Int J Adv Manuf Technol (2009) 43:33–39
DOI 10.1007/s00170-008-1695-y

ORIGINAL ARTICLE

Three-dimensional stability lobe and maximum material removal rate in end milling of thin-walled plate

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Received: 9 May 2008 / Accepted: 1 August 2008 / Published online: 26 August 2008

Abstract Chatter phenomenon often occurs during end milling of thin-walled plate and becomes a common limitation to achieve high productivity and part quality. For the purpose of chatter avoidance, the optimal selection of the axial and radial depth of cut, which are decisive primary parameters in the maximum material removal rate, is required. This paper studies the machining stability in milling of the thin-walled plate and develops a three-dimensional lobe diagram of the spindle speed, axial, and radial depth of cut. Through the three-dimensional lobe, it is possible to choose the appropriate cutting parameters according to the dynamic behavior of the chatter system. Moreover, this paper studies the maximum material removal rate at the condition of optimal pairs of the axial and radial depth of cutting.

Keywords Chatter · Three-dimensional stability lobe · Maximum material removal rate · Thin-walled plate

1 Introduction

With the development of high-speed cutting technology, many aircraft parts are composed of monolithic components to form ribs and thin-walled plates. Due to the large area and low rigidity, the thin-walled plates are always machined in numerical-control end-milling processes. At the high removal rate conditions, the milling of thin-walled plate leads a lot of dynamic and static problems consecutively. The main dynamic problem is the self-excited vibration called regenerative chatter. Due to the regenerative chatter, the cutting thickness is changed with time, which leads to the dynamic cutting force. Therefore, chatter is one of the major limitations on high productivity and part quality even for high-speed and high-precision milling machines.

In workshops, machine tool operators often select conservative cutting parameters to avoid chatter. Moreover, in some cases, additional manual operations are required to clean chatter marks left on the part surface. The studies on chatter can go back to the 1950s with Tobias [1], Tlusty and Polacek [2], and Merrit [3], who explained the regenerative chatter in orthogonal cutting and developed the two-dimensional stability lobe theory. The two-dimensional stability lobe theory deals with the stability of solutions for dynamical cutting systems, which usually stands for the spindle speed and axial depth. As a function of these two cutting parameters, the border between a stable cut (i.e., chatter-free) and an unstable one (i.e., with chatter) can be visualized in a chart called stability lobes diagram (SLD). In the middle of the 1990s, Altintas [4] presented an analytical form of the stability lobe theory for milling. Both of these stability lobe theories can help to select the appropriate cutting parameters of the spindle speed and axial depth to avoid chatter in machining processes.

In the end milling of thin-walled plates, due to the change of cutting position, mass and stiffness of cutting system, and other factors, the two-dimensional stability lobe diagram sometimes is not comprehensive to describe the stability of chatter system. Therefore, several researches on the three-dimensional stability lobe diagram of chatter system have then been done and achieved some developments, such as that of Vincent Thevenot et al. who introduced the dynamical behavior variation of the part with respect to the tool position in order to determine optimal cutting conditions during the machining process.

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Accordingly, they constructed a three-dimensional stability lobe diagram of the spindle speed, axial depth, and tool position with a constant radial depth [5]. U. Bravo and S. Herranz considered the dynamic parameters variation of the workpiece along the milling process of the thin-walled plate as both mass and rigidity were reduced. Sequentially, they developed a three-dimensional lobe diagram that based on the relative movement of the chatter systems to cover all the intermediate stages of machining the walls [6, 7]. Tony L. Schmitz considered the change of system dynamics with the tool overhang length and developed the three-dimensional lobe diagram of the spindle speed, axial depth, and tool overhang length [8].

Material removal rate (MRR) is a critical parameter for describing the machining process productivity at the condition of ensuring the quality. MRR in end milling is proportional to the spindle speed, the axial and radial depth of cut, the feed per revolution per tooth, and the number of cutting teeth. Therefore, in order to improve the machining efficiency, this paper studies the three-dimensional stability in milling of the thin-walled plate and develops a three-dimensional lobe diagram of the spindle speed, and axial and radial depth of cut. According to the developed three-dimensional lobe diagram, one can select the appropriate cutting parameters to avoid chatter at the condition of ensuring the maximum MRR. The main flow chart of this research is shown in Fig. 1.

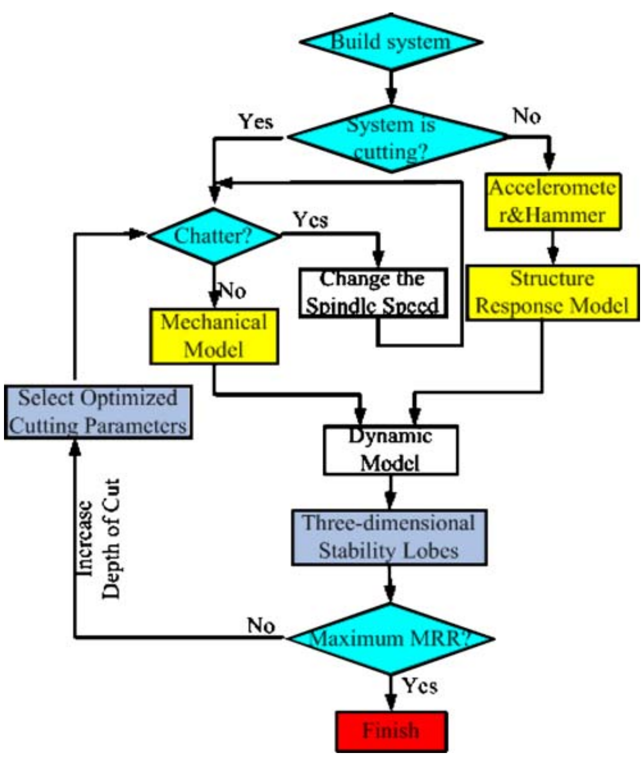
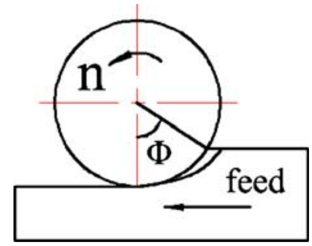


Fig. 1 The main flow chart of this research

Fig. 2 Model of up milling



2 Mathematic models of three-dimensional stability lobe

This study is mainly based on the work of Altintas and Budak [4], and pays more attention on the end milling of thin-walled plates. Predicting the dynamic behavior for the chatter system requires a cutting force model, a structural response model, and a solution procedure.

For simplification, it applies the following assumptions for the mathematic models of three-dimensional stability of thin-walled plate:

1. The workpiece is a thin, isotropic, and cantilever plate.
2. The tool is much more rigid than the workpiece. Therefore, the workpiece can be considered as a flexible body and can then be modeled as a single significant mode of vibration
3. The variation of dynamic parameters due to the material removal is neglected.
4. The end-milling process is up milling, as shown in Fig. 2.
5. The deflection at different locations of the workpiece is neglected.

2.1 Mechanical model

According to the assumption 1, the workpiece is modeled as a thin, isotropic plate, as shown in Fig. 3. It is assumed to be clamped at one end, free along the two parallel sides, and is subjected to the cutting force of the cutter. It is also assumed that the interactions between the cutter and the workpiece could be approximately represented by a cutting region with stiffness k and damping coefficient c as shown in Fig. 4. The cutter is represented by a rigid impactor. When the cutter impinges upon the cutting region, the workpiece experiences additional elastic and damping forces proportional to the cutting volume.

1. The axial stability limit and spindle speed

In order to analyze easily, the dynamic model in Fig. 4 can be simplified as follows:

1. The chatter system is a linear one.

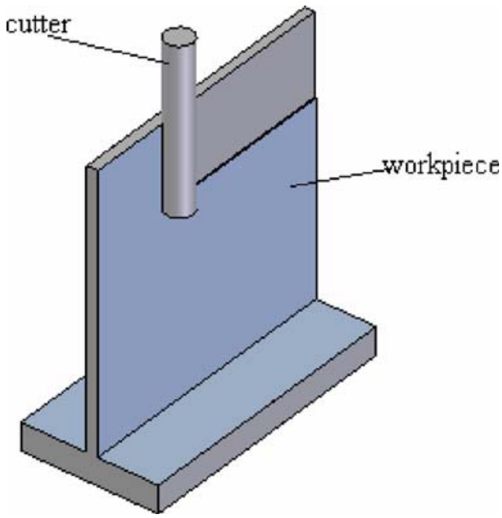


Fig. 3 Workpiece in end milling

2. The direction of dynamic cutting force is consistent with that of the steady cutting force, and only the effect of cutting thickness is considered.

The initial dynamic equation can be established according to the dynamic model in Fig. 4.

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) \tag{1}$$

where $y(t)$ is the chatter displacement (mm), $F(t)$ is the dynamic cutting force (N), m is the modal mass of the chatter system (N·s^2/mm), c is the modal damp of the chatter system (N·s/mm), and k is the modal stiffness of the chatter system (N·s/mm).

Equation 1 is treated with Laplace transform. The transfer function matrix at the cutter–workpiece contact zone is defined as follows:

$$\Phi_{yy}(s) = \frac{y(s)}{F(s)} = \frac{1}{k} \cdot \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\xi s}{\omega_n} + 1} \tag{2}$$

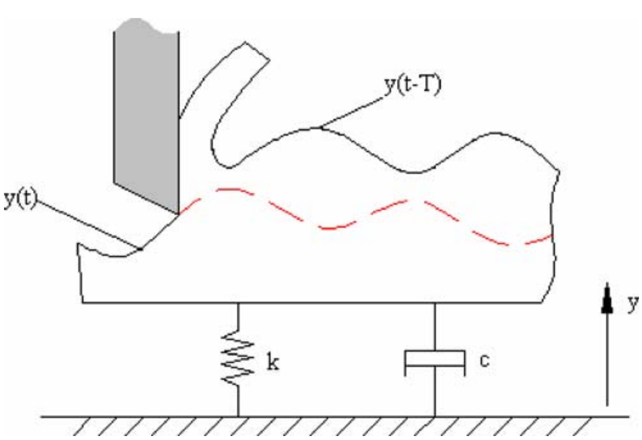


Fig. 4 Regenerative chatter model

where $\omega_n = \sqrt{\frac{k}{m}}$ is the natural pulsation of the chatter system.

According to the automatic control theory, the time domain characteristics of the chatter system depend on the root of the characteristic equation of the transfer function. When the real part of the root is zero, the system is the critical state, and the root of characteristic equation can be expressed as follows:

$$s = i\omega \tag{3}$$

By substituting Eq. 3 with Eq. 2, the transfer function is expressed as follows:

$$\Phi_{yy}(i\omega) = \frac{1}{k} \cdot \frac{1 - r^2 - i2\xi r}{(1 - r^2)^2 + (2\xi r)^2} \tag{4}$$

where $r = \frac{\omega}{\omega_n}$, ω is the chatter pulsation, ξ is the damping ratio, and $\xi = -\frac{c}{2m\omega_n}$.

On the basis of milling equations of Altinats and Budak, the transfer function of chatter system is expressed as follows.

$$\Phi_{yy}(i\omega) = \frac{4\pi}{Za_pK_t\alpha_{yy}[1 - e^{-i\omega T}]} \tag{5}$$

where Z is the number of teeth, K_t is the tangential milling force coefficient (N/mm^2), a_p is the axial depth of cutting (mm), and α_{yy} is the directional dynamic milling coefficient in the y direction and can be defined as follows:

$$\alpha_{yy} = \frac{1}{2}[-\cos 2\phi - 2K_r\phi - K_r \sin 2\phi]_{\phi_{st}}^{\phi_{ex}} \tag{6}$$

where K_r is the radial milling force coefficient and ϕ_{st}, ϕ_{ex} is the start and exit immersion angles of the cutter to and from the cutting zone, respectively. In the case of up milling, the start immersion angle is zero, and the exit immersion angle is ϕ_{ex}. Then, the directional dynamic milling coefficient can be expressed as follows:

$$\alpha_{yy} = \frac{1}{2}(1 - \cos 2\phi_{ex} - 2K_r\phi_{ex} - K_r \sin 2\phi_{ex}) \tag{7}$$

Using the Eqs. 4 and 5, and the Euler's formula, the axial stability limit is obtained as follows:

$$a_{p\,\lim} = \frac{2\pi k}{ZK_t\alpha_{yy}} \cdot \frac{(1 - r^2)^2 + (2\xi r)^2}{1 - r^2} \tag{8}$$

Here, the spindle speed n_s corresponding to the axial stability limit can be obtained with the following expression:

$$n_s = \frac{60\omega}{Z\left(2m\pi + \pi - 2\arctan\frac{2\xi r}{r^2 - 1}\right)} \tag{9}$$

where m is the whole number of full vibration cycles between passages of two teeth.

2. Radial depth of cut

The start immersion angle of the cutter to the cutting zone is zero, and the exit immersion angle of the cutter from the cutting zone is ϕ_{ex}. According to Fig. 2, the relationship between the radial depth of cut and exit angle is defined as follows:

$$b/R = 1 - \cos\phi_{ex} \tag{10}$$

where b is the radial depth of cutting (mm), and R is the tool radius (mm).

2.2 Structure response model

Before stability and accuracy predictions can be made, the dynamic parameters (natural frequency, damping ratio, and stiffness) of the workpiece for each natural mode must be developed. The most common method for testing the dynamic parameters is to impact the structure with an instrumented hammer and measure the system response with an accelerometer. The basic flow chart applied to test the dynamic parameters is shown in Fig. 5.

Firstly, one needs to exert the exciting force on several points of the thin-walled plate and to measure the corresponding acceleration. Then, the transfer function between the impacted points and the measured points can be obtained by the signal analysis equipment. Lastly, dynamic parameters of chatter system are derived by curve fitting.

The size of the thin-walled plate used in this paper is 100×100×10 mm. The material of the part is aluminum alloy with Young's modulus of 70 Pa and Poisson's ratio of 0.33. Through the developed method, the dynamic parameters of chatter system can be obtained. The natural frequency is 502.5 rad/s, the damping ratio is 0.038, and the stiffness is 4,500 N/mm.

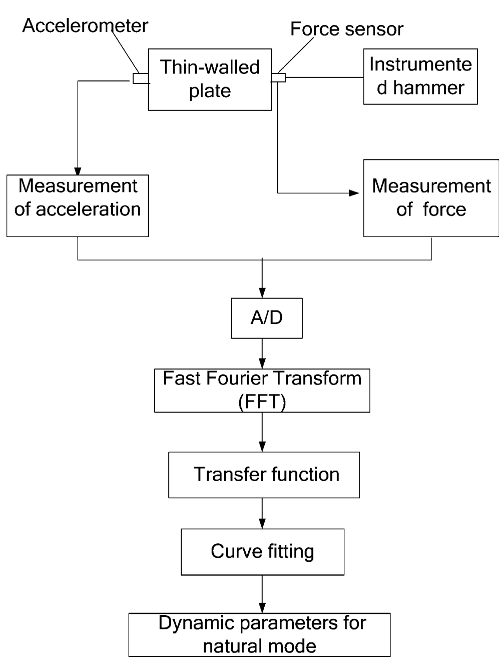


Fig. 5 Flow chart of testing the dynamic parameters

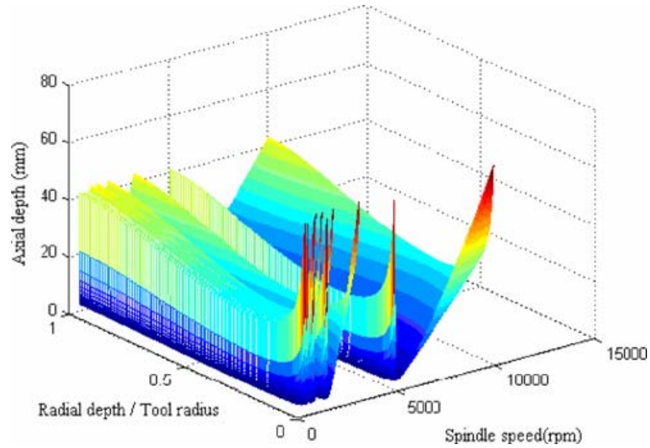


Fig. 6 Three-dimensional stability lobe

2.3 Three-dimensional stability lobe

Through the cutting experiments, the tangential milling force coefficient is 2,400 N/mm^2, and the radial milling force coefficient is 0.9 N/mm^2. The cutter is a two-tooth end-milling cutter. By using Matlab7.0 software, the stability lobe in three-dimensional of the spindle speed, axial, and radial depth of cut is represented in Fig. 6.

According to Fig. 6, when the radial depth is half of the tool radius, the two-dimensional stability lobe diagram of the axial depth and spindle speed is shown as Fig. 7. The curve in Fig. 7 is the boundary between chatter-free machining operations and unstable process. According to

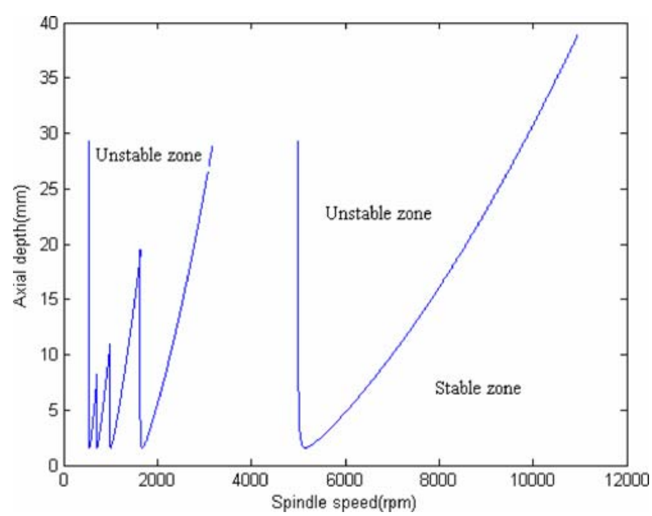


Fig. 7 Two-dimensional stability lobe

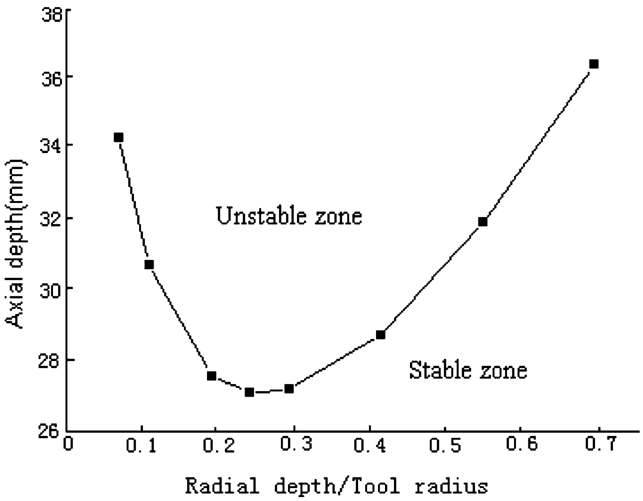


Fig. 8 Relationship between the axial and radial depth

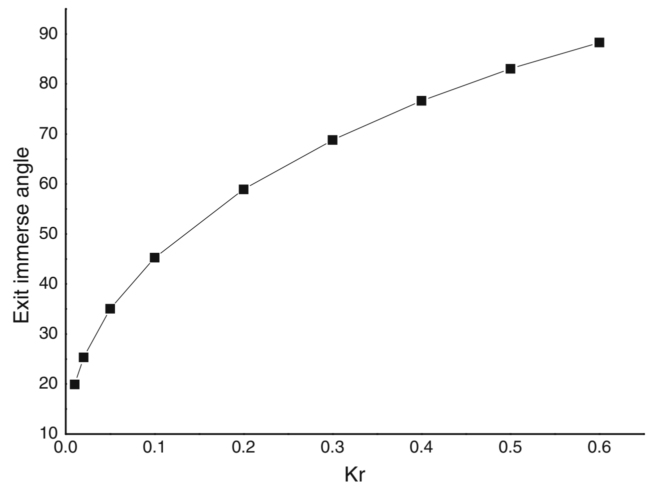


Fig. 10 Relationship between the ϕ_{ex} and K_r

Fig. 7, one can select chatter-free combinations of machining parameters.

According to Fig. 6, when the spindle speed is 10,000 rpm, the relationship between the axial and radial depth under the condition of chatter-free is shown as Fig. 8. The upper side of the curve in Fig. 8 is unstable zone, and the down side of that is stable zone. Figure 8 illustrates that the axial depth at the beginning gradually decreases with the increment of the radial depth. When the radial depth increases to a certain extent, the axial depth begins to increase with the increment of the radial depth. When the spindle speed is fixed, one can select the appropriate parameters to avoid chatter according to the curve in Fig. 8.

3 Material removal rate

Maximizing productivity is a primary goal in the machine shop environment. For milling processes, this frequently translates into maximizing the MRR. Possible mechanisms for increasing MRR include increasing the spindle speed, cutting depths, and feed rate. There are practical limits on the spindle speed and feed rate at which a milling machine can be operated. Therefore, the increasing MRR can be achieved through increasing the cutting depth, the axial, and radial depth of cut.

The mathematic equation for MRR can be expressed as follows:

$$\mathrm{MRR} = a_p \cdot b \cdot n_s \cdot Z \cdot f_t \tag{11}$$

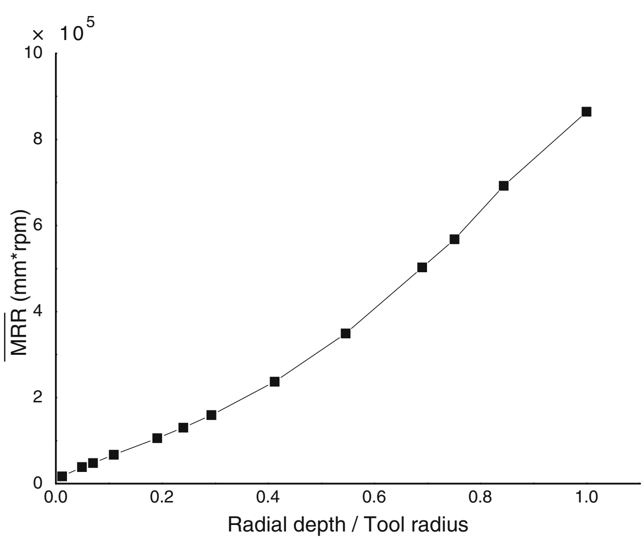


Fig. 9 Relationship between the radial depth and $\overline{\mathrm{MRR}}$

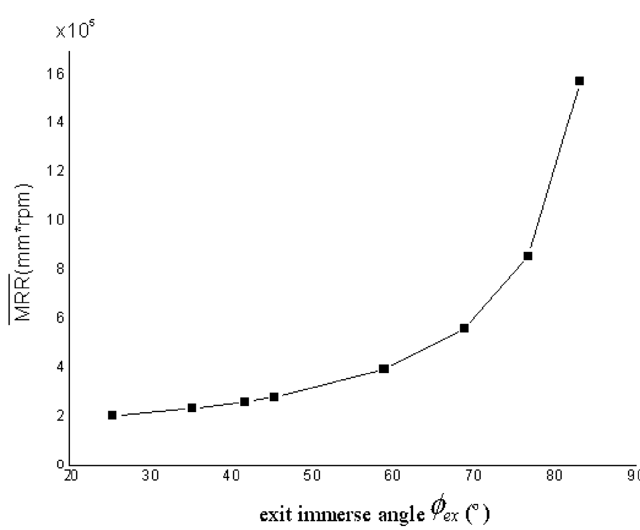


Fig. 11 Relationship between $\overline{\mathrm{MRR}}$ and ϕ_{ex}

where f_t is the feed per revolution per tooth. In general, the effect of f_t on stability is small and can be neglected [9]. Hence, MRR is proportional to the multiplication of the axial and radial depth of cut, and the spindle speed. It is interesting to find out at which combination of the axial and radial depth of cut the spindle speed is fixed, where the maximum value of MRR may be achieved. Therefore, a normalized value of MRR is used hereafter.

$$\overline{\mathrm{MRR}} = \frac{\mathrm{MRR}}{R \cdot f_t} = Z \cdot a_p \cdot \frac{b}{R} \cdot n_s \tag{12}$$

If the spindle speed is fixed at 10,000 rpm, at the condition of chatter-free cutting for the milling of thin-walled plate with two-tooth end milling, the relationship between the radial depth and $\overline{\mathrm{MRR}}$ is shown in Fig. 9. Figure 9 illustrates that $\overline{\mathrm{MRR}}$ is proportional to the radial depth, and $\overline{\mathrm{MRR}}$ is increased with the higher radial depth.

4 Maximum material removal rate of chatter-free

To obtain the maximum MRR, the radial depth should be maximal. The acceptable (chatter-free) maximum radial depth of cut depends on the geometry of the tool, the direction of cutting, and the mode of milling (up or down milling).

By substituting Eqs. 7, 8, and 10 with Eq. 12, the $\overline{\mathrm{MRR}}$ can be expressed as follows:

$$\overline{\mathrm{MRR}} = \frac{4\pi k n_s}{K_t} \cdot \frac{\left(1 - r^2\right)^2 + (2\xi r)^2}{1 - r^2} \cdot \frac{1 - \cos\phi_{ex}}{\left(1 - \cos 2\phi_{ex} - 2K_r\phi_{ex} - K_r \sin 2\phi_{ex}\right)} \tag{13}$$

If the geometry and material of the tool, the material of the workpiece, and the mode of the milling are made sure, the milling force coefficients K_t and K_r will be certain. According to the assumption (3) that the variation of dynamic parameters due to the material removal is neglected, the modal parameters k, ξ, and r are invariant. In order to obtain the maximum MRR of the chatter-free, Eq. 13 is the computed derivatives of the exit immerse angle ϕ_{ex}; the differential equation can be obtained as follows.

$$\frac{d\left(\overline{\mathrm{MRR}}\right)}{d\phi_{ex}} = \frac{4\pi k n_s}{K_t} \cdot \frac{\left(1 - r^2\right)^2 + (2\xi r)^2}{1 - r^2} \cdot \frac{\sin\phi_{ex}\left(1 - \cos 2\phi_{ex} - 2K_r\phi_{ex} - K_r \sin 2\phi_{ex}\right) - \left(1 - \cos\phi_{ex}\right)\left(2\sin 2\phi_{ex} - 2K_r - 2K_r\cos 2\phi_{ex}\right)}{\left(1 - \cos 2\phi_{ex} - 2K_r\phi_{ex} - K_r \sin 2\phi_{ex}\right)^2} \tag{14}$$

If $\frac{d\left(\overline{\mathrm{MRR}}\right)}{d\phi_{ex}} = 0$, the exit immerse angle ϕ_{ex} can be obtained. From Eq. 14, it is known that the exit immerse angle is only related with the radial milling force coefficient K_r. Through Matlab7.0, the exit immerse angle can be obtained if K_r is made sure. That is to say, for a certain cutting condition, one can select the appropriate ϕ_{ex} to obtain the maximum MRR of the chatter-free. The relationship between the exit immerse angle and the radial milling force coefficient K_r is shown as Fig. 10. Figure 10 illustrates that the exit immerse angle is increased with the higher radial milling force coefficient. Consequently, according to the three-dimensional stability lobe, one can obtain that the relationship between the maximum $\overline{\mathrm{MRR}}$ of the chatter-free and the exit immerse angle ϕ_{ex}, which is shown as Fig. 11. Figure 11 illustrates that the maximum MRR of the chatter-free is increased with the larger exit immerse angle ϕ_{ex}.

5 Conclusions

Chatter has always been a significant problem in high-speed machining, which is one of the major limitation on productivity and part quality. In order to avoid chatter, one needs to select the appropriate cutting parameters through the stability lobe. In this paper, a three-dimensional stability lobe theory has been presented. A three-dimensional, radial depth of cut has been introduced in the stability lobe diagram. This method is based on the identification of the optimal pairs of axial and radial depth of cut. Thus, the optimal cutting conditions for increased chatter-free MRR can be obtained through the three-dimensional stability lobe.

For the purpose of chatter avoidance, the primary parameters are the axial and radial depth of the cut, which are decisive in the productivity. Therefore, at the condition of stability cutting, one needs to select the optimal pairs of axial and radial depth to obtain the maximum MRR. If the geometry and material of the tool, the material of the workpiece, and the mode of milling are made sure, the axial and radial depth are both related with the exit immerse angle. Therefore, in order to obtain the maximum MRR, one needs to know the appropriate exit immerse angle. This paper studies the relationship between the maximum MRR and exit immerse angle, and finds that the maximum MRR of chatter-free is increased with the larger exit immerse angle.

Acknowledgement The authors are grateful to the National High Technology Research and Development Program of China for supporting this work under grant no. 2008AA042405, the National Natural Science Foundation of China for supporting this work under grant no. 50675122, and National Science Foundation of Shandong of China for supporting this work under grant no. Z2007F03.

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