) \tag{7}$$

Reese (1978) suggested the relationship expressed by Eq. (8-a) between $q(\text{for } z_e/D_e=0.05)$ and SPT N-value for bored piles in sands based on seven data, while Meyerhof (1956) proposed Eq. (8-b) for piles in saturated very fine or silty sands (*ton* unit used in the paper is short ton, private correspondence and Meyerhof, 1976):

$$q(\text{for } z_e/D_e=0.05)=4\,N/6\ [\text{short tf/ft}^2] =64\,N\ [\text{kPa}]\ (N\leq 60) \tag{8-a}$$

$$q_{ult}=q_c=4\,N\ [\text{short tf/ft}^2] \tag{8-b}$$

All the above-mentioned results; i. e., Table 1, Eq. (7), and Eq. (8); are expressed by the following relationship:

$$q(\text{for } z_e/D_e=0.05)=q_{ult}/6=0.17\,q_{ult} \tag{9}$$

From Eqs. (2) and (9), we obtain the following value of the reference displacement for end resistance $z_{ref.e}$, where $q=0.5\,q_{ult}$ from its definition:

$$z_{ref.e}=0.25\,D_e \quad (D_e=\text{base diameter}) \tag{10}$$

Thus so-called ultimate end bearing capacity determined from relative base settlement $z_e/D_e=0.10-0.25$ is estimated to be $0.29\,q_{ult}-0.50\,q_{ult}$ from Eqs. (2) and (10), and the range of values appropriately explains $q(\text{for } z_e/D_e=0.10-0.25)=(1/3-1/2)q_c$ (Jamiolkowski et al., 1984; De Beer, 1988). Therefore, the value of a_e for a sandy layer may be determined from Eq. (10) as follows:

$$a_e=0.25\,D_e/q_{ult} \tag{11}$$

However, the values of $z_{ref.e}$ range $0.31\,D_e$-$0.16\,D_e$ according to Eqs. (2) and (7).

Table 2. A recommendation for estimating f_{ult} and q_{ult} of bored piles

Ultimate skin resistance : f_{ult} [kPa]	sand	$5N$ (≤ 200 [kPa])
	clay	c_u or $10N$ (≤ 150 [kPa])
Ultimate end resistance* : q_{ult} [kPa]	gravel	$\bar{q}_c$ or $600\bar{N}$
	sand	$\bar{q}_c$ or $400\bar{N}$
	silt	$\bar{q}_c$ or $250\bar{N}$
	clay	$9\bar{c}_u$, $\bar{q}_c$, (or $100\bar{N}$)

* The values of q_c, N, or c_u within $1D_e$ above and $1D_e$ below from the pile base (D_e=base diameter) are averaged as far as the load-settlement curve of $z/D_e < 0.10$ is concerned.

The value of $z_{ref,e}$ may have to be modified around this range in the back analysis.

The values of $z_{ref,e}$ for stiff clays seem to be much smaller than those of Eq. (10) judging from data summarized by Reese (1978). Thus Eq. (11) is a safe assumption for preliminary analysis of bored piles whose bases are embedded in cohesive soils.

Estimation of f_{ult} and q_{ult}

The constants b_f, b_e, a_f, and a_e are determined from Eqs. (3), (4), (6), and (11), respectively. Thus actual data required are f_{ult} and q_{ult}. These values may be estimated from local experience in the conventional design based on ultimate bearing capacity.

As to f_{ult}, recommendations shown in Table 2 (J. R. A., 1980) based on the results of field measurements in Japan are useful for preliminary analyses of bored cast-in-place piles in Japan.

The value of q_{ult} may be estimated from CPT-q_c, SPT-N, and/or c_u (or $q_u/2$) taking account of the scale effect, i. e. the difference in size of the influence zone between in-situ tests and actual piles. In the ultimate state, the region of $4D_e$ above and $6D_e$ below the pile tip may be the influence zone for end bearing capacity (Hirayama, 1988). We practically need, however, the load-settlement curve for a bored pile up to the settlement at the pile head being at most 10% of the pile diameter. Consequently, the influence zone is smaller than that of the ultimate state. For calculating the settlement of a rigid circular footing, Schmertmann et al. (1978) proposed an influence factor which is a simplified function of depth between the base and $2D_e$ below. The majority of the settlement is, however, due to the strain in the region between the base and $1D_e$ below except when the soil between $1D_e$ and $2D_e$ is much weaker than the upper soil. A. I. J. (1988) recommeded $1D_e$ above and $1D_e$ below the pile tip as the influence zone for evaluating the bearing capacity of cast-in-place piles defined by the settlement of $0.1D_e$. Therefore, the region of $1D_e$ above and $1D_e$ below the pile tip is regarded as the influence zone. However, the range up to $2D_e$ below the tip has to be taken into account if there is rather a weaker stratum below the above-mentioned region. The values of q_c, N, or c_u within the region are averaged for estimating q_{ult}. Since the q_c/N ratio varies according to the grain size distribution or soil type, the values shown in Table 2 (Barrett et al., 1985) may be used for preliminary analyses.

ONE-DIMENSIONAL FINITE ELEMENT FORMULATION

A pile is discretized into one-dimensional finite elements taking into account surrounding strata or corresponding measuring points of a loading test as shown in Fig. 3. The pile is modeled by elastic bar elements, and the skin resistance and end resistance are represented by the non-linear springs acting at the midpoint of each element and the tip.

The tangential spring constant for skin resistance of the i th element k_{fi} is obtained from Eq. (1) as follows :

$$k_{fi}=C_iL_i\frac{df_i}{d\bar{z}_i}=\frac{C_iL_ia_{fi}}{(a_{fi}+b_{fi}\bar{z}_i)^2} \qquad (12)$$

where C_i and L_i are the circumferential length and the segment length of the i th element, and $\bar{z}_i=(z_i+z_{i+1})/2$. Hence the element tangential stiffness matrix of the i th element is given by

$$\boldsymbol{k}_i=\begin{bmatrix} P_i+2S_i & -P_i+S_i \\ -P_i+S_i & P_i+2S_i \end{bmatrix} \qquad (13)$$

where $P_i=A_iE_i/L_i$, $S_i=k_{fi}/6$; A_i and E_i are

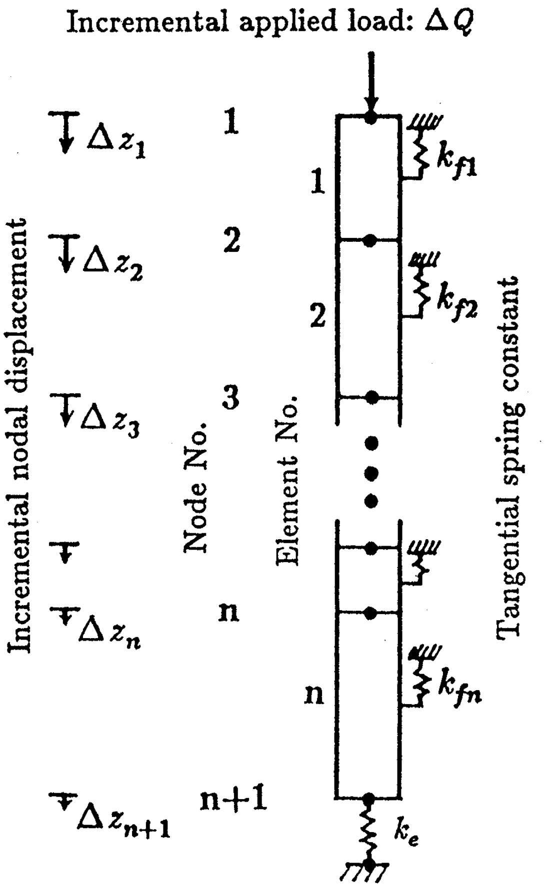


Fig. 3. One-dimensional non-linear finite-element model for a pile

the pile cross-sectional area and the pile elastic modulus of the ith element (Trochanis et al., 1987). If there are no data of E_i, about 2.6×10^7 [kPa] may be used for preliminary analyses.

The tangential spring constant for end resistance k_e is obtained from Eq. (2) as follows:

$$k_e=A_n\frac{dq}{dz_e}=\frac{A_n a_e}{(a_e+b_e z_{n+1})^2} \qquad (14)$$

where A_n=the cross-sectional area of the nth element (i. e. base area).

The global tangential stiffness matrix $\boldsymbol{K}$ is formulated from Eqs. (13) and (14), and the following incremental form of non-linear algebraic equations is obtained:

$$\boldsymbol{K}\Delta\boldsymbol{z}=\Delta\boldsymbol{Q} \qquad (15)$$

where $\Delta\boldsymbol{z}^T=(\Delta z_1, \Delta z_2, \cdots, \Delta z_{n+1})$,

$\Delta\boldsymbol{Q}^T=(\Delta Q, 0, \cdots, 0)$

(ΔQ=incremental applied load).

A computer program termed PILELS has been written. If ΔQ is small enough; i. e., less than 1/200 of the ultimate bearing capacity, displacements of the practically same magnitude (differences are less than 0.5%) can be calculated for a given problem. The computing time for the pile divided into 10–15 elements is about one minute by a personal computer. Since the parameters required can be determined from usual in-situ and/or laboratory tests and personal computers are enough for calculations, the application of the model to analysis and design is very practical.

ANALYSES OF LARGE-DIAMETER BORED PILES

Analysis Based on SPT Results

Loading tests for large bored piles (2–3 m in diameter and 40–70 m in length) for Bannosu Viaduct of Honshu-Shikoku Bridge, west Japan (Takahashi et al., 1981) were analyzed. Firstly, according to the results of site investigation shown in Fig. 4, f_{ult} and q_{ult} were estimated as shown in Table 3(a). The input constants required were calculated from Eqs. (3), (4), (6), and (11). Secondary, the results of the above-mentioned preliminary analysis were compared with the measured results, and input data were modified as shown in Table 3(b). The calculated results and measured results are shown in Figs. 5 and 6. The ratios of the end resistance to total applied load (i. e., 40 [MN]) are about 0% (T_1–pile), 35% (T_2–pile), and 0% (T_3–pile) according to both measurements and calculations. The results indicate the importance of skin resistance of long and large-diameter bored piles, as has been pointed out.

Analysis Based on CPT Results

Summarizing the results of six loading tests of large-diameter bored piles ($D_s=D_e=1.2-2.0$ m) and CPT tests (Fugro cone) in the Osaka area, Sibata et al. (1987) and Horikoshi (1987) suggested the following relationships between ultimate skin friction

Table 3. Input data for analyses of Bannosu site

	EL. from [m]	+3	−2	−10	−14	−17	−20	−28	−32	−38	−48	−53
	to [m]	−2	−10	−14	−17	−20	−28	−32	−38	−48	−53	−67
	Soil type	sand	sand	sand	sand	sand	clay	clay	sand	clay	clay	sand
	SPT-$\bar{N}$	5	10	20	30	40	20	8	30	20	50	50
(a) Preliminary analysis	f_{ult} [kPa]	25	50	100	150	200	150*	80	150	150*	150*	200*
	q_{ult} [kPa]								12,000			20,000
(b) Back analysis**	f_{ult} [kPa]	30	60	100	120	160	120	80	225	375	375	500
	q_{ult} [kPa]								18,000			20,000

* Reduced according to Table 2 ** Reference displacements for skin friction (i.e., $z_{ref,f}$) were changed from $0.0025D_s$ (cf. Eq. (5)) to $0.00125D_s$ for T_1-pile and T_3-pile, and to $0.005D_s$ for T_2-pile

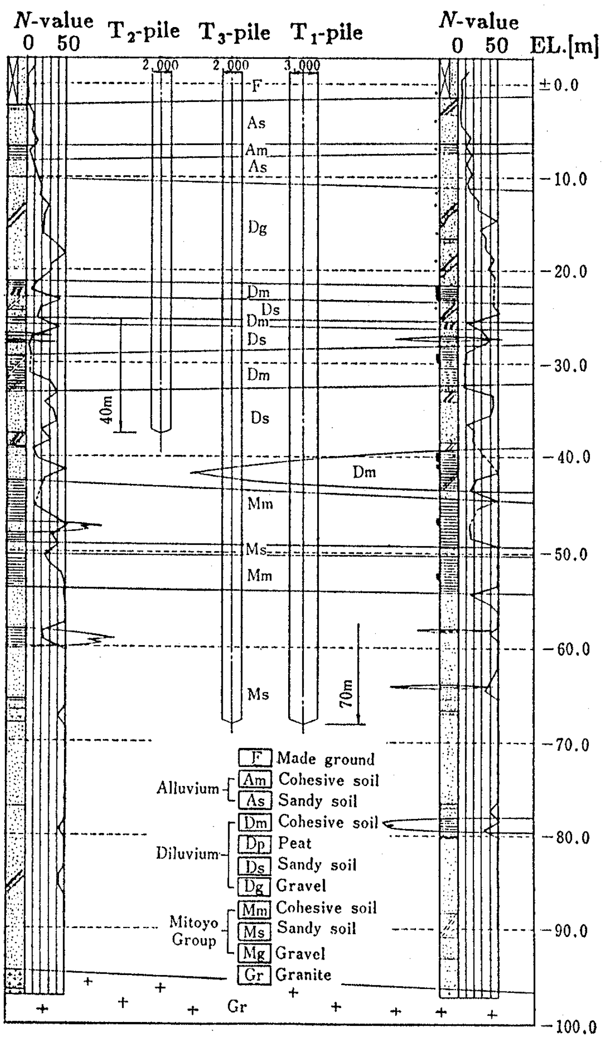


Fig. 4. Geological section of Bannosu site (from Takahashi et al., 1981)

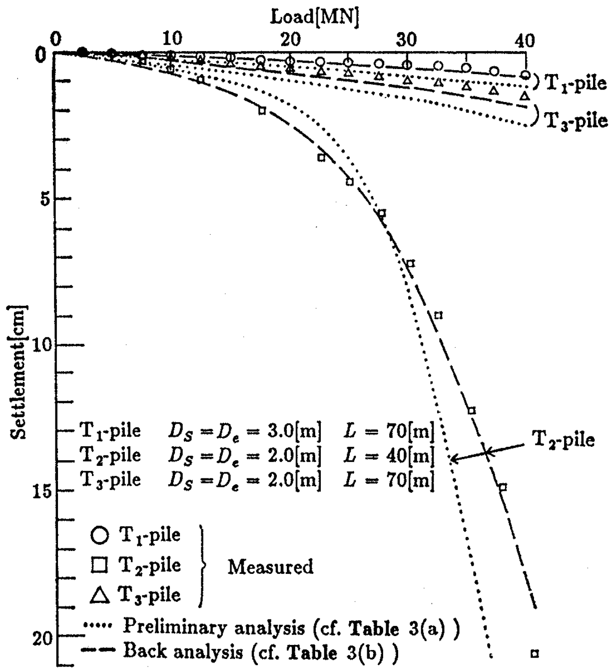


Fig. 5. Comparison of load-settlement curves at pile heads

f_{ult} and sleeve friction of CPT f_c :

$$f_{ult}=f_c \quad : \text{for sand} \qquad (16\text{-a})$$

$$f_{ult}=2f_c \quad : \text{for clay} \qquad (16\text{-b})$$

A loading test at North Osaka Bay (D_s= 1.5 m, D_f=44 m) was analyzed by using Eq. (16). The analytical and experimental results are shown in Figs. 7 and 8. Because the input data based on local experience were used, the results of the analysis are thought to be more reliable than those based on general data.

Application to Design

There are a number of factors in the design of bored piles beyond computations, and then loading tests as follows are significant : loads are over ultimate skin resistance, and the distribution of axial loads as a function of depth is measured (Reese, 1978). Another proper way to check the quality of a bored pile is to compare the CPT-q_c

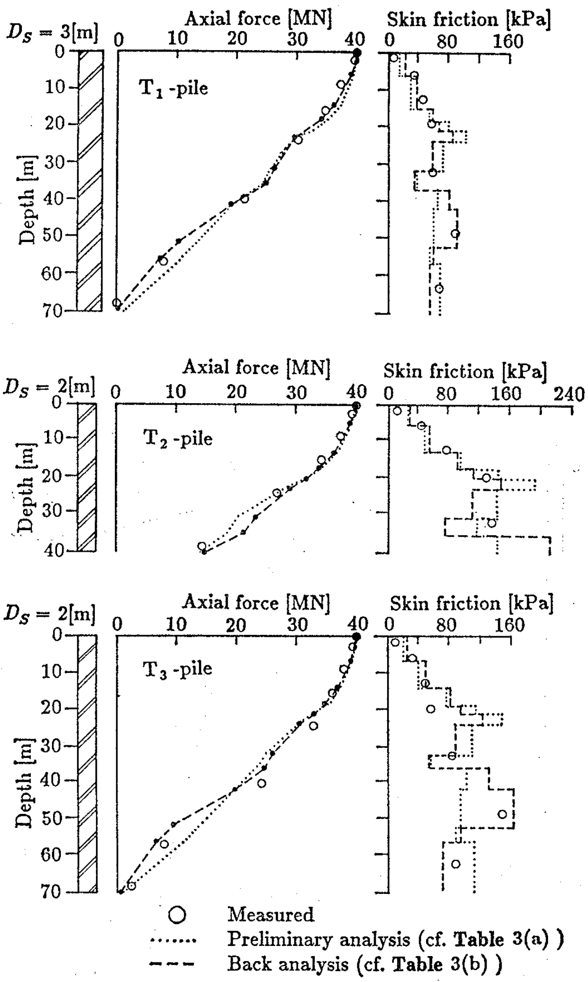


Fig. 6. Comparison of axial forces and skin frictions along piles

diagrams in the immediate vicinity of a bored pile with those obtained before the installation of the pile (De Beer, 1988).

In general, the model proposed may be useful for the following situations (Reese, 1978):

1) For sizing of test shafts. The preliminary analysis will allow the appropriate selection of shaft diameters and penetrations for a loading-test program.

2) For adjusting the size of the shafts after the results from test loading are available. The input data modified by back analysis will allow the reliable estimation of load-settlement behaviour for piles whose sizes are different from those of test piles,

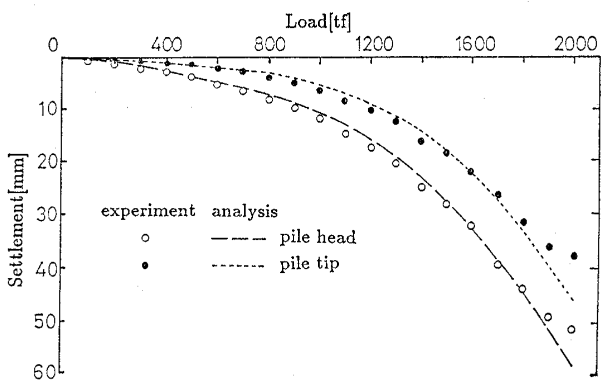


Fig. 7. Comparison of load-settlement curves (1tf=9.8kN)

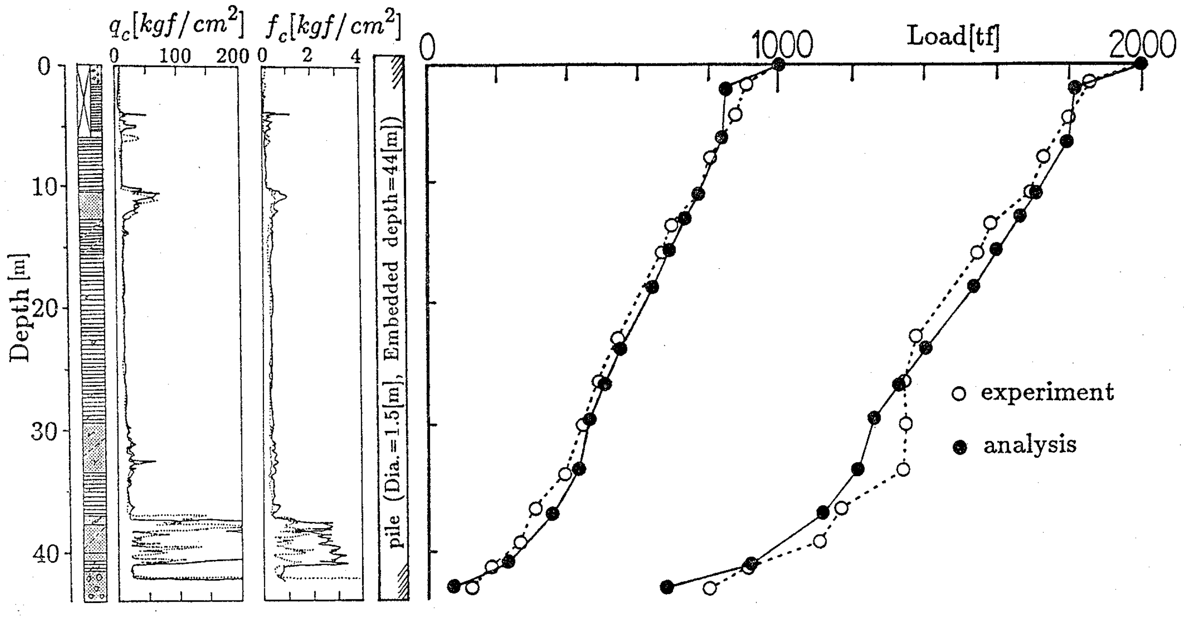


Fig. 8. Comparison of axial forces (1kgf/cm²=98kPa, 1tf=9.8kN)

and these results may be used for optimizing the design. For instance, large-diameter bored piles have often been used as underpinning for subway excavation in Japan. The allowable settlement is very severe, and settlement prediction is essential for the design in such cases. The model demonstrated the usefulness in such a case.

The better local data for estimating f_{ult}, q_{ult}, $z_{ref,f}$, and $z_{ref,e}$ we can have, the more reliable preliminary analyses are. Such examples are the studies on large-diameter bored piles in London clay (e. g. Whitaker, 1976) and in the Osaka area (Shibata et al., 1987), etc. These kinds of data may allow probabilistic determination of the input constants, so that the analytical results are discussed in terms of reliability or factors of safety.

CONCLUSIONS

Displacements needed to mobilize half of the ultimate skin resistance are as small as 1/100 of those of the ultimate end resistance as indicated by Eqs. (5) and (10) when $D_s = D_e$. In the cases of under-reamed piles, the difference could be larger. The mobilized end resistance in a sandy layer is only about 1/6 of the ultimate end resistance when the base settlement is 5% of the base diameter, while the skin resistance has been fully mobilized.

Thus, it is essential to take the settlement into consideration for the design of bored piles, especially long or large-diameter ones. In the method proposed, the parameters required can be determined from the results of usual in-situ and/or laboratory tests and personal computers are enough for calculations. The method is, therefore, practically useful for analysing the load-settlement behaviour of bored cast-in-place piles.

NOTATION

a_e, a_f, b_e, b_f = constants used in hyperbolic transfer functions (cf. Fig. 2)
D_e = base diameter
D_s = shaft diameter
f = skin friction (per unit area)
f_c = sleeve friction of cone penetration test
f_{ult} = ultimate skin friction
q = end resistance (per unit area)
q_c = cone bearing capacity
q_{ult} = ultimate end resistance (cf. Fig. 1)
q (for $z_e/D_e = 0.05$) = end resistance when z_e is 5% of D_e
z = pile shaft displacement at a certain point
z_e = tip displacement
$z_i (i = 1, 2, \cdots, n+1)$ = displacement of the i th nodal point in the one-dimensional finite element model
$z_{ful,f}$ = displacement at which maximum skin friction is mobilized
$z_{ref,e}$ = reference displacement for end resistance (i. e. tip displacement z_e required to mobilize half of q_{ult})
$z_{ref,f}$ = reference displacement for skin friction (i. e. shaft displacement z required to mobilize half of f_{ult})

REFERENCES

1) Architectural Institute of Japan (1988) : Recommendations for Design of Building Foundations (in Japanese).
2) Anderson, W. F., Young, K. Y. and Sulaiman, J. I. (1985) : "Shaft adhesion on bored and cast-in-situ piles," Proc. 11 th ICSMFE, Vol. 3, pp. 1333-1336.
3) Barrett, A. J., Wrench, B. P. and Legge, J. D. (1985) : "Back analysis of test piles driven into estuarine sands," Proc. 11 th ICSMFE Vol. 3, pp. 1347-1352.
4) BCP Committee (1971) : "Field tests on piles in sand," Soils and Foundations, Vol. 11, No. 2, pp. 29-49.
5) Clough, G. W. and Duncan, J. M. (1971) : "Finite element analyses of retaining wall behavior," Proc. ASCE, Vol. 97, No. SM 12, pp. 1657-1673.
6) De Beer, E. (1988) : "Different behaviour of bored and driven piles," Deep Foundations on Bored and Auger Piles, Van Impe (ed.), Balkema, Rotterdam, pp. 47-82.
7) Focht, J. A. Jr. and O'Neill, M. W. (1985) : "Piles and other deep foundations," Theme Lecture 4, Proc. 11 th ICSMFE, Vol. 1, pp. 187-209.
8) Goeke, P. M. and Hustad, P. A. (1979) : "Instru-

mented drilled shafts in clay-shale," Proc. Symp. on Deep Foundations, ASCE, pp. 149-165.

9) Hirayama, H. (1988) : "A unified base bearing capacity formula for piles," Soils and Foundations, Vol. 28, No. 3, pp. 91-102.

10) Horikoshi, K. (1987) : "Study on skin friction resistance of piles in multi-layered ground," M. Eng. Thesis, Kyoto Univ. (in Japanese).

11) Jamiolkowski, M., Lancellotta, R. and Pasqualini, E. (1984) : Discussion on "Scale effects of ultimate pile capacity," Proc. ASCE, Vol. 110, No. GT 8, pp. 1156-1159.

12) Japan Road Association (1980) : Specifications and Comments for Highway Bridge ; IV Substructure (in Japanese).

13) Kerisel, J. (1964) : "Deep foundations basic experimental facts," Proc. North American Conf. on Deep Foundations, Mexico City, pp. 5-44.

14) Kishida, H. and Takano, A. (1977) : "Distribution of contact pressure under base of bored piles in sand, Part II," Trans. of AIJ, No. 261, pp. 25-40 (in Japanese).

15) Komada, K. (1975) : "A proposal on bearing capacity formula for piles," Tsuchi-to-Kiso, JSSMFE, Vol. 23, No. 7, pp. 57-61 (in Japanese).

16) Meyerhof, G. G. (1956) : "Penetration tests and bearing capacity of cohesionless soils," Proc. ASCE, Vol. 82, No. SM 1, pp. 866. 1-866. 19.

17) Meyerhof, G. G. (1976) : "Bearing capacity and settlement of pile foundations," 11 th Terzaghi Lecture, Proc. ASCE, Vol. 102, No. GT 3, pp. 197-228.

18) Poulos, H. G. and Davis, E. H. (1980) : Pile Foundation Analysis and Design, John Wiley and Sons, N. Y., pp. 71-108.

19) Reese, L. C. (1978) : "Design and construction of drilled shafts," 12 th Terzaghi Lecture, Proc. ASCE, Vol. 104, No. GT 1, pp. 95-116.

20) Sabini, G. and Sapio, G. (1981) : "Behaviour of small diameter bored piles under axial load," Proc. 10 th ICSMFE, Vol. 2, pp. 823-828.

21) Schmertmann, J. H., Hartman, J. P. and Brown, P. R. (1978) : "Improved strain influence factor diagrams," Proc. ASCE, Vol. 104, No. GT 8, pp. 1131-1135.

22) Semple, R. M. and Rigden, W. J. (1984) : "Shaft capacity of driven pipe piles in clay," Analysis and Design of Pile Foundations, J. R. Mayer (ed.), ASCE, pp. 59-79.

23) Sharma, H. D., Harris, M. C. and Scott, J. D. (1986) : "Bearing capacity of bored cast-in-place concrete pile on oil sand," Proc. ASCE, Vol. 112, No. GT 12, pp. 1101-1116.

24) Shibata, T., Yashima, A., Emi, S. and Horikoshi, K. (1987) : "The estimation of the shaft friction of bored piles," Proc. 32 nd SMFE Symp., JSSMFE, pp. 7-10 (in Japanese).

25) Takahashi, K., Miyajima, K., Kashima, S., Yamamoto, N., Yamagata, M. and Aizawa, R. (1981) : "Investigation of bearing capacity of foundation ground of Honshu-Shikoku Bridge," Proc. 9 th ICSMFE, Case History Vol., pp. 107-156.

26) Trochanis, A. M., Bielak, J. and Christiano, P. P. (1987) : "Hysteretic dissipation of piles under cyclic axial load," Proc. ASCE, Vol. 113, No. GT 4, pp. 335-350.

27) Vesic, A. S. (1977) : Design of Pile Foundations, National Cooperative Highway Research Program, Synthesis of Highway Practice 42, Transportation Research Board, National Research Council, Washington, D. C.

28) Whitaker, T. (1976) : The Design of Piled Foundations, 2 nd Ed., Pergamon Press, Oxford.